An improved ICM method based on sine-type filter-polish function for topology optimization with frequency constraints

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Abstract. To prevent the numerical instabilities about checkerboards, mesh-dependency and localized eigenmodes in structural topology optimization with frequency constraints , an improved ICM method based on the new sine-type filter-polish function is presented. Moreover, sensitivity filter of difference of the strain energy and the kinetic energy of *ith* element corresponding to the *jth* eigenmode is adopted in dynamic optimization process. The sequential quadratic approximate algorithm is applied to solve the optimal model for continuum structure established in this paper.

Introduction

Continuum structure is widely used in practical engineering structure. In order to avoid the occurrence of resonance, dynamic damage and failure, the design considering frequency constraints must be considered. The continuum topology optimization with frequency constraints is one of important work in the engineering design.

In recent years, the study on dynamic topology optimization is relatively few since it is more complicated comparing with static topology optimization. Frequency topology optimization is of great importance in dynamic topology optimization and engineering fields. Topology Optimization with respect to frequencies of structural vibration was first considered by Diaz and Kikuchi[9], who studied the topology optimization of eigenvalues by using the homogenization method where reinforcement of a structure is optimized to maximize eigenvalues. In 2007 , Du and Olhoff [2] introduced SIMP method for maximization of first eigenvalue and frequency gaps. Huang et al.[3] investigated the maximization of fundamental frequency of beam, plane and three-dimensional block by applying a new bi-directional evolutionary structural optimization (BESO) method, and dealt with localized modes by modifying the traditional penalization function of SIMP method. Qi et al.[4] presented a level set based shape and topology optimization method for maximizing the simple or repeated first eigenvalues of structure vibration. The topology optimization models of continuum structure that minimizes weight and subjects to static displacement and frequency or stress constraints are also established by the ICM method [5,6].

The ICM method abstracts the topology variables from low-level design variables such as the cross-section, thickness and density, etc., so as to be independent of the traditional physical quantities and use continuous variables in [0,1] instead of 0 and 1 binary discrete variables to avoid the difficulties of discrete optimization. In this paper, a new sine-type filter-polish function is presented as filter function of elemental weight and stiffness so as to weaken the low-topological-variable elements while enhancing the high-topological-variable elements by polarization,and then the topological optimization model for the continuum structure is constructed which has the minimized weight objective and frequency constraints . The sequential quadratic programming is adopted to solve it. Some numerical problems, such as the checker board patterns, the mesh dependence, the local modal and modal switch problems ,are eliminated .

The new sine-type filter-polish function

Filter function is the core of the ICM method which is used to identify element solid or void to represent the nonlinear dependency between independent continuous topological variables $t_i \in [0,1]$ and material properties. The definition and selection of the filter function largely decides the establishment and solving of the optimization model and will affect the final topology optimization results. Denote $f_w(t_i)$, $f_k(t_i)$ and $f_m(t_i)$ as filter functions for frequency topology optimization and they are given as follows:

$$w_i = f_w(t_i)w_i^0, \mathbf{k}_i = f_k(t_i)\mathbf{k}_i^0, \mathbf{m}_i = f_m(t_i)\mathbf{m}_i^0 \quad (1)$$

where w_i^0, $\boldsymbol{k}_i^0$ and $\boldsymbol{m}_i^0$ are the element weight, element stiffness matrix and element mass matrix of original structure before the process of topology optimization, respectively. w_i, $\boldsymbol{k}_i$ and $\boldsymbol{m}_i$ are the ones in the process of topology optimization, respectively.

In the classical ICM method, Sui Yun-kang [7] suggested three kinds of simple filter functions as following:

(a) Power function $f(t) = t^a$, $a = 3$ (2)

(b) Modified Sigmoid function $f(t) = \dfrac{1}{22}\ln\dfrac{1+t/b}{1-t/b}$, $b = 1$ (3)

(c) Composite exponential function $f(t) = (e^{t/g} - 1)/(e^{1/g} - 1)$, $g = 0.0621$. (4)

In fact, the above three kinds of simple functions (2),(3) and (4) could push the intermediate densities towards its binary bounds (0/1) and recover the original 0 and 1 discrete material distribution (see Fig.1). Fig.1 shows that during the filtering or identifying in the ICM method, most of the intermediate topological variables t_i tend to 0/1 .

