

A Lucid and fast Algorithm for N-k Contingencies Based on Low-Rank Update

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Abstract. In this paper, the low rank update mathematics was employed to N-k contingencies computation. In the method, the two low-rank matrixes correspond respectively location and modification values were constructed. The amount of calculation was decreased greatly due to the node admittance matrix was revised by the product of two low-rank matrixes and the inversion of the modified matrix is obtained by computation of low-rank matrixes base on original admittance matrix. The test result indicates that the method is lucid and fast due to avoid re-factorization, and can be used flexibility in multi-outages as can be calculated at once. Accordingly, it is suitable for N-k contingencies computation of large-scale power system, which the whole calculation process is on sparse admittance matrix and calculations of low-rank matrix. Finally, an S-1047 system was employed analyze the validity and practicality of this proposed method.

1 Introduction

N-k contingencies analysis is a problem of increasing importance due to the increasing complexity of the power system infrastructure [1]- [5]. Recent large-scale power grid failures have highlighted the need for effective and fast computational tools for N-k contingencies analysis. Since the situations, such as the simultaneous outage of multiple lines due to the increase of the electric network scale and complicity, or the maintenance bus with the multiple components, the simultaneous variations of parameters resulted from integrating stochastic renewables or switching of operation conditions of substations or plants and so on, become more and more common. The Transmission System Operators therefore need to be able to perform N-2, N-3 or even deeper N-k analyses in order to operate safely the transmission system they are responsible for.

When running N-k ($k > 1$) analyses, a computational problem arises. In another hand, less computation time and better adaptability for flexible operation condition are required for the rapid development of smart grid.

In many cases, linear models are considered satisfactory and the principle of superposition is then employed. Methods of contingency analysis which use the system Z_{bus} without having to modify it become attractive from the computational viewpoint—especially if loads can be treated as constant current injections into the various buses of the system [6]. But the Z_{bus} method shouldn't be followed when tripping three branches at the same time such as breaking three-winding transformer, autotransformer, and multicomponent on bus, etc., due to the recursion makes the calculation too complicated. The re-factorization method is poor at calculation for large-scale power system, because of the heavy computation burden caused by the re-factorization of the revised admittance matrix in every outage, although there is no limitation to the multi-fold outage. The general view of the compensation method is fast and no limitation to the multi-fold outage, but the compensating vector calculation is elusive and complicated [7]- [11].

In this paper, an alternative lucid and fast linear method based on the low rank update is proposed. In this method, two low-rank matrixes were constructed, and the product of the two matrixes was used to revise the original admittance matrix. Afterwards according to the originally admittance matrix and the result of calculation of low-rank matrixes, the revised admittance matrix was avoided be refactored, so the amount of calculation was decreased greatly, and it can be used

flexibility in multi-fold outages.

2 The Low-Rank Updates Mathematics

The low-rank updates method is related the inverse of a matrix after a small-rank perturbation to the inverse of the original matrix. The most common applications have the following structure [12] [13].

2.1 The Solution Procedure of the Low-rank Updates

We are given a linear system $\mathbf{Ax} = \mathbf{b}$, where unknown vector $\mathbf{x} \in \mathbb{R}^n$, \mathbf{A} deviates slightly from matrix \mathbf{A}_0 and the difference $\mathbf{A} - \mathbf{A}_0$ can be expressed in the form \mathbf{LM} (their construction will be presented in section 3), where $\mathbf{L} \in \mathbb{R}^{n \times k}$ and $\mathbf{M} \in \mathbb{R}^{k \times n}$ have relatively small rank ($k \leq n$).

Before changing of coefficient matrix, the original linear equation system given by

$$\mathbf{A}_0 \mathbf{x}_0 = \mathbf{b} \quad (1)$$

Where $\mathbf{b} \in \mathbb{R}^n$, $\mathbf{A}_0 \in \mathbb{R}^{n \times n}$ are given, \mathbf{x}_0 has been solved. while the coefficient matrix is changed, the equation $\mathbf{Ax} = \mathbf{b}$ be written

$$\mathbf{Ax} = (\mathbf{A}_0 + \mathbf{LM})\mathbf{x} = \mathbf{b} \quad (2)$$

We can rewrite Eq. (2) as

$$\mathbf{A}_0 \mathbf{x} = \mathbf{b} - \mathbf{LMx} = \mathbf{b} - \mathbf{Lc} \quad (3)$$

This equation leads us to define $\mathbf{c} = \mathbf{Mx}$.

