

Hybrid Biogeography/Complex-based Optimization

Chen Wang

Dept of Mechanical Engineering
Hubei University of Automotive Technology
Shiyan, China
e-mail:wangc_jx@huat.edu.cn

Yang Yang

Shanghai Key Laboratory of Intelligent Manufacturing and Robotics
Shanghai University
Shanghai, China

Abstract—The optimization of complex systems is a very difficult problem in modern engineering technology. It is with multi-subsystems, multi-objectives and multi-constraints. In this paper, a novel solution to the complex systems optimization called HBBO/Complex. HBBO/Complex adapted from biogeography-based optimization (BBO) and combined the simulated annealing (SA). The inferior migrated islands will not be selected unless they pass the Metropolis criterion of SA. This method can prevent the local optimal solution. Compared with typical existing many-objective optimization algorithms, HBBO/complex has better convergence characteristics. The results confirm the HBBO/complex provides the best performance in the benchmark problems.

Keywords—biogeography; SA; many-objectives optimization

I. INTRODUCTION

Multi-objective evolutionary algorithms (MOEAs) are well-suited for solving numerous multi-objective problems with two or three objectives. However, as the number of conflicting objectives increases, the performance of most MOEAs is badly deteriorated [1]. In case of Pareto-based MOEAs, these difficulties are intrinsically related to the fact that as the number of objectives increase, the proportion of non-dominated elements in the population grows, being increasingly difficult to discriminate among solutions using only the dominant relation [2].

Many-objective optimization evolutionary algorithms (MaEAs) refer to optimization problems greater than 4 [3]. Due to minimize and maximization problem can be mutual transformation, therefore, without loss of generality, this article mainly describes minimize multi-objective problem and its related concepts MaEAs can be defined as follows:

$$\begin{aligned} &\text{Minimize } F(x) = (f_1(x), (f_2(x) \dots (f_m(x))T \\ &\text{Subject to } x \in \Omega \end{aligned}$$

Where $\Omega \subset R^n$ is the feasible search region, $x=(x_1, x_2 \dots x_n)$ T is the decision variable vector, $f_i : R_n \rightarrow R$, $i=1,2 \dots m$ are the m objective functions, and R_m is the objective space.

Classical optimization methods may fail to do so especially when the objective functions are nondifferentiable and without closed forms. For this reason, people resort to heuristic optimization methods such as evolutionary (EAs). Multi-objective evolutionary algorithms (MOEAs) have been attracting considerable attention. The number of MOEAs can be classified as three categories: (1) the decomposition-based approaches [4], [5] and [6]; (2) the indicator-based approaches

[7], [8] and [9]; and (3) the objective aggregation-based approaches [10], [11] and [12].

Biogeography/Complex based optimization(BBO/complex) algorithm is a kind of adaptive decomposition method. Detailed explanations of BBO/complex are introduced in Section II. However, the performance and convergence rate of BBO/complex is still to be further improved. With the migration flow of n SIV between rich and poor islands, we need a method to enhance its exploration and evaluate the badly modified whether to be accepted or not, it can prevent the past features always be overwritten by the newly emigrated features from other islands. On the other hand, since there are plenty of targets and constraints in the subsystem, when sharing information in the subsystem, we need a new method to reduce the computation time of the CPU. The simulated annealing (SA) algorithm was presented by Kirkpatrick et al. [13] and Valdo Cerny [14], SA algorithm is an intelligent algorithm that randomly search optimization based on probability. It is having the capacity of probabilistic jumping and it is able to accept non-inferior solutions and inferior solutions. Thus, effectively avoid falling into minimal local solutions. We are inspired here by Metropolis criterion of SA algorithm to solve the problem posed above. Details about SA will be introduced in next section.

II. BBO FOR COMPLEX SYSTEMS AND SIMULATED ANNEALING

A. BBO for Complex Systems

BBO was invented less than a decade ago, but according to [15] to provide competitive optimization performance with ACO [16], differential evolution (DE) [17], particle swarm optimization (PSO) [18], and many other algorithms. Complex systems contain more than one subsystem, each of which is partially independent of the others. BBO/Complex is extending BBO to systems with multi-subsystems, where each subsystem contains multi-objectives and multi-constraints. The environment of BBO/Complex includes n archipelagos, where n is the number of subsystems. Every archipelago consists of islands. The islands represent possible solutions to the problem. The structure of BBO/Complex is conceptually different from other typical algorithm. It includes both the framework and the optimization algorithm, as showed in Figure 1. It provides an efficient way to communicate between subsystems and provides a unique migration strategy to share information both within and across subsystems.

