

Multi-criteria Optimization of Electromechanical Modules:

Part 2 - RAZOR method

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Abstract — The paper presents the second part a study conducted on multi-criteria optimization of electromechanical modules. This part focuses on the use of RAZOR method, which is a multi-criteria optimization outranking method, based on the so-called Z-score in statistics. The conducted study is motivated by understanding the fact that the task for designing such modules, as well as the task for selecting an existing module for a given application are multivariate. This leads to the need for implementing optimization based decision-making on specified criteria. The used method offers a new approach to solving multi-criteria optimization tasks and provides another tool to facilitate the decision-making process.

Keywords — optimization, multi-criteria optimization, RAZOR, electromechanical modules, z-score, z-standardization

I. INTRODUCTION

Electromechanical modules (EMM) are widely used in many different industrial applications as a basic part of machine/system drive [1]. Examples for usage of EMM include aircraft industry (all the flaps on the wings of planes are equipped with their own motor and gear reducer), hoisting equipment, automotive industry, robotics, textile machinery (for example a combination between AC squirrel cage motor and worm gear reducer), conveyer systems helical bevel geared motors (in belt conveyors), water treatment facilities, even automated garage doors, etc..

As the name indicates, an EMM represents the unification of the electrical and mechanical part of a drive. In most of the cases this is a combination between an electric motor and a gear reducer, also called geared-motor. Many companies manufacture geared motors that usually use adapters as connection elements. Where needed, a clutch may also be used. Figure 1 shows a general view of the structure of an EMM.

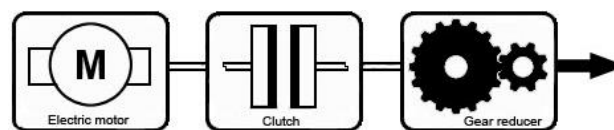


Fig. 1. Structural scheme of EMM

Because of the various possible applications of these modules, as well as the many different types of electric motors and gear reducers, a wide variety of EMM systems are available. Many different types of motors and gear reducers have been developed, some of which have been closely associated with a particular application [2]. In theory, every available type of electric motor and gear reducer can be used for the realization of different variants of EMM. In practical application, however, there are number of requirements that need to be addressed such as [3], [4]:

- unified mounting dimensions;
- geometrical and constructive limitations;
- adherence of the kinematic condition for two consecutively connected transmissions, for instance $T_{out(n)} = T_{in(n+1)}$, where $T_{out(n)}$ is the torque of the output shaft of the prior gear reducer and $T_{in(n+1)}$ is the torque on the input shaft of the next gear reducer;
- combinations of excessively large electric motor with a small gear unit that will overload the gear reducer;
- combination of an excessively small motor with a large gear reducer that will not utilize the full capability of the gear reducer, etc.

Although all these requirements limit the geared-motor combinations, there are still many possible variants. Depending on the type of components used (electric motor, gear reducer and clutch (if needed)) and the values for the input data, i.e. rotational speeds and torque, a significant number of modules to can be realized. When faced with such a

large number of alternatives, the task for designing new EMM and/or the task for selecting a suitable drive for any given application becomes very difficult. This justifies the need for conducting optimization.

In this paper, the application principle of using RAZOR method and the standard score approach for conducting multi-criteria optimization of existing electromechanical modules, part of the manufacturing program of the German company KEB Antriebstechnik GmbH is presented.

The paper first provides the description of the problem in Section II. The approaches used to solve the problem are discussed in Section III. Then Section IV discusses the main part of the article, where the solution method is demonstrated using a numerical example. Finally, concluding remarks are given in Section V.

II. DESCRIPTION OF THE PROBLEM

An EMM consists of electric motor, gear reducer and/or a clutch. As part of this study, a database has been developed, which gives information about different types of electric motors, gear reducers and the possible combinations between them. It consists of three interconnected tables, i.e. tables for (1) electric motors, (2) gear reducers and (3) EMM. Using this interconnected tables, the decision supporting information can be extracted by the means of a query and the search results are presented in a user-friendly search form. The possible EMM combinations, entered in the database, correspond to particular values for the input data. There is a large number of variants of EMM, so in order for the user to be able to select a suitable electromechanical drive for a given application, optimization of the selected alternatives, based on predefined criteria, need be conducted.