Solve $\mathbf{A}_0 \mathbf{W} = \mathbf{L}$ for the unknown \mathbf{W} , where $\mathbf{W} \in \mathbb{R}^{n \times k}$.

Substituting \mathbf{L} into Eq. (3)

$$\mathbf{A}_0 \mathbf{x} = \mathbf{b} - \mathbf{Lc} = \mathbf{A}_0 \mathbf{x}_0 - \mathbf{A}_0 \mathbf{Wc}$$

We obtain

$$\mathbf{x} = \mathbf{x}_0 - \mathbf{Wc} \quad (4)$$

Where

$$\mathbf{c} = \mathbf{Mx} = \mathbf{M}(\mathbf{x}_0 - \mathbf{Wc}) \quad (5)$$

We can rewrite Eq. (5) as

$$(\mathbf{E} + \mathbf{MW})\mathbf{c} = \mathbf{Mx}_0 \quad (6)$$

Where \mathbf{E} is a $k \times k$ unit matrix. The unknown \mathbf{c} can be solved.

2.2 The calculation steps of low-rank updating

As mentioned before, we can rewrite the calculation steps of low3 rank updating as follows:

Step1: Solve $\mathbf{A}_0 \mathbf{x}_0 = \mathbf{b}$ for the unknown \mathbf{x}_0 .

Step2: Construct the matrix \mathbf{L} and \mathbf{M} to satisfy the equation $\mathbf{A} = \mathbf{A}_0 + \mathbf{LM}$.

Step3: Solve $\mathbf{A}_0 \mathbf{W} = \mathbf{L}$ for the unknown \mathbf{W} .

Step4: Solve $(\mathbf{E} + \mathbf{MW})\mathbf{c} = \mathbf{Mx}_0$ for the unknown \mathbf{c} .

Step5: Finally, $\mathbf{x} = \mathbf{x}_0 - \mathbf{Wc}$.

If the matrix \mathbf{A}_0 is a sparse matrix, then the linear systems associated with \mathbf{A}_0 in Step1 and Step3 are solved quickly. If $\mathbf{M} \in \mathbb{R}^{k \times n}$, $k \ll n$, then the rank of the modification \mathbf{LM} is small relative to the dimension n of the \mathbf{A}_0 and the system of k linear equations $(\mathbf{E} + \mathbf{MW})\mathbf{c} = \mathbf{Mx}_0$ is solved quickly due to $\mathbf{W} \in \mathbb{R}^{n \times k}$, $(\mathbf{E} + \mathbf{MW}) \in \mathbb{R}^{k \times k}$. In many applications, $k=1$ and the \mathbf{c} is the scalar. Obviously, the low-rank update method avoided the factorization of the \mathbf{A} matrix when solving the equation $\mathbf{Ax} = \mathbf{b}$ due to manipulate the calculation of the small rank matrix \mathbf{L} , matrix \mathbf{M} and the sparse matrix \mathbf{A}_0 . On this account, the calculation work is much lesser.

3 N-k contingency analysis with low rank update

The linear model based on sparse admittance-matrix generally presented as $\mathbf{YV} = \mathbf{I}$. Where \mathbf{I} is

the column vector of the injection currents, \mathbf{V} is the column vector of the nodal voltages and \mathbf{Y} is the nodal admittance matrix.

Assume that \mathbf{Y}_0 is the original admittance-matrix; consider some system equipment sustains forced outages. Because the outage of branch only affects the self and mutual admittance, it relates a small rank perturbation to the original admittance matrix. We can obtain the new admittance matrix \mathbf{Y} by modifying the original admittance matrix \mathbf{Y}_0 . The equation can be written as $\mathbf{Y} = (\mathbf{Y}_0 + \Delta\mathbf{Y})$, $\Delta\mathbf{Y} = \mathbf{L}\mathbf{M}$. The key question involved is how to construct the low rank matrix \mathbf{L} and \mathbf{M} . the method for different situations is to do as follows.

