Prediction of velocity distribution of laminar circular jet with an EMMS model

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Abstract. The energy minimization multi-scale model is applied to the laminar circular jet. The shear dissipation tends to minimum, which can be used as the stability condition of laminar circular jet. The stability condition is adopted to predict the velocity distribution of laminar circular jet. Studies showed that the velocity distribution index of the laminar circular jet is 2.1, which is difference from the traditional Gaussian distribution.

Introduction

EMMS (Energy-minimization Multi-scale) model is a mechanism model, which is widely used in gas-solid two-phase flow, and has achieved great success [1-13]. Li Jinghai eatl. found that the coordination mechanism between the inertial and viscous effect in single-phase turbulence is very similar to that in a gas-solid system. Therefore, the EMMS model can be extended to turbulence field and the turbulence stability conditions is proposed. In this model, the shear viscous dissipation rate tends toward minimum and the total dissipation tends to maximum at the same time. This model predicted the turbulent pipe flow velocity distribution successfully [14, 15].

Jet flow is a common flow besides pipe flow. However, the EMMS model has not yet been applied to the study of jet flow. In this paper, we try to extend the energy minimum scale model to the research of circular jet. By using the stability condition of the circular jet, the velocity distribution of laminar and turbulent circular jet is predicted.

Assumed velocity distribution in a circular jet

In the fully developed region of the circular jet flow in the main section, the velocity distribution of each section is similar [16].

$$\frac{u}{u_m} = f\left(\frac{r}{b_e}\right) \tag{1}$$

where b_e is the characteristic half thickness of the jet cross section, which is shown in Fig.1.

Based on the experimental results and the random nature of the turbulence, the assumption of reasonable velocity distribution:

$$u = u_m e^{-\frac{r^n}{b_e^n}} \tag{2}$$

where n is a variable to be determined.

The basic characteristics of circular jet

Kinetic energy of arbitrary cross section. Kinetic energy

$$E = \int_0^\infty 2\pi r \rho u.\frac{u^2}{2} dr = \rho\pi b_e^2 u_m{}^3 \int_0^\infty \eta e^{-3\eta^n} d\eta = \rho\pi b_e^2 u_m{}^3 e(n) \tag{3}$$

where $e(n) = \int_0^\infty e^{-3\eta^n} \eta d\eta$, is a function of the variable n.