Optimization can be defined as a process or methodology for finding compromised solution to a given problem. In mathematics, optimization is defined as finding the minimum or the maximum of a given function. In practice, optimization finds application in many areas such as different manufacturing processes, parameter optimization, planning, system modeling, etc. as a method for solving different real-life problems. Depending on the complexity of the task and the involved parameters, optimization can be single criteria or multi-criteria.

The single criteria optimization tasks are always very well defined and concrete, as they offer only one single solution. Most real world problems require the simultaneous optimization of multiple, often competing, criteria or objectives [5]. If these objectives are conflicting, then they do not share the same optimum value [6]. Moreover, there may not exist just one single solution, but rather several incomparable alternatives. Problems with multiple objectives (criteria) are generally known as multiple criteria optimization or multiple criteria decision-making problems.

The general view of a single criteria optimization task is given as dependence (1):

$$\text{Extr}_{a \in A} \{k(a) \mid k : A \rightarrow R\} \rightarrow ? \quad (1)$$

where A – finite set of m vectors, which represent possible solution; $k(\cdot)$ – a function (criteria), which evaluates the elements of A , in a way that it presents itself as an image of A in R^n ; **Extr** – substitutes **Max** (maximum) or **Min** (minimum) and means search of an extremum of the function $k(\cdot)$.

The multi-criteria optimization task, on the other hand, can generally given as dependence (2):

$$\text{Opt} \{k(a)\} = \text{Extr} \{k_1(a), k_2(a), \dots, k_h(a), \dots, k_n(a) \mid a \in K\} \quad (2)$$

where **Opt** symbolizes the optimal alternative, which defines $k(\cdot)$ as the best criteria, $k_h(a)$, $h=1, 2, \dots, k$ is k criteria and **Extr** substitutes **Max** (maximum) or **Min** (minimum) and means search of an extremum for every component function $k_i(\cdot)$.

Multi-criteria decision analysis (MCDA) can be formulated as a general term for methods providing a systematic quantitative approach to support decision making in problems involving multiple criteria and alternatives [7]. The multi-criteria optimization process involves decision-making with a number of factors (criteria) in order to find the most suitable solution among several alternatives. These alternatives are evaluated with respect to each criterion, as the criteria are weighted by the decision-maker's assessment. Figure 2 shows a general view of a flowchart at solving multi-criteria optimization problems.

There are different classifications of the methods for solving multi-criteria optimization tasks. The classical MCDA methods can be divided into three main classes [8], [9]:

1. *Multi-attribute value theory* – in which the global preferences of the decision maker are generalized based on the synthesis of one generalized criterion;
2. *Outranking methods* - in which the global preferences of the decision maker are generalized based on the synthesis of one or several generalized relations of the preferences between the alternatives;
3. *Interactive methods* – in which the local preferences of the decision maker are collected iteratively via direct or indirect comparison between two and more alternatives.

The family of outranking methods use the so-called “outranking relations” to rank the given set of alternatives [10]. This is implemented by systematically comparing all pairs of alternatives (pairwise comparison) on each criterion and determining which alternatives are preferred to the others.

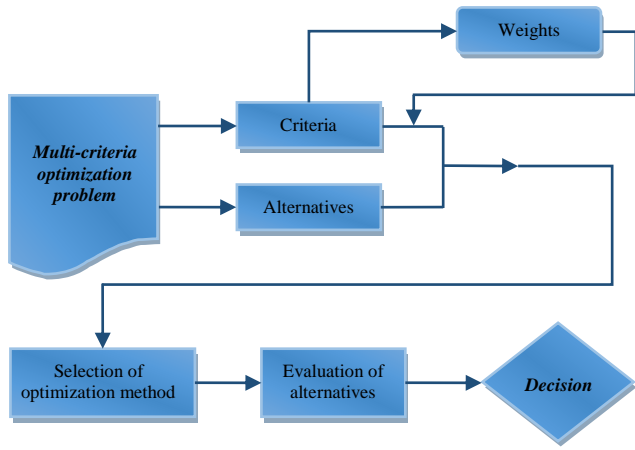


Fig. 2. Flow chart of multi-criteria optimization task solving

Typical representatives of the outranking family of methods include:

- ELECTRE - **E**Limination **E**t Choix Traduisant la **R**Ealité [11];
- PROMETHEE - **P**reference **R**anking **O**rganization **M**ETHod for **E**nrichment **E**valuations) [12];
- MAPPACC - **M**ulticriterion **A**nalysis of **P**references by means of **P**airwise **A**ctions and **C**riterion **C**omparisons [13].