3.1 The matrix \mathbf{L} and \mathbf{M} of single contingencies

Removing a branch with admittance y_{ij} between node i and j from the original n node network, the admittance matrix should be modified by

$$\Delta\mathbf{Y} = \begin{bmatrix} 0 & i & & j & 0 \\ \hline -y_{ij} & 0 & \cdots & 0 & y_{ij} \\ 0 & 0 & & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & & & 0 & 0 \\ y_{ij} & 0 & \cdots & 0 & -y_{ij} \\ \hline 0 & & & & 0 \end{bmatrix} \begin{matrix} \\ i \\ \\ j \\ \\ \end{matrix}$$

Then the following relationships hold according to the equation $\Delta\mathbf{Y} = \mathbf{L}\mathbf{M}$

$$\mathbf{L}^T = [0, \cdots, l_i, \cdots, l_j, \cdots, 0]$$

$$\mathbf{M} = [0, \cdots, m_i, \cdots, m_j, \cdots, 0]$$

Evidently,

$$l_i m_i = -y_{ij}; \quad l_i m_j = l_j m_i = y_{ij}; \quad l_j m_j = -y_{ij}.$$

The admittance y_{ij} is known, so that

$$\left. \begin{aligned} l_i &= -\frac{y_{ij}}{m_i} = \frac{y_{ij}}{m_j} \\ l_j &= \frac{y_{ij}}{m_i} = -\frac{y_{ij}}{m_j} \end{aligned} \right\} \quad (7)$$

Observe that the elements l_i , l_j , m_i , m_j are correlation, which can obtain through a given value of an arbitrary element. That is, the matrix \mathbf{L} and matrix \mathbf{M} been decomposed from $\Delta\mathbf{Y}$ have the characteristics such as being multi-valued and having randomness. According to the feature of power systems, we put forward a standard normative structure of matrix \mathbf{L} and matrix \mathbf{M} for making the calculation easier and more specific.

If we set $m_i = -y_{ij}$, then the following relationships hold according to Eq.(7), $m_j = y_{ij}$, $l_i = 1$, $l_j = -1$. So that $\mathbf{L}^T = [0, \cdots, 1, \cdots, -1, \cdots, 0]$, $\mathbf{M} = [0, \cdots, -y_{ij}, \cdots, y_{ij}, \cdots, 0]$

These results are universally applicable. Then we can obtain quickly

$$\mathbf{L}\mathbf{M} = \begin{bmatrix} 0 & i & & j & 0 \\ \hline -y_{ij} & 0 & \cdots & 0 & y_{ij} \\ 0 & 0 & & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & & & 0 & 0 \\ y_{ij} & 0 & \cdots & 0 & -y_{ij} \\ \hline 0 & & & & 0 \end{bmatrix} \begin{matrix} \\ i \\ \\ j \\ \\ \end{matrix} \quad \mathbf{Y} = \mathbf{Y}_0 + \mathbf{L}\mathbf{M} = \mathbf{Y}_0 + \begin{bmatrix} 0 & i & & j & 0 \\ \hline -y_{ij} & 0 & \cdots & 0 & y_{ij} \\ 0 & 0 & & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & & & 0 & 0 \\ y_{ij} & 0 & \cdots & 0 & -y_{ij} \\ \hline 0 & & & & 0 \end{bmatrix} \begin{matrix} \\ i \\ \\ j \\ \\ \end{matrix}$$

Apparently, the new admittance \mathbf{Y} can be obtained through the original admittance \mathbf{Y}_0 modified by $\mathbf{L}\mathbf{M}$. The matrix \mathbf{L} can be defined as a location matrix and the matrix \mathbf{M} as a corresponding

modification values matrix. In this case, the \mathbf{L} is a one-column vector, the \mathbf{W} solved from $\mathbf{Y}_0\mathbf{W}=\mathbf{L}$ is $n \times 1$, obviously, $(\mathbf{E}+\mathbf{M}\mathbf{W})$ is a scalar. The \mathbf{c} from the scalar equation $(\mathbf{E}+\mathbf{M}\mathbf{W})\mathbf{c}=\mathbf{M}\mathbf{V}_0$ can be solved quickly. $\mathbf{V}=\mathbf{V}_0-\mathbf{W}\mathbf{c}$.