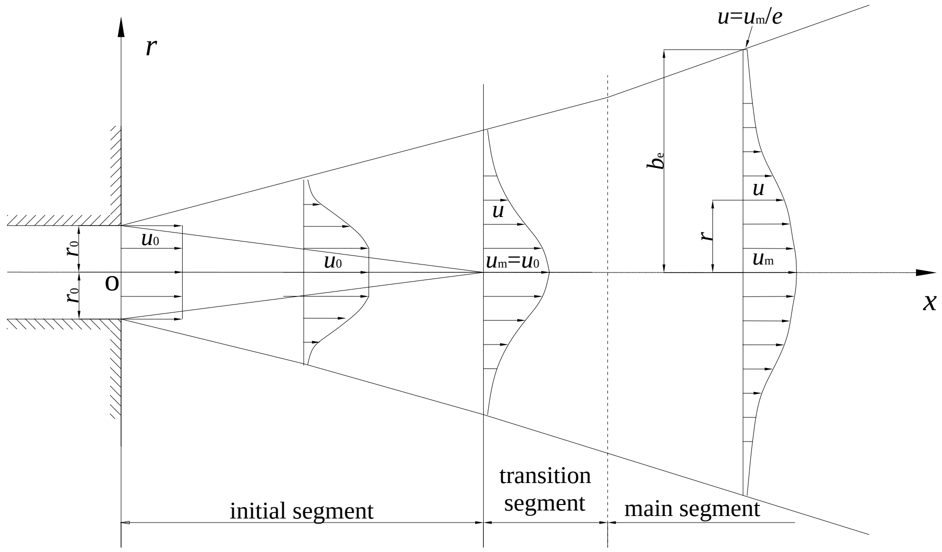

Fig.1.Main section of circular jet

The change of kinetic energy along the flow direction

$$\frac{dE}{dx} = \rho\pi e(n)\frac{d(b_e^2 u_m{}^3)}{dx} \tag{4}$$

Momentum of arbitrary cross section. Jet momentum

$$M = 2\pi\int_0^\infty \rho r u^2 dr = 2\rho\pi b_e^2 u_m{}^2 \int_0^\infty \eta e^{-2\eta^n} d\eta = 2\rho\pi b_e^2 u_m{}^2 m(n) \tag{5}$$

where $m(n) = \int_0^\infty e^{-2\eta^n}\eta d\eta$, is a function of the variable *n*.

The momentum of each section along the jet is equal to the momentum of the exit $M_0 = \frac{\pi}{4}\rho D^2 u_0^2$ where *D* is the diameter of circular jet and u_0 is the jet exit velocity.

Therefore

$$8b_e^2 u_m{}^2 m(n) = D^2 u_0^2 \tag{6}$$

Flux of arbitrary cross section. Jet flux

$$Q = \int_0^\infty 2\pi r\rho u dr = 2\rho\pi b_e^2 u_m \int_0^\infty \eta e^{-\eta^2} d\eta = 2\rho\pi b_e^2 u_m q(n) \tag{7}$$

where $q(n) = \int_0^\infty e^{-\eta^n}\eta d\eta$, is a function of the variable *n*.

The change of flux along the flow direction

$$\frac{dQ}{dx} = -2\rho\pi q(n) b_e^2 \frac{du_m}{dx} \tag{8}$$

It can be expressed as

$$dQ = 2dx\rho\pi b_e \alpha u_m \tag{9}$$

where α is entrainment factor.

$$\alpha = \frac{-2\rho\pi q(n) b_e^2 \frac{du_m}{dx}}{2\rho\pi b_e u_m} = -q(n)\frac{b_e}{u_m}\frac{du_m}{dx} \tag{10}$$

The relationship among parameters. For a circular jet [16],

$$\frac{u_m}{u_0} = a\frac{D}{x} \quad (11)$$

$$b_e = \varepsilon x \quad (12)$$

where a and ε are constants.

According to the Equation (7), the relationship between these constants is

$$a\varepsilon = \frac{1}{\sqrt{8m(n)}} \quad (13)$$

According to the Equation (10), entrainment factor can be expressed as:

$$\alpha = q(n)\varepsilon \quad (14)$$

Stability analysis of laminar circular jet flow

In a circular jet, the local shear dissipation of unit volume can be expressed as [17]

$$W_v(y) = \mu\left(\frac{du}{dy}\right)^2 \quad (15)$$

Its integral over the whole jet section

$$\overline{W}_v = 2\pi\int_0^\infty rW_v dr = 2\pi n^2 \mu u_m{}^2 \int_0^\infty \eta^{2n-1} e^{-2\eta^n} d\eta = \frac{n\pi\mu u_m{}^2}{2} \quad (16)$$

According to the stability condition of the laminar flow [18]

$$\overline{W_v} \to \min \quad (17)$$

From Equations (11), (13), (14) and (16), Equation (17) can be expressed as:

$$\frac{\pi\mu D^2 u_0^2}{16x^2\alpha^2}\frac{nq^2(n)}{m(n)} \to \min \quad (18)$$

If α is a constant, the stability condition of laminar circular jet can be expressed as:

$$W(n) = \frac{nq^2(n)}{m(n)} \to \min \quad (19)$$

Results analysis and discussion

In order to obtain the velocity distribution of laminar circular jet, the Equation (19) can be used as the optimizing condition. According to Equation (19), the value of W is dependent on the value of n, which is shown in Fig.2.