The RAZOR method is also part of the outranking methods family and is used in diverse applications including in combination with recent evolutionary techniques such as genetic algorithms [14], neural networks [15] and simplification of particle swarm optimization [16]. The methodology is based on the statistics Z-standardization approach. The basics of the method is the possibility to measure the distance to the arithmetic average by the means of the standard deviation, which has a special name **Z-standardization**. The measured values using this method are called **Z-values** or most commonly referred to as **standard score** or **Z-score**.

The standard score is a very useful statistic because it allows the probability of a score occurring within our normal distribution to be calculated and enables the comparison of two scores that are from different normal distributions [17]. This statistical approach finds application in various fields, such as medicine [18], financial and banking sector [19], computer technology [20] and the like. Main advantage of this approach is that it allows non-comparable distribution values to be equalized to one scale, so they can be compared.

III. APPROACH TO SOLVING THE PROBLEM

The first step in solving an optimization task is to define the target functions and their requirements and limitations. The optimization criteria as multi-criteria analysis of EMM can be differentiated into two main groups: (1) static criteria and (2) dynamic criteria.

The static criteria represent some geometrical, energy and economical characteristics of the EMM, such as:

- V_{Σ} - total volume of the EMM, including the volume of the electric motor, of the clutch and of the gear reducer: $V_{\Sigma} = V_{\text{mot}} + V_{\text{gear}} + V_{\text{clutch}}$, [mm³];
- $L \times B \times H$ - overall dimensions of the EMM, [mm³];
- η_{total} - total efficiency of the EMM, including the efficiency of the electric motor, of the clutch and of the gear reducer: $\eta_{\text{total}} = \eta_{\text{mot}} * \eta_{\text{gear}} * \eta_{\text{clutch}}$, [-];
- m_{total} - total weight of the EMM, [kg];
- w - comparative value assessment, [-];
- a_w - center distance of the gear reducer, [mm], etc.

The dynamic criteria, on the other hand, represent some of the EMM dynamic characteristics, such as:

- fast performance – the time needed for reaching a stationary regime;
- degree of uniformity in starting regime – evaluation of the maximal amplitude of the deviation of the speed of its stationary values;
- coefficient of dynamic overload in starting regime;
- the deviation of the torque - the amplitude of the deviation of the torque in stationary regime compared to its nominal values, etc.

After the decision maker has defined the alternatives, which will be optimized and has selected the needed criteria, the RAZOR method can be applied. The method consists of several steps, the sequence of which is shown in Fig. 3.

The multi-criteria optimization task, given as dependence (2) is considered. As depicted in Fig. 3, the first step is to define the arithmetic average for the criteria $k_i(\cdot)$:

$$\bar{x}(k_s) = \frac{1}{m} \sum_{i=1}^m k_s(a_i) \quad \forall s = 1, 2, \dots, n \quad (3)$$

where m is the number of the selected criteria.

Based on the calculated value for the arithmetic average, the standard deviation for the criteria $k_i(\cdot)$ is calculated in step 2 (Eq. (4)):

$$\sigma(k_s) = \sqrt{\frac{1}{m-1} \sum_{i=1}^m (k_s(a_i) - \bar{x}(k_s))^2} \quad \forall s = 1, 2, \dots, n \quad (4)$$

At step 3 of the procedure the Z-values are calculated, according to the following equation:

$$z_x = \frac{x - \bar{x}}{\sigma} \quad (5)$$

where x is a concrete value from the distribution (the data row), \bar{x} is the arithmetic average of this distribution, and σ is the standard deviation of the same distribution.

