Semi-Hertzian Theory and its Application in Wheel Wear Calculation

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Abstract. With the rapid development of the railway industry, the speed and load of the railway vehicle has been significantly increasing but operation environment of the railway vehicle becomes worse. All these will cause problems for the wheel-rail wear. And all the problems will cause potential risk to the operation of the railway vehicle. Wheel-rail relation has always got a lot of attention as the most important and most complicated project in the field of vehicle dynamics. This article will research a fast algorithm to calculate the non-Hertzian contact situation based on the basic contact mechanics model which is called semi-Hertzian algorithm. It also uses this model for some further application like the wear calculation.

1. Introduction

1.1 Wheel-rail Contact Mechanics

The main aim for wheel-rail rolling contact theory is to calculate the distribution of normal and tangential contact force and stress between the wheel and rail. Many theories have been established for this purpose. *H.Hertz*[1] creatively given the analytical solution of the normal contact problem between two elastic objects in 1882. The *J.J.Kalker*[2] established the three dimensional non-Hertzian contact theory and its program Contact. In 2006 *AYASSE .etc* [3,4] put forward an algorithm called Semi-Hertzian theory in order to solve the contact problems when the lateral curvature of the contact object is not constant. For the tangential problem, there are also several practical models such as *Carter*[5] two dimensional rolling contact theory, *Vermeulen-Johnson*[6] non-spin three dimensional rolling contact model; *Kalker*[7] linear theory and *Kalker*[8] simplified theory.

1.2 Calculation and Prediction of Wheel Wear

In order to predict wear of the wheels, many models have been developed based on many kinds of wheel-rail wear experiments and the rolling contact theory. Some typical ones such as *Archard* model, *Barghin*[9] model, *Pearce*[10] model, *Zobery*[11] model and *Jendel*[12] model。In this paper, wheel wear is calculated by Archard and Zobery model combined with semi-Hertzian theory.

2. Wheel-rail Contact Mechanics

2.1 Normal Contact Force and Semi-Hertzian Theory

The main concept for Semi-Hertzian Theory was to divide the hertzian contact patch into strips (shown in the figure) and calculated the normal contact force and stress independently in each strip with the equation

$$\delta_{zz}(x, y_i) = \frac{1}{\pi}\frac{1}{nr}\frac{E}{1-v^2}\sqrt{1-\left(\frac{x}{a_i}\right)^2}\,\frac{h_i}{a_i}\frac{1}{\varepsilon} \tag{1}$$

$$N_i = \frac{1}{2}\frac{1}{nr}\frac{E}{1-v^2}\frac{1}{\varepsilon}h_i\delta y \tag{2}$$

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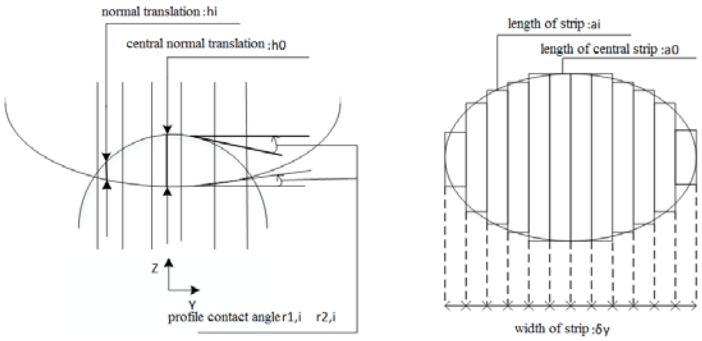

Fig. 1 the discrete method of the contact patch

And the calculate result with a LM tread and R60 rail and 100KN axis load was shown below

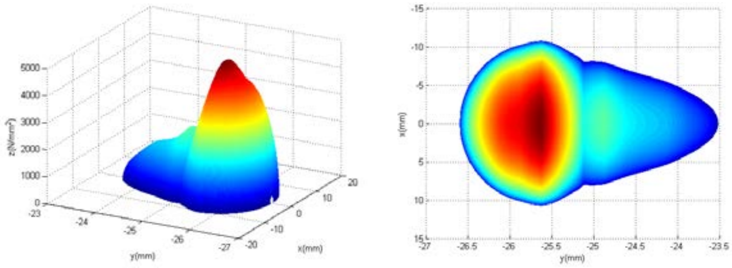

Fig. 2 the normal contact stress distribution

As can be seen from the figures, the maximum contact stresses was $4983.5N/mm^2$, the total stress in the contact patch was $1.0445\times10^5 N$.

2.2 Rolling Contact mechanics

For the calculation of the tangential creep force in this model, a modified FASTSIM algorithm was adopted as the equations written as

$$\hat{\sigma}_{zz,i}(x)=\frac{4}{3\pi}\frac{1}{n_i r_i}\frac{E}{1-v^2}(1-(x/a_i)^2)\frac{h_i}{a_i}\frac{a_i}{a}\frac{1}{\varepsilon_i} \quad (3)$$

$$\sigma_{zx,i}=-\frac{3}{8}Gc_{11,i}v_{x,i}(1-x/a_i)\frac{a_i}{a} \quad (4)$$

$$\sigma_{zy,i}=-(\frac{3}{8}Gc_{22,i}v_{y,i}+\frac{2}{\pi}\sqrt{\frac{n_i}{m_i}}Gc_{23,i}\phi_i(a_i+x))(1-\frac{x}{a_i})\frac{a_i}{a} \quad (5)$$

When the creepage $v_x=v_y=1e-3$, the result is shown in the figure below,

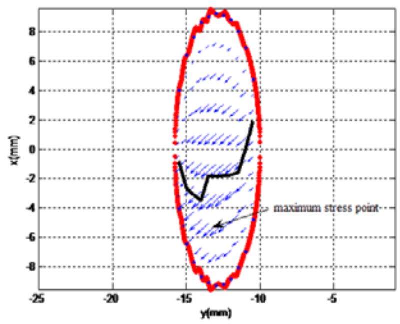

Fig. 3 the real distribution of the wheel-rail tangential contact force

3. Prediction of Wheel Wear

In order to calculate the wear condition of the wheel, two models were applied in this chapter, Archard and Zobery model. The result of Semi-Hertzian Theory was used in the simulation of the wheel wear.