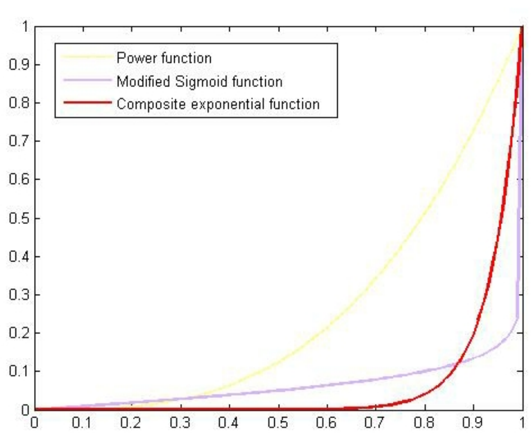

Fig. 1 Three kinds of simple filter functions in the classical ICM method

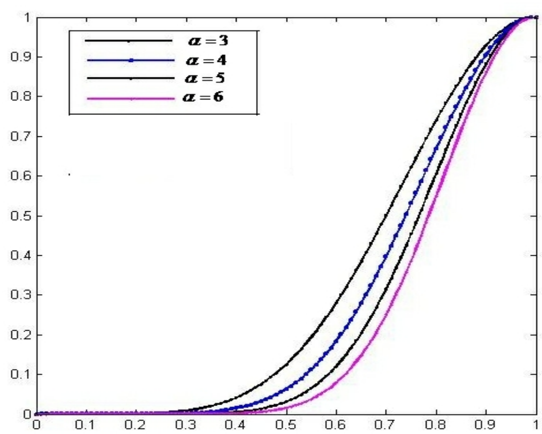

Fig.2 The filter function $f(t)$, $a = 3,4,5,6$, $D = 0.5$.

After many times of iteration during the filtering or identifying in the ICM method, Fig.1 shows that nearly all these topological variables t_i corresponding to the *i-th* element will quickly tend to 0/1 , but there are still a few intermediate-topological variables which lead to no clear boundaries. Since the intermediate-topological variables is inevitably to produce many gray-scale elements in the final optimal topology after the filtering or identifying, a new sine-type filter-polish function $f(t)$ is presented as follows (see Fig.2) :

$$s(x)=\begin{cases} y=\frac{1}{2}+\frac{1}{2}\sin\frac{p}{2}(\frac{x}{D}-1), 0\le x<D \\ \frac{1}{2}, \qquad\qquad x=D \\ y=\frac{1}{2}+\frac{1}{2}\sin\frac{p}{2}(\frac{x-D}{1-D}), D<x\le 1 \end{cases} \quad (5)$$

and

$$f(t)=s^{a}(t) \ \ (a>0) \quad (6)$$

As indicated in Fig.2, the proposed new sine-type filter-polish function $f(t)$ can polarize the majority of elements to 0 or 1.0 more reasonable at the same probability. So the number of elements with the intermediate-topological variables in the final topology will be sharply reduced, so as to get a topological design with distinct boundaries.

The Formulation and solution based on the improved ICM method

In this paper, we denote f_i as the frequency of *i-th* order, and $\underline{f_i}, \overline{f_i}$ are the low and up bound of frequency respectively. The usual relation between frequency f and eigenvalue is $l=(2pf)^2$ in the elastic structure, and then the frequency constraints can apparently be transformed into eigenvalue constraints based on above the $l=(2pf)^2$.

If we set $f_w(t)=f_m(t)=s^3(t)$ and $f_k(t)=s^6(t)$ as the filter function of the element weight, element mass matrix and element stiffness matrix, respectively, then the model of continuum topology optimization with frequency constraints can be formulated as follows

$$\begin{cases} \text{find} \quad \mathbf{t}=(t_1,t_2,\mathbf{L},t_N) \\ \text{make} \ W=\sum_{i=1}^{N} f_w(t_i)w_i^0 \to \min \\ \text{s.t.} \quad \underline{l}_j \le g(l_j(f_k(t_i),f_m(t_i))) \le \overline{l}_j (j=1,\mathbf{L},J) \\ \qquad 0\le t_i \le 1 \ \ (i=1,\mathbf{L},N) \end{cases} \quad (7)$$

where $\boldsymbol{t}$ and W denote the topological design variable vector and the weight of structure, $\boldsymbol{i}$ and j are the ***i-th*** element and the ***j-th*** order frequency respectively, J and N represent the total number of constraints and elements.