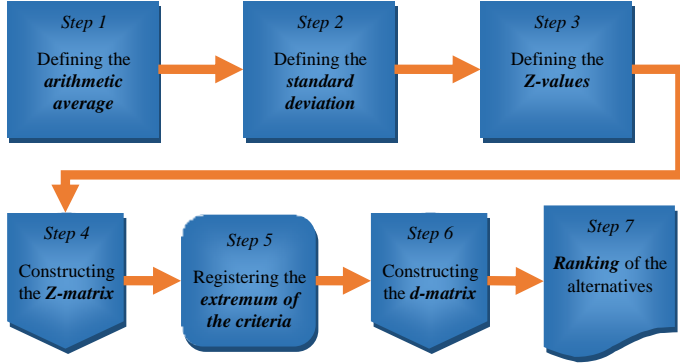


Fig. 3. Steps for implementing RAZOR method

Based on these values, the so-called Z -matrix is created $\{z(k_s)_i\}$:

$$z(k_s)_i = \frac{k_s(a_i) - \bar{x}(k_s)}{\sigma(k_s)} \quad \forall s=1,2,\dots,n \quad \forall i=1,2,\dots,m \quad (6)$$

Then in step 5, the extremum above A of the criteria $k_i(\cdot)$ is registered:

$$\bar{k}_s = \text{Extr}\{k_s(a_1), k_s(a_2), \dots, k_s(a_m)\} \quad \forall s=1,2,\dots,n \quad (7)$$

The so-called “Ideal alternative” vector is obtained, which is built from the extremums of the components $k_i(\cdot)$ of the criteria $k(\cdot)$:

$$\overline{K(A)} = (\bar{k}_1, \bar{k}_2, \dots, \bar{k}_n) \quad (8)$$

At step 6, the so called d -matrix is constructed. It is a matrix with the distances from each alternative to the “Ideal alternative”:

$$d_{si} = \left| z(k_s)_i - \bar{z}(k_s)_i \right| = \left| \frac{k_s(a_i) - \bar{k}_s}{\sigma(k_s)} \right| \quad \forall i=1,2,\dots,m \quad \forall s=1,2,\dots,n \quad (9)$$

The final, i.e. step 7, is to define the ranking of the alternatives using the following expression:

$$R_i^n = \sum_{s=1}^n d_{si} \quad (10)$$

While conducted the calculations according to the above outlined steps, the alternative a_i dominates the alternative a_j regarding the optimality on the n criteria if $R_i^n \geq R_j^n$.

IV. DEMONSTRATIVE EXAMPLE

A multi-criteria optimization of EMM is conducted using RAZOR method. To demonstrate the method, the following input data are used: $n_{\text{out}} = 12 \text{ min}^{-1}$ and $M_{\text{out}} = 510 \text{ Nm}$. Based on these values the input power P_{in} and the output power P_{out} are calculated as $P_{\text{in}} = 0.67 \text{ kW}$, $P_{\text{out}} = 0.64 \text{ kW}$ respectively. An electric motor with nominal power $P_{\text{nom}} = 0.75 \text{ kW}$ will be able to ensure that the values of the input data can be achieved.

Existing geared motors, produced by the company KEB are used as a case study in this example. Their structural components are 0.75 kW asynchronous squirrel cage motor (2-, 4-, 6- and 8-pole motors are available) and a gear reducer (helical, bevel, worm, with parallel shafts and combined gear units). The above-given values for the input data are achieved with 52 different combinations.

To conduct the optimization, the following static criteria are used:

- the total volume of the EMM V_{Σ} in $[\text{cm}^3]$;
- the overall dimensions of the EMM $LxBxH$ in $[\text{cm}^3]$;
- the total efficiency of the module η_{total} ;
- the total weight of the module m in $[\text{kg}]$.

Following the RAZOR methodology, after the arithmetic average and the standard deviation of the alternatives for all four criteria are calculated, the Z -matrix can be constructed. Based on the defined extremums for every criteria and on the calculated standard deviations, the d -matrix can then be constructed. The technical data of the EMM, as well as the results from the optimization are presented in Table I.