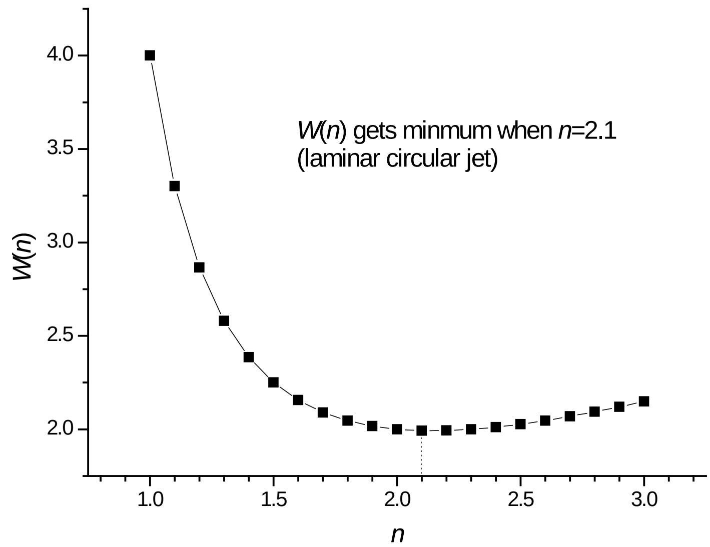

Fig.2. The relationship between of $W(n)$ and n in laminar circular jet

From Fig.2, with the increment of *n*, *W*(*n*) first decreases and then increases. When *n* equal 2.1, *W*(*n*) obtains minimum. It is obvious that *n*=2 will lead to recognized Gaussian distribution. Therefore, this result is different from Gaussian distribution [16, 19].

Xu *eatl*. found that the velocity distribution is different from the Gaussian normal distribution when the Reynolds number is small through the experiment. In order to analyze the results of the EMMS model and the Gauss distribution, we can make the corresponding velocity distribution curve, as shown in Fig.3. From the Fig.3, the EMMS model shows better agreement with the experimental results (Fig. 4 in Ref [19])than Gauss distribution.

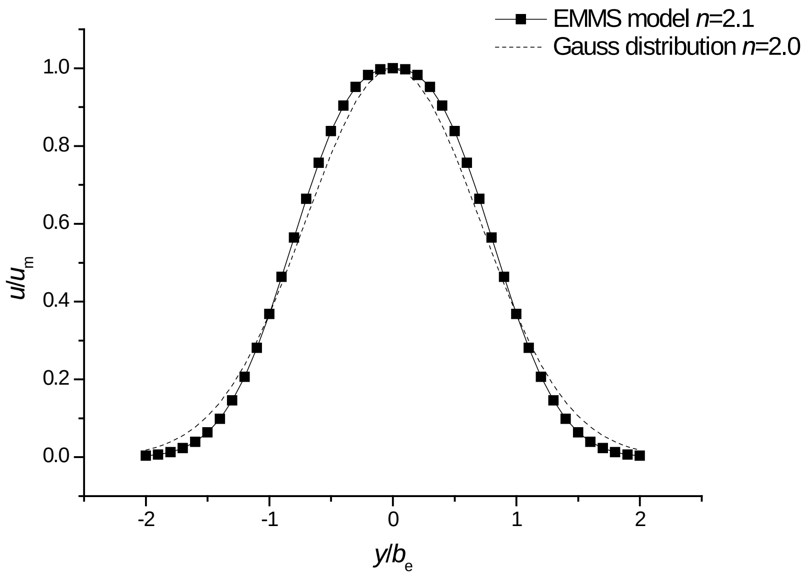

Fig.3. Comparison of velocity distribution by an EMMS model and Gauss distribution in laminar circular jet

Summary

In this paper, the EMMS model is applied to the research of circular jet. The stability condition is used to predict the velocity distribution of the laminar circular jet. The results show that the velocity distribution index of the laminar circular jet is 2.1. The conclusion is different from the traditional Gauss distribution function, but is more consistent with the experimental results.

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