In the finite element analysis, the dynamic behavior of a continuum structure can be represented by the following general eigenvalue problem

$$(\mathbf{K}-l_j\mathbf{M})\mathbf{u}_j=0 \quad (8)$$

where, $\boldsymbol{K}$ is the global stiffness matrix and $\boldsymbol{M}$ is the global mass matrix. λ_j is the ***j-th*** eigenvalue and $\boldsymbol{u}_j$ is the eigenvector corresponding to λ_j. The eigenvalue λ_j and the corresponding eigenvector $\boldsymbol{u}_j$ are related to each other by Rayleigh quotient

$$l_j=\frac{\mathbf{u}_i^T\mathbf{K}\mathbf{u}_i}{\mathbf{u}_i^T\mathbf{M}\mathbf{u}_i} \quad (9)$$

Since eigenvalue λ_j is implicitly related with topology variable $\boldsymbol{t}$, we use first-order Taylor series expansion for eigenvalue to express their relationship explicitly as follows[16,17]:

$$\sum_{i=1}^{N} c_{ij}x_i \le d_j \quad (10)$$

Take the reciprocal of stiffness filter function as design variables as follows[16,17]

$$x_i=\frac{1}{f_k(t_i)} \quad (11)$$

then we have $$t_i = f_k^{-1}(x_i) \quad (12)$$

Therefore, the stiffness matrix filter function, mass matrix filter function and weight filter function are given as

$$f_k(t_i) = \frac{1}{x_i} \ ; \ f_m(t_i) = f_m[f_k^{-1}(\frac{1}{x_i})] = F(x_i) \ ; \ f_w(t_i) = f_w[f_k^{-1}(\frac{1}{x_i})] = F(x_i) \quad (13)$$

The global stiffness matrix $\boldsymbol{K}$ and the mass matrix $\boldsymbol{M}$ can be calculated by

$$\mathbf{K} = \sum_{i=1}^{N} \mathbf{k}_i = \sum_{i=1}^{N} f_k(t_i)\mathbf{k}_i = \sum_{i=1}^{N} \frac{1}{x_i}\mathbf{k}_i^0 \quad (14)$$

$$\boldsymbol{M} = \sum_{i=1}^{N} \boldsymbol{m}_i = \sum_{i=1}^{N} f_m(t_i)\boldsymbol{m}_i^0 = \sum_{i=1}^{N} F(x_i)\boldsymbol{m}_i^0 \quad (15)$$

$$W = \sum_{i=1}^{N} w_i = \sum_{i=1}^{N} f_m(t_i) w_i^0 = \sum_{i=1}^{N} F(x_i) w_i^0 \quad (16)$$

However, model (7) is a programming with nonlinear objective and linear constraints following the explicit process of constraints. We use second-order Taylor series expansion to approximate the objective function and ignore the constant item. Let $F(x_i) = [s_k^{-1}(\frac{1}{x_i})]$,

$a_i = \frac{1}{2} w_i^0 F''(x_i^{(v)})$, $b_i = w_i^0[F'(x_i^{(v)}) - x_i^{(v)} F''(x_i^{(v)})]$,then the model (7) is transformed into the following quadratic programming model:

$$\begin{cases} find \quad \mathbf{t} = (t_1, t_2, \mathbf{L}, t_N) \\ \min \quad W = \sum_{i=1}^{N} (a_i x_i^2 + b_i x_i) \\ s.t. \quad \sum_{i=1}^{N} c_{ij} x_i \le d_j (j = 1, \mathbf{L}, J) \\ \quad 1 \le x_i \le \overline{x_i} \ (i = 1, \mathbf{L}, N) \end{cases} \quad (17)$$

The sequential quadratic approximate algorithm is applied based on exact dual mapping to solve the optimal model for continuum structure established in this paper.

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