TABLE I. TECHNICAL DATA OF THE EMM AND THEIR RANKING

MR_ID	Gear_ID	MotID	i calc (-)	V_{Σ} (cm^3)	$LxBxH$ (cm^3)	η_{total} (-)	m (kg)	R_i^n	rank
MR0001	G33G12	DM80K2	250.00	17670.40	23 242.53	0.73	29.40	0.00616	6
MR0002	G43G22	DM80K2	250.00	24872.02	34 582.60	0.73	42.40	0.01516	25
MR0003	G53G22	DM80K2	250.00	39339.66	54 428.98	0.73	67.40	0.03213	42
MR0004	K43G12	DM80K2	250.00	22803.36	24 478.44	0.73	40.40	0.01090	16
MR0005	K53G22	DM80K2	250.00	35247.69	40 788.35	0.73	61.40	0.02517	36
MR0006	K63G22	DM80K2	250.00	51933.89	60 350.16	0.73	87.40	0.04318	48
MR0007	S32G12	DM80K2	250.00	21494.26	28 197.08	0.67	38.40	0.01203	18
MR0008	S42	DM80K2	250.00	24082.40	34 496.55	0.70	51.40	0.01713	26

MR_ID	Gear_ID	MotID	i calc (-)	V_{Σ} (cm ³)	LxHxB (cm ³)	η_{total} (-)	m (kg)	R_i^n	rank
MR0009	S42G22	DM80K2	250.00	34206.64	44 428.89	0.67	57.40	0.02597	37
MR0010	F33G12	DM80K2	250.00	20915.55	28 718.26	0.76	33.40	0.00946	13
MR0011	F43G12	DM80K2	250.00	29529.66	43 189.86	0.76	46.40	0.01996	30
MR0012	F53G22	DM80K2	250.00	47364.12	70 669.55	0.76	72.40	0.04079	46
MR0013	F63	DM80K2	250.00	56223.13	78 157.67	0.76	99.40	0.05219	50
MR0014	F63G22	DM80K2	250.00	73634.06	104 617.75	0.76	104.40	0.06825	52
MR0015	G33	DM80GC4	125.00	13394.31	16 429.14	0.76	26.00	0.00120	2
MR0016	G43	DM80GC4	125.00	17650.88	31 491.62	0.76	37.00	0.00966	14
MR0017	G53	DM80GC4	125.00	28130.08	42 076.16	0.76	64.00	0.02265	33
MR0018	K43	DM80GC4	125.00	17915.40	23 896.30	0.77	38.00	0.00757	9
MR0019	K43G12	DM80GC4	125.00	22803.36	29 799.84	0.74	42.00	0.01264	20
MR0020	K53	DM80GC4	125.00	26714.91	37 691.84	0.77	56.00	0.01897	28
MR0021	K63	DM80GC4	125.00	40088.26	54 667.83	0.77	84.00	0.03525	44
MR0022	S22	DM80GC4	125.00	12403.06	15 009.48	0.72	25.00	0.00081	1
MR0023	S32	DM80GC4	125.00	16616.93	22 304.10	0.72	36.00	0.00696	7
MR0024	S42	DM80GC4	125.00	24082.40	34 496.55	0.71	53.00	0.01729	27
MR0025	F33	DM80GC4	125.00	14850.91	21 595.22	0.77	30.00	0.00398	4
MR0026	F43	DM80GC4	125.00	20471.00	32 885.06	0.77	43.00	0.01230	19
MR0027	F53	DM80GC4	125.00	30866.40	52 212.47	0.77	67.00	0.02723	39
MR0028	G33	DM90SC6	83.33	14275.58	18 916.88	0.72	28.90	0.00354	3
MR0029	G43	DM90SC6	83.33	18532.15	27 311.65	0.72	39.90	0.01003	15
MR0030	G53	DM90SC6	83.33	29011.35	43 228.91	0.72	66.90	0.02459	34
MR0031	G63	DM90SC6	83.33	45228.35	65 054.39	0.72	97.90	0.04410	49
MR0032	K43	DM90SC6	83.33	18796.67	24 437.28	0.73	40.90	0.00933	12
MR0033	K53	DM90SC6	83.33	27596.18	38 607.30	0.73	58.90	0.02084	32
MR0034	K63	DM90SC6	83.33	40969.53	41 724.76	0.73	86.90	0.03303	43
MR0035	K73	DM90SC6	83.33	65196.54	86 413.39	0.73	138.90	0.06690	51
MR0036	S32	DM90SC6	83.33	17498.20	23 089.29	0.68	38.90	0.00880	11
MR0037	S42	DM90SC6	83.33	24963.67	35 718.25	0.68	55.90	0.01910	29
MR0038	F33	DM90SC6	83.33	15732.18	22 741.15	0.73	32.90	0.00593	5
MR0039	F43	DM90SC6	83.33	21352.27	34 478.22	0.73	45.90	0.01438	22
MR0040	F53	DM90SC6	83.33	31747.67	54 742.79	0.74	69.90	0.02942	40
MR0041	G33	DM100L8	62.50	16656.69	21 529.66	0.67	40.00	0.00838	10
MR0042	G43	DM100L8	62.50	20913.26	30 740.27	0.67	51.00	0.01511	24
MR0043	G53	DM100L8	62.50	31392.46	48 257.12	0.67	78.00	0.03014	41
MR0044	K43	DM100L8	62.50	21177.78	27 306.72	0.67	52.00	0.01441	23
MR0045	K53	DM100L8	62.50	29977.29	42 542.26	0.67	70.00	0.02623	38
MR0046	K63	DM100L8	62.50	43350.64	61 110.02	0.67	98.00	0.04298	47
MR0047	S22	DM100L8	62.50	15665.44	18 016.02	0.63	39.00	0.00737	8
MR0048	S32	DM100L8	62.50	19879.31	26 306.28	0.63	50.00	0.01381	21
MR0049	S42	DM100L8	62.50	27344.78	40 070.08	0.62	67.00	0.02461	35
MR0050	F33	DM100L8	62.50	18113.29	26 218.92	0.67	44.00	0.01118	17
MR0051	F43	DM100L8	62.50	23733.38	39 483.68	0.67	57.00	0.02009	31
MR0052	F53	DM100L8	62.50	34128.78	61 974.96	0.67	81.00	0.03595	45

