Prediction of the Number of the Infective People By Using SIR Method

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Abstract. We establish the SIR model, in which we define three groups，i.e. the Susceptive, the Infective and the Remove. The sum of the proportion of the three groups is 1. The three groups' rate of change can be obtained. We can get the phase path, with the initial rate of the Susceptive—S(t)，and the rate of the Infective—I(t). We can get the best touch rate and cure rate by curve fitting. Finally we can get the variation of the number of the Infective,

Introduction

Ebola virus disease (EVD), formerly known as Ebola hemorrhagic fever, is a severe,The first EVD outbreaks occurred in remote villages in Central Africa, near tropical rainforests, but the most recent outbreak in West Africa has involved major urban as well as rural areas.In order to eradicate Ebola,predicting the number of the infective people is necessary.

SIR Model

Model Overview

Different kinds of model have its own characteristic when we describe the process about the spread of infectious diseases. There is a SIR model which we study the propagation mechanism of infectious diseases, and don't consider the medical knowledge.

According to the transformation rules of the number of susceptible, the infected and the removed, we can establish a SIR model. We calculate the answer of SIR model with Runge-Kutta differential equation. After curve fitting, we can get the effective touch rate. With the data we already find, we analysis the accuracy of the model. Finally, we can predict the changes of the number of the patients.

Table 1. Model parameters

Parameters	meaning
N	The total number of the population in the country
S (t)	The quantitative proportion of susceptible at time t
I (t)	The quantitative proportion of the infective at time t
R (t)	The quantitative proportion of the removed at time t
B	The effective number of contagions of a patient
γ	The average t recovery rates of the patient every day
α	The mortality proportion of the patient every day
βS (t)	The ratio of the number of infections that per patient infect per unit time.

The establishment of the model

1）According to the assumption, we can get the conclusion as follows::

A patient can infect $\beta S(t)$ susceptible person. The total number of patient is $NI(t)$. So there are $\beta S(t)I(t)$ healthy people infected. Then the increasing rate of patient is:

$$\frac{\partial s(t)}{\partial t} = -\beta s(t) I(t) \tag{1}$$

2) The number of patient equal to the number of increasing patient $S(t)\beta NI(t)$ subtracts the number of removed patient $(\gamma+\alpha)I(t)$. Thus, the rate of the increasing patient number can be expressed as:

$$\frac{\partial I(t)}{\partial t} = \beta S(t) I(t) - (\alpha+\gamma) I(t) \tag{2}$$

3) We can get the transform between the Infective and the Removed. The decreasing rate of Removed is equal to the number of Infective.

$$\frac{\partial R(t)}{\partial t} = \gamma I(t) \tag{3}$$

There are three groups in the SIR model, that is the Susceptive (S), the Infective (I) and the Remove (R). So we can get the relation among the three parameters：

$$\begin{cases} S(t)+I(t)+R(t)=1 \\ \dfrac{\partial S(t)}{\partial t} = -\beta S(t) I(t) \\ \dfrac{\partial I(t)}{\partial t} = \beta S(t) I(t) - (\alpha+\gamma) I(t) \\ \dfrac{\partial R(t)}{\partial t} = \gamma I(t) \end{cases} \tag{4}$$

Fundamental theorem

If $\sigma s_0 < 1$, I(t) will monotone decrease , and $i(\infty)=0$.

If $\sigma s_0 > 1$, I(t) will increase firstly to the maximum---

$i_m = i_0 + s_0 - 1/\sigma - 1/\sigma \ln(\sigma s_0)$, where $s = 1/\sigma$. And then decrease to zero.

S(t) is a monotone decrease fundamental, and $s(+\infty)$ is the root of

$$0 = (s_0 + i_0) - s + \frac{1}{\sigma} \ln \frac{s}{s_0} \tag{5}$$

Phase-plane methods

Let's define $\sigma = \beta / \gamma$, and $\frac{1}{\sigma}$ is the rate of Relative removal.

The Phase trajectory's definition domain is $(s,i) \in D$, where

$$D = \{(s,i) \mid s \ge 0, i \ge 0, s+i \le 1\}$$

According to the equation set (4), we can get the equation (5) by remove dt.

$$\frac{di}{ds} = \frac{1}{\sigma s} - 1 \qquad (6)$$

And the answer of equation set (4) is

$$i_m = (s_0 + i_0) - s + \frac{1}{\sigma} \ln \frac{s}{s_0} \qquad (7)$$

The phase plane is shown in the Figure 1. The direction of the arrow presents the tendency of S(t) and I(t).