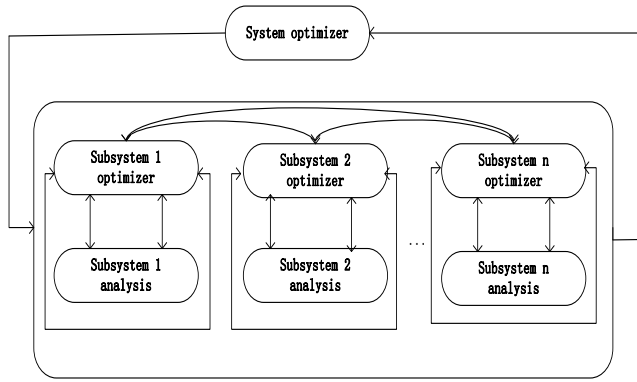


Fig 1: BBO/Complex formulation

B. Migration algorithm

Migration in BBO/Complex needs to be modified due to the fact that the environment of BBO/Complex contains more than one subsystem. Each subsystem contains multi-objectives and multi-constraints. The BBO/Complex's migration stage contains two types: with-subsystem and cross-subsystem. The islands with better part distance will have a better chance to be selected as the emigrating island. The migration processes are described by Algorithm 1.

migration

- 1: Initialization λ_i and μ_i for each member.
- 2: Perform with-subsystem migration: probabilistically choose the immigrating islands based on the islands ranks. Use roulette wheel selection based on the emigration rates to select the Emigrating island.
- 3: Perform cross-subsystem migration: find suitable pairs of subsystems Based on similarity levels. Calculate distances between each pair of Islands from different subsystems. Use roulette wheel selection Based on partial distances to select the emigration islands.

C. Mutation algorithm

In BBO/Complex, there events are modeled as SIV mutation. The mutation rate m_i can be determined by involving the species count probabilities P_i into the following equation:

$$m_i = m_{\max} \left(1 - \frac{P_i}{p \max}\right)$$

Where the $P_{\max} = \max(p_i)$ and m_{\max} is a user-defined maximum mutation rate that m_i can reach. The mutation is described by Algorithm 2.

mutation

- 1: for $i \leftarrow 1$ to k do
 - { where $k = \text{number of islands or individuals}$ }
- 2: Calculate probability P_i based on λ_i and μ_i
 - {by iterative or eigenvector method}
- 3: Calculate mutation rate m_i
- 4: if $\text{rand} < m_i$ and $i \geq R_m$ then { R_m is a user defined mutation range}
- 5: Replace n SIV vector of ISI_i with a randomly generated n SIV vector
- 6: end if
- 7: end for

D. Simulated annealing algorithm

Algorithm SA is a meta-heuristic technique based on a thermodynamic process of the annealing of materials [22], [23], and [24]. The SA algorithm is constructed based on the statistical mechanics, which was demonstrated by Metropolis et al. in 1953[25] through the concept of Boltzmann's probability

distribution. It means if a system is maintained in a thermal equilibrium at temperature T , then the probability distribution p of its energy E can be achieved by [26]:

$$P(E) = e^{-\frac{\Delta E}{k_B T}}$$

Where k_B is a Boltzmann's constant. The difference in energy ΔE means the difference in cost function between the past and current iterations, which can be determined as follows:

$$\Delta E = f(x_n) - f(x_o)$$

For minimization problems, $\Delta E \leq 0$ means $f(x_n) \leq f(x_o)$, so the new design point is directly accepted. Otherwise, the Metropolis criterion will be enabled to decide whether to accept or reject X_o . For this case where $\Delta E > 0$, the acceptance is treated probabilistically according with the

relation $P = \frac{1}{1 + e^{(\Delta E / \max(T))}}$. It can be viewed the influence of temperature in the acceptance process. For the highest magnitudes of T , The acceptance probability to choose a worse state is likewise higher. This process will avoid trapping into local optima. As the temperature decrease, the SA algorithm accepts only states which minimize the FO cost. Therefore, the way that temperature decreases during the iteration of the algorithm is an important parameter, this parameter is named cooling schedule [26].