3.2 Grounding branches outage

According above discussion, tripping a grounding branch with admittance y_{i0} from bus i , the \mathbf{L} , \mathbf{M} structures are as follow

$$\mathbf{L}^T = [0, \dots, \overset{i}{1}, 0, \dots, 0]$$

$$\mathbf{M} = [0, \dots, -y_{i0}, 0, \dots, 0]$$

Then,

$$\mathbf{LM} = \begin{bmatrix} 0 & \dots & \overset{i}{0} & \dots & 0 \\ \vdots & \ddots & & & \vdots \\ 0 & & -y_{i0} & & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & \dots & 0 & \dots & 0 \end{bmatrix} i$$

Apparently, the value of modification is correct.

3.3 Analysis of multiple contingencies

The method for a single branch tripped can extend to more than one operation occurs simultaneously in the network. In order to simplify notation, we only present the structures of the matrix \mathbf{L} and matrix \mathbf{M} when tripping two lines at the same time. It is assumed that the branches are to be tripped between node-pairs ij and kh and are to have admittance of y_{ij} and y_{kh} corresponding. The \mathbf{L} , \mathbf{M} structures are as follow

$$\mathbf{L}^T = \begin{bmatrix} 0, \dots, \overset{i}{1}, \dots, \overset{k}{0}, \dots, \overset{j}{-1}, \dots, \overset{h}{0}, \dots \\ 0, \dots, 0, \dots, 1, \dots, 0, \dots, -1, \dots \end{bmatrix}$$

$$\mathbf{M} = \begin{bmatrix} 0, \dots, \overset{i}{-y_{ij}}, \dots, \overset{k}{0}, \dots, \overset{j}{y_{ij}}, \dots, \overset{h}{0}, \dots \\ 0, \dots, 0, \dots, -y_{kh}, \dots, 0, \dots, y_{kh}, \dots \end{bmatrix}$$

$$\mathbf{LM} = \begin{bmatrix} 0 & \overset{i}{-y_{ij}} & 0 & \dots & 0 & \dots & \overset{j}{y_{ij}} & \dots & 0 & \dots & 0 \\ \vdots & 0 & 0 & \ddots & & & 0 & & \vdots & & \vdots \\ \vdots & \ddots & \ddots & 0 & & & & & 0 & & \vdots \\ \vdots & & 0 & -y_{kh} & & & y_{kh} & & & & \vdots \\ 0 & & 0 & 0 & \dots & 0 & 0 & & & & \vdots \\ \vdots & y_{ij} & \ddots & & & -y_{ij} & & & \vdots & & \vdots \\ \vdots & & 0 & & & 0 & \dots & 0 & & & \vdots \\ 0 & \dots & 0 & y_{kh} & & & -y_{kh} & & & & 0 \\ 0 & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & 0 \end{bmatrix} \begin{matrix} i \\ k \\ j \\ h \end{matrix}$$

Observe that the $\mathbf{M} \in \mathbb{R}^{n \times 2}$, the \mathbf{W} solved from $\mathbf{Y}_0\mathbf{W}=\mathbf{M}$ is $n \times 2$, $(\mathbf{E}+\mathbf{M}\mathbf{W})$ is a 2×2 low rank matrix.

3.4 Tripping three-winding transformers or auto-transformers

Application of the compensation method and Z_{bus} method are more complicated when three or more branches would be tripped. Therefore, the grounding branches are usually neglected because the errors are not very significant when they are used to analyze line outage states. If the transformer ground branches are neglected, the two-winding transformer can be deal as a branch. However, in modern power system, besides these two-winding transformers, there are lots of three-winding transformers and autotransformers (Fig.1). How to trip these types transformers that have three branches conveniently?