In the above table, the following designations are used:

- MR_ID – identification for every alternative, used as primary key in the database
- Gear_ID – gear reducer identification
- Mot_ID – electric motor identification
- i calc (-) – calculated gear reducer ratio
- G33 – helical gear unit coaxial, size 3, 3-stage
- K43 – helical bevel gear unit, size 4, 3-stage
- F33 – helical gear unit with parallel shafts, size 3, 3-stage
- S22 – helical worm gear unit, size 2, 2-stage
- DM100L8 – asynchronous squirrel cage motor series DM, size 100L, 8-pole.

The results show that alternative **a22** (MR0022-S22DM80GC4 – helical worm geared motor with 4-pole asynchronous squirrel cage motor) is the optimal solution

among all alternatives, according to the predefined criteria. As seen from Table I, the difference in the values of the calculated sum R_i^n for the alternatives ranked from 1st to 14th is not significant. Therefore, if after the conducted optimization, the decision maker is still not convinced on which one of the alternatives to select for a given application, further optimization of the selected alternatives can be done. In such a case, other criteria can be introduced into the optimization process. For example, another optimization procedure can be run for the 14 alternatives, which have values for the sum R_i^n in the range between 0.0008 ÷ 0.001, at which not only other criteria can be introduced, but also another multi-criteria optimization method can be used.

V. CONCLUSION

While most of the other outranking methods require for the decision maker to have prior detailed knowledge about their methodology, the RAZOR method can be mastered and applied very easily without such a priori. When conducting optimization for example with PROMETHEE method, the decision maker has to select suitable preference functions and values for the so-called indifference and preference thresholds, which directly influences the final ranking of the alternatives. The RAZOR method does not have these disadvantages.

Another advantage of the optimization method presented in this paper is that the usage of the Z-score for forming the ranking of the alternatives allows the realization of the idea for visualization of the multidimensional data in the two-dimensional space by which the better alternatives can be geometrically visualized.

In short, the RAZOR method can be easily applied at solving multi-criteria optimization tasks and permits automation of the optimization process by means of a different software programs.

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