III. THE HYBRID BBO/COMPLEX ALGORITHM

The proposed hybrid BBO/complex (HBBO/complex) is described by Algorithm 3. When the migration stage is completed, the features (n SIV) of the islands will not be directly overwritten with the new values that come from the probabilistically selected source inlands. Instead, there n SIV of the islands is saved in two temporary matrices. Each row of their matrices represents one individual. The old independent variables are used again if and only if the modified individual shows lower solution quality and does not meet the Metropolis criterion. With this restriction on the migration stage, the overall performance of the HBBO/Complex algorithm can be enhanced. By this method, the exploration of the BBO/complex algorithm is greatly improved.

HBBO/Complex

- Initialization stage with all the parameters,
- 1: Decompose the complex systems based on the system requirements;
- 2: Compute the constraint violations of all islands;
- 3: Do migration.
- 4: **While** ($T > T_{\min}$)
- 5: Calculate $\Delta E = V_2(i) - V_1(i)$
- 6: **if** $\Delta E > 0$ **then**
- 7: Apply the Metropolis criterion
- 8: **if** $P(E) > \text{rand}$ **then**
- 9: Re-select the past ($1, 1 \rightarrow n$) vector of matrix as an updated population for ISI_i
- 10: **end if**
- 11: **end if**
- 12: **end for**
- 13 Update the population with sorting and mapping.
- 14: Do mutation.
- 15: Do clear duplicated SIV.
- 16: Replace the worst ISI with the good ISI saved in the elitism stage

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17: Update the population with sorting and mapping
18: end if
19: end for
20: Display the best population.

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IV. SIMULATION RESULTS

In this section, we compare the performance of HBBO/Complex in real-world benchmark problems with Original BBO/Complex and Collaborative optimization (CO). The benchmark problems are obtained from [22] and include the speed reducer problem. It contains several subsystems and multi-constraints. The speed reducer problem is a gear box design problem. The objective is to minimize the gear box weight and the von Mises stresses for shafts 1 and 2. It contains 3 objectives, 11 constraints, and 7 design variables. This problem is defined as follows:

$$\text{Min F1} = 0.7854X_1 X_2^2(3.3333X_3^2 + 14.9334 X_3 - 43.0934) - 1.05079 X_1(X_6^2 + X_7^2) + 7.477(X_6^3 + X_7^3) + 0.7845(X_4 X_6^2 + X_5 X_7^2),$$

$$\text{Min F2} = \sqrt{\left(\frac{745x_4}{x_2 x_3}\right)^2 + 1.69 \times 10^7},$$

$$\text{Min F3} = \sqrt{\left(\frac{745x_5}{x_2 x_3}\right)^2 + 1.575 \times 10^8},$$

Such that the following constraints hold:

$$g_1 = \frac{27}{x_1 x_2^2 x_3} - 1 \leq 0$$

$$g_2 = \frac{397.5}{x_1 x_2^2 x_3} - 1 \leq 0.$$

$$g_3 = \frac{1.93x_4^3}{x_2 x_3 x_6^4} - 1 \leq 0$$

$$g_4 = \frac{1.93x_5^3}{x_2 x_3 x_7^4} - 1 \leq 0.$$

$$g_5 = \sqrt{\frac{(745x_4 / x_2 x_3) + 1.69 \times 10^7}{0.1x_6^3}} - 1100 \leq 0.$$

$$g_6 = \sqrt{\frac{(745x_5 / x_2 x_3) + 1.575 \times 10^8}{0.1x_6^3}} - 850 \leq 0.$$