3.1 Wheel Wear Prediction Models

In the Archard model, the stick area and the slip area were calculated independently. This model assumed that the wear only occurred in the slip area, as the equation shows

$$V_{wear} = k_w \frac{Pd}{H} \tag{6}$$

where V_{wear} is the colume of wear, s is the sliding distance, P is the normal force, H is the hardness of the softer material and k is a wear coefficient.

To combine this model with the semi-hertzian model, the following equation was derived

$$\Delta z = k_w \frac{\sigma_{zz} d}{H} \tag{7}$$

where

$$\sigma_{zz,i} = \frac{4}{3\pi} \frac{1}{n_i r_i} \frac{E}{1-v^2} (1-(\frac{x}{a_i})^2) \frac{h_i}{a_i} \frac{a_i}{a} \frac{1}{\varepsilon_i} \tag{8}$$

In the Zobery model, the energy flow dentsity $\dot{E}_d$ in each element can be written as

$$\dot{E}_d(i,j) = \begin{cases} \tau_x(i,j)V_x(i,j) + \tau_y(i,j)V_y(i,j) & (i,j) \in A_s(t) \\ 0 & (i,j) \notin A_s(t) \end{cases} \tag{9}$$

And then this equation could be simplified as

$$\dot{E}_d(r_p,t) = \frac{F_x V_x + F_y V_y + M_z w}{A(t)} \frac{p(r_p,t)}{\overline{p}(t)} \tag{10}$$

where F_x, F_y, M_z were the contact force in the contact patch, $p(r_p,t)$ was the normal contact stress, and $\overline{p}(t)$ was the mean contact stress.

3.2 Simulation Result of Real Wheel Profile

In order to calculate the wear condition with longitudinal and lateral creep, the creepage is settled as 0.002.

The wear distribution was calculated with the Semi-Hertzian-Zobery model。The results were shown below.

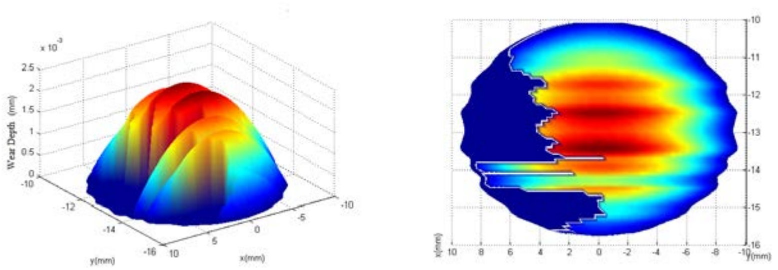

Fig. 4 Wear Distribution with Zobery Model (creepage=0.002)

As is shown in the figures, when $v_x = v_y = 2e-3$, the contact patch can be divided into adhesion area and slide area. Wear only happened in the slide area. The maximum wear was at the center of the contact patch and the wear depth is 2.3*10^-3(mm).

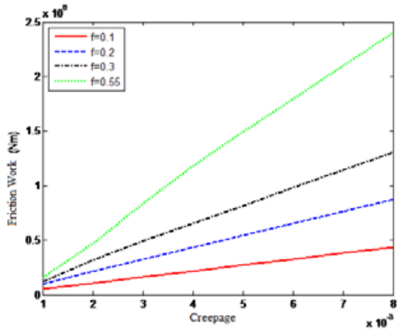

Fig. 5 the curve of friction work with Zobery Model (creepage=0.002)

The result of wear distribution calculated with the Semi-Hertzian- Archard model were shown below

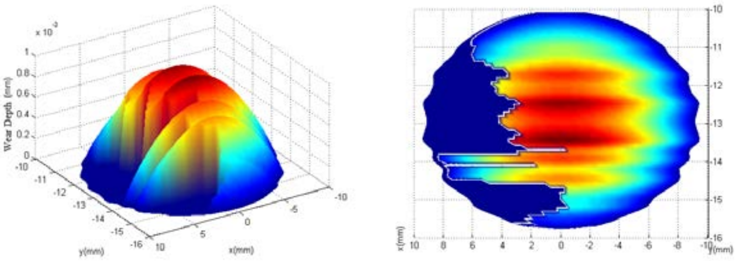

Fig. 6 Wear Distribution with Archard Model (creepage=0.002)

As is shown in the figures, when $v_x = v_y = 2e-3$, the contact patch can be divided into adhesion area and slide area. Wear only happened in the slide area. The maximum wear was at the center of the contact patch and the wear depth is 2.3*10^-3(mm).

4. Conclusion

In this paper, semi-Hertzian algorithm was studied in order to calculate the normal contact between wheel and rail. Then Archard model and Zobery-Jendel model was used to simulate the wear depth of wheel with real profile. As is shown in the paper, semi-Hertzian theory can be a good application in the prediction of wheel wear problems.

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