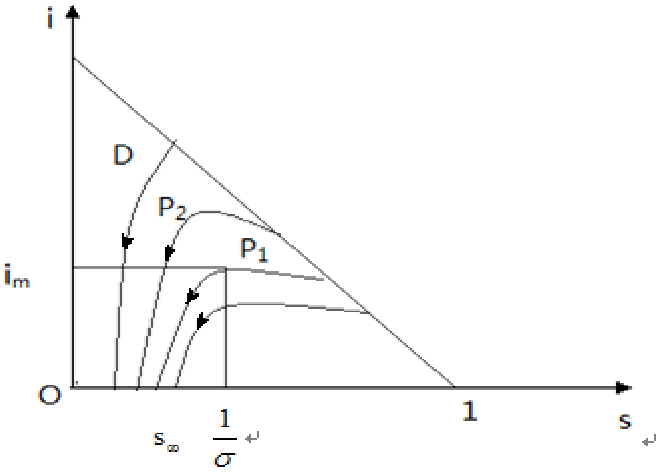

Figure 1. The Phase-plane of the SIR model

Analysis the answer of equation (4) and the Figure 1, we can get four conclusions:

Eventually the proportion of healthy who are uninfected is $s(+\infty)$. And $s(+\infty)$ is the single root of equation (5). And $s(+\infty)$ is the abscissa of the crossover point

No matter how many the initial number of susceptible and infective person, the number of the Ebola patients will be zero.

If $s_0 > 1/\sigma$, I(t) will increase firstly to the maximum---
$i_m = i_0 + s_0 - 1/\sigma - 1/\sigma \ln(\sigma s_0)$, where $s = 1/\sigma$.And then I(t) decrease to zero.

If $\sigma s_0 < 1$, I(t) will monotone decrease , and $i(\infty) = 0$.

So it can be seen that the disease is spreading when the proportion of patient I(t) is growth. And $1/\sigma$ is the threshold. Only when $s_0 > 1/\sigma$, the disease will be spread. Reduce the number of σ, we can curb the spread of infectious diseases.

With the improvement of people's living standard, the rate of contact is growing small. And the higher the medical level is, the greater the cure rate is. So improve the level of medical and health can help us to control the spread of infectious diseases.

Curve fitting

With the MATLAB helping, we can get a curve which can tally the data we find. The figures are as following:

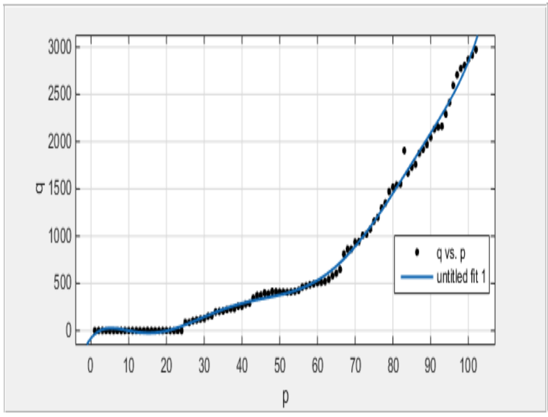

Figure 2 Guinea

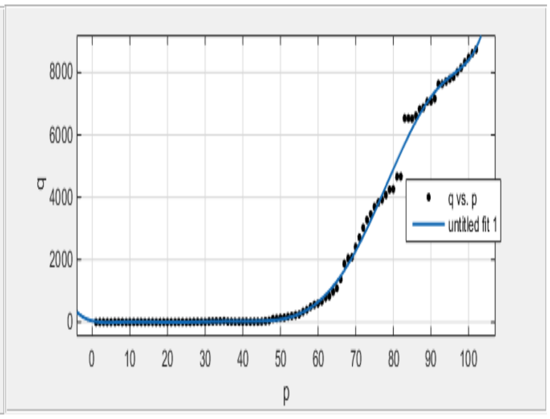

Figure 3 Liberia

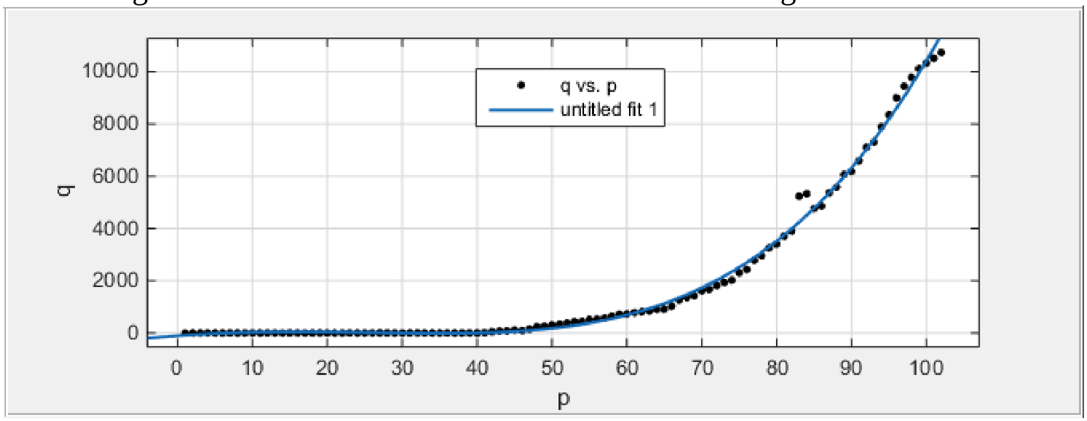

Figure 4 Sierra Leone

We use the curve fitting to get a reasonable β to describe the spread of Ebola. We can not ensure the accurate value of β. We change β to get the satisfactory answer---- β=0.098

As the differential equation of SIR model can not be solved directly, so we choose Runge-Kutta to get the arithmetic solution.

Analysis of the SIR model

We can only obtain the differential equation for SIR model, however, the model can not get the numerical calculation. Therefore, the numerical solution of differential equations can be used to find the solution of the model 1. The Fourth Order Runge-Kutta method is:

$$y_{n+1} = y_n + \frac{h}{6}\left(K_1 + 2K_2 + 2K_3 + K_4\right) \tag{8}$$

Where:

$$K_1 = f\left(t_n, y_n\right);$$

$$K_2 = f\left(t_n + \frac{h}{2}, y_n + \frac{hK_1}{2}\right);$$

$$K_3 = f\left(t_n + \frac{h}{2}, y_n + \frac{hK_2}{2}\right);$$

$$K_4 = f\left(t_n + h, y_n + hK_3\right);$$

$$y_0 = \chi$$

So

$$f(t,i) = \beta S(t) I(t) - (\alpha + \gamma) I(t)$$
$$f(t,s) = -\beta S(t) I(t)$$
$$f(t,r) = \gamma I(t)$$

The solution of SIR Model

Because of different environments and regions, there are varies infection rate and recovery rate and mortality. We take Guinea as example.
so we can get that:

$$\begin{aligned} &\beta = 0.098, \alpha + \gamma = 0.027, N = 0.112 \times 10^8, \\ &I(0) = 86 / N = 86 / 0.112 \times 10^8, \\ &S(0) = 1 - S(0) = 1 - 86 / 0.112 \times 10^8 \end{aligned} \tag{9}$$

Then we can predict the amount of infective in the future.

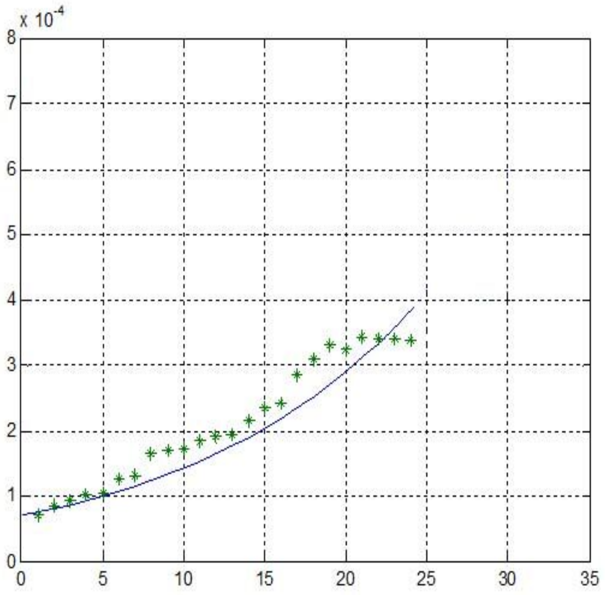

Figure 5. the amount of infective in the future in Guinea

Reference

[1] Qiyuan Jiang,JinXing Xie,Jun Ye.Mathematical Model[M].Beijing Higher Education Press,2003.