$$g_7 = x_2 x_3 - 40 \leq 0$$

$$g_8 = \frac{x_1}{x_2} - 12 \leq 0.$$

$$g_9 = \frac{-x_1}{x_2} + 4 \leq 0.$$

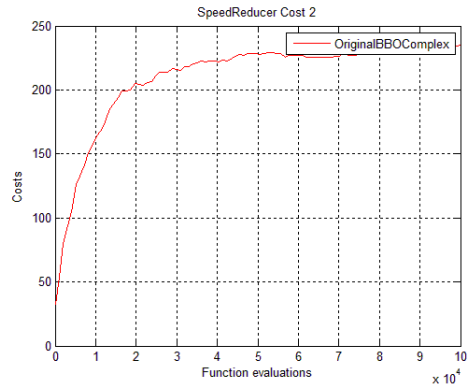
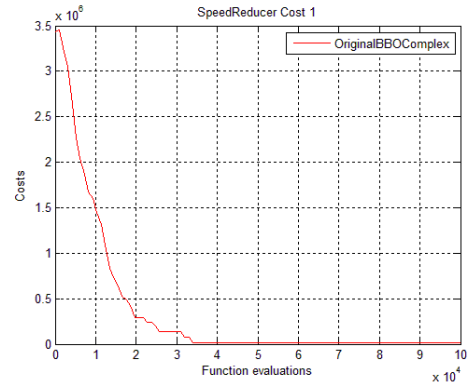
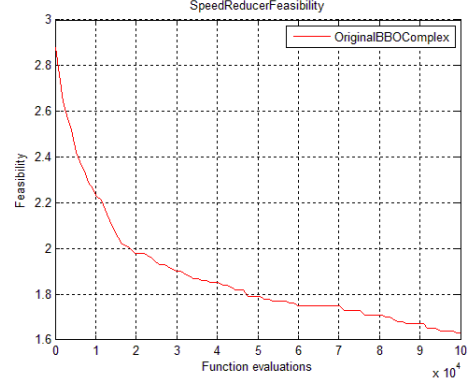
$$g_{10} = \frac{1.5x_6 + 1.9}{x_4} - 1 \leq 0.$$

$$g_{11} = \frac{1.1x_7 + 1.9}{x_5} - 1 \leq 0.$$

Table 1 show the parameters used in the HBBO/Complex.

Table 1: Simulation parameters of the HBBO/Complex algorithms

parameter	value
Population	10
P_{mutate}	0.05
$P_{\text{migration}}$	0.5
T_0	1000
T_{end}	0.01
q	0.9



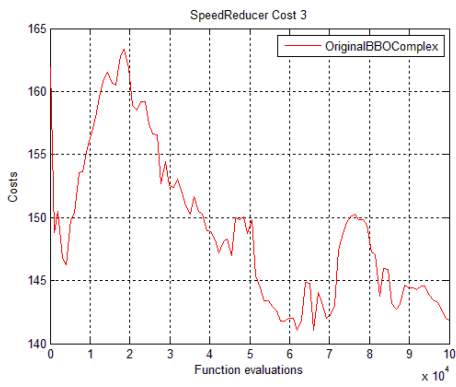


Fig 2: The original BBO/Complex algorithm feasibility and cost of each objective for the speed reducer problem