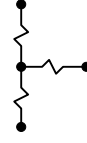


Fig.1 Buses of three-winding transformer and autotransformer

In low rank update method, the pivotal work is constructing the matrix \mathbf{L} and \mathbf{M} . According the above constructing principle, let we set

$$\mathbf{L}^T = \begin{bmatrix} 0, \dots, 1, 0, \dots, 0, 0, \dots, 0, 0, \dots, -1, 0, \dots \\ 0, \dots, 0, 0, \dots, 1, 0, \dots, 0, 0, \dots, -1, 0, \dots \\ 0, \dots, 0, 0, \dots, 0, 0, \dots, 1, 0, \dots, -1, 0, \dots \end{bmatrix}$$

$$\mathbf{LM} = \begin{bmatrix} 0 & & & & 0 \\ & -y_{io} & 0 & 0 & y_{io} \\ & 0 & -y_{jo} & 0 & y_{jo} \\ & 0 & 0 & -y_{ko} & y_{ko} \\ & y_{io} & y_{jo} & y_{ko} & -\sum_{t=io, jo, ko} y_t \\ 0 & & & & 0 \end{bmatrix}$$

We can see that the complex issue in the compensation method and \mathbf{Z}_{bus} method is easy in the low rank update method. $(\mathbf{E}+\mathbf{MW})$ is a 3×3 low rank matrix.

4 case test

A S-1047 network which is an real system (its features is listed in table 1.) is used to evaluate the performance of the low rank update method in N-k contingency computation. The test was performed on Windows7 3.2GHz, 4-core processor, 4GB RAM; using Matlab 2011a code base. The result of the calculation time comparison with the fastest Mid-Compensation method and Re-factorization method listed in table2 when tripping 1~6 branches at the same time. The comparison of the \mathbf{Z}_{bus} method is not done since the method is not practical for N-k ($k \geq 3$) contingencies.

TABLE I PARAMETERS OF THE S-1047 TEST SYSTEMS

Test case	Number of buses	Number of lines	Number of Transformer s branches	Number of Grounding branches
S-1047	1047	1182	164	8

The results show that the rate of computations of low rank update method is 30 times faster than the re-factorization method and slightly faster than mid-compensation method when tripping a branch. However, with the increasing the number of k, the speed of the low rank update method exhibited. When k=6, the low rank update method is 12 times faster than the re-factorization method and 4.65 times faster than the mid-compensation method. That is, the bigger the number of k, the more calculation speed has the advantage.

TABLE II
CALCULATION TIME COMPARISON WITH
THE DIVERSE LINES COMBINATION OUT IN S 1047-BUS SYSTEMS

Number of outages	RE-FACTORIZAT ION	Mid-Compensation	The Low-rank Update
1	0.009101	0.000337	0.000301
2	0.009268	0.000891	0.000573
3	0.009411	0.001392	0.000627
4	0.009984	0.001822	0.000689
5	0.010272	0.002783	0.000811
6	0.010983	0.004311	0.000927

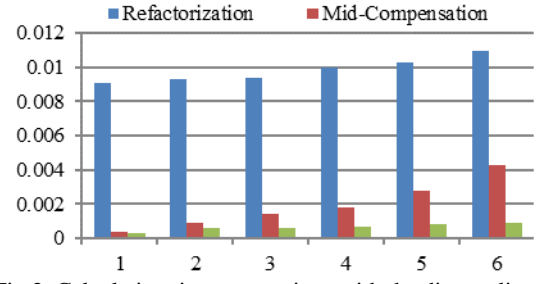


Fig.2. Calculation time comparison with the diverse lines combination outages in S 1047-bus systems

5 Conclusion

The numerical example demonstrated that the proposal method speed up the computation largely, with the scale of the power network and the risk of multi-contingencies increasing, and the combinations mounting. Review the procedure of the method; it is easy to construct the low rank matrix \mathbf{L} and \mathbf{M} for all contingencies situations; the solution procedure is lucid for only five steps base on low rank matrix and original sparse admittance matrix. Therefore, it is suitable for N-k contingencies computation of large-scale power system.

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