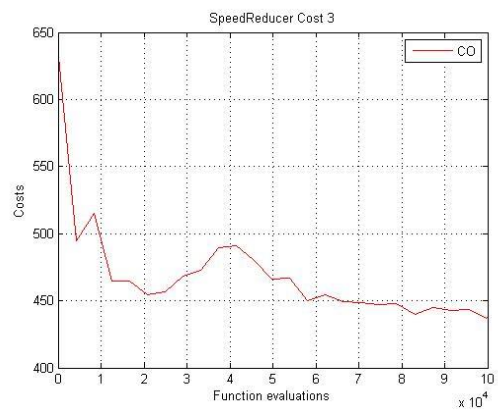
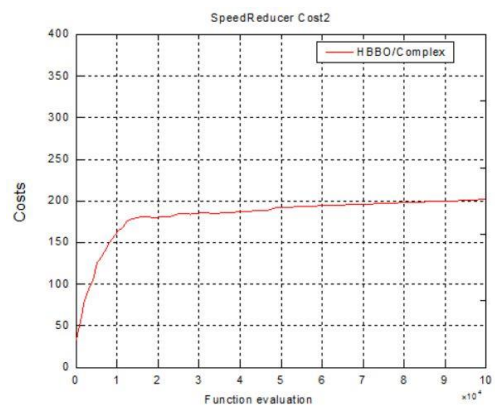
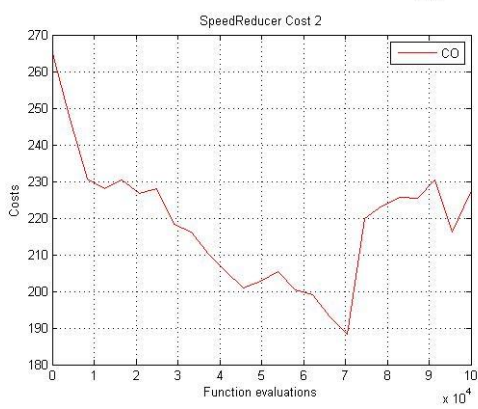
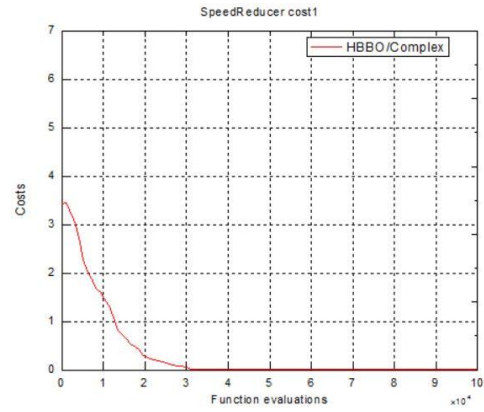
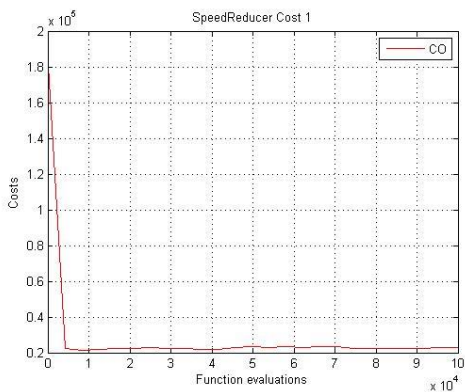
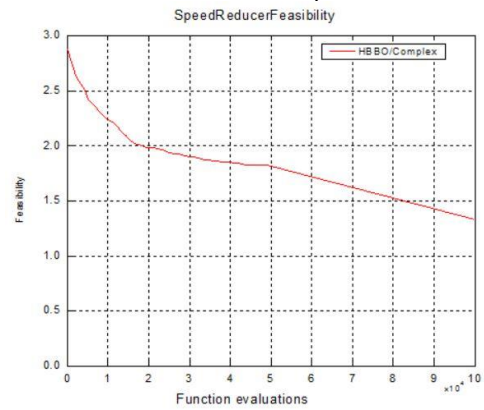
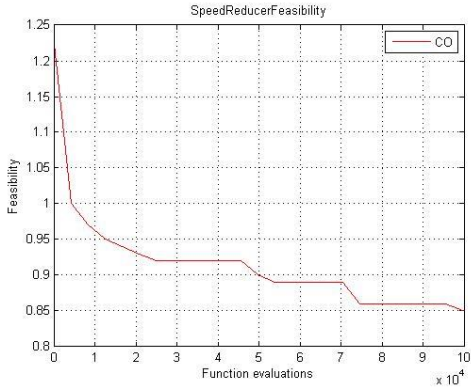


Fig 3: The CO algorithm feasibility and cost of each objective for the speed reducer problem



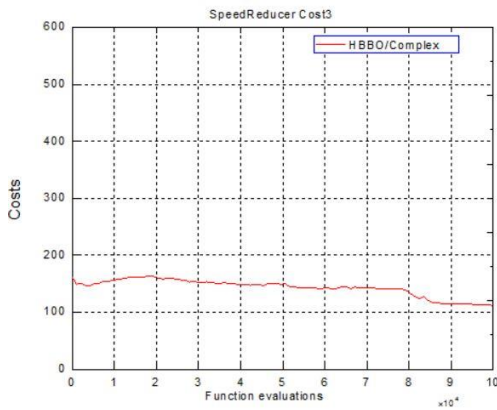


Fig 4: The HBBO/Complex algorithm feasibility and cost of each objective for the speed reducer problem

V. CONCLUSIONS

In this paper, we propose a novel complex system solution called HBBO/Complex. We compare the HBBO/Complex, CO and original BBO/Complex algorithm. The figures also show that the performance of HBBO/Complex is superior to other two algorithms. With this process, the old features will not always be overwritten by the newly emigrated features from other islands. Instead, the Metropolis criterion is used to evaluate the badly modified populations whether they can be accepted or not. It has more flexible decomposition optimization options compared to CO and original BBO/Complex algorithm.

The obtained results show the performance of HBBO/Complex is markedly affected by introduced the SA algorithm. In general, aiming at many subsystems, many-objectives and many-constraints problems for complex systems this hybrid algorithm raises the algorithm's immunity level against trapping into local optima.

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