Synchronization of Two Four-wing Fractional-Order Chaotic Systems and Its Applications in Secure Communication

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Abstract. This paper discusses the issues related to chaos synchronization between two four-wing fractional-order chaotic systems with identical structure by using state observers. The effectiveness and feasibility of the proposed synchronization scheme are verified by numerical simulations and experimental records, on the basis of which, a synchronization fractional-order circuit for illustrating the application in secure communication is obtained and analyzed.

Introduction

Chaos is a special form of motion of nonlinear dynamical systems, which has been widely studied in the past few decades [1,2]. Chaotic signal is very hard to predict. How to use and control the chaotic signal, and make it generate better application values, which would be a challenge that many researchers are facing with [3,4]. Therefore, synchronous control, which generally is used in the study on secure communication, has attracted increasing interest from application point of view [5].

Since PC chaotic synchronization method was proposed in 1990, the synchronization of chaos has aroused extensive interest of scholars, and subsequently many synchronization methods have been represented, such as drive-response synchronization [6], active-passive synchronous [7], coupling synchronization [8], feedback synchronization [9], and so on [10,11]. Chaos synchronization can be applied in many areas, such as secure communication [13], spread spectrum communication, information compression and storage [5,14] etc.

With the rapid development of digital and network technology, the significance of information security has been important in the past few decades. Intrinsic properties of chaotic systems such as sensitivity to initial conditions, and a wide spectrum bandwidth of system signal, it makes the chaotic system have an outstanding advantages and important application prospect in the field of secure communication. Especially the fractional order systems are more common in the field of secure communication. So it has important theoretical significance and practical value to investigate the application of fractional order chaotic system in secure communication field.

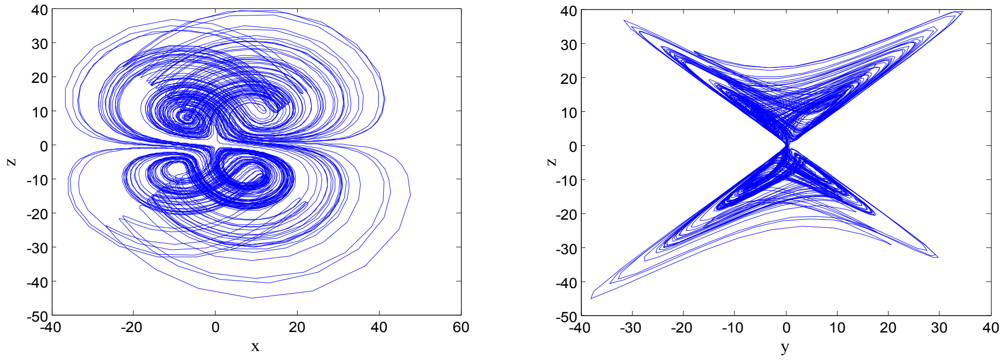
Based on the observer, a synchronization control of two fractional-order four-wing chaotic systems with identical structure is discussed in this paper, and the numerical simulation results verify the effectiveness of the synchronization method. In addition, an analog circuit is designed to implement the synchronization of two fractional-order four-wing chaotic systems. The results from circuit experiments also show the synchronization method is feasible. Based on the above the fractional-order synchronization circuit, a secure communication circuit is also designed to study secure communication. All of the results show that the synchronization method and its application for secure communication are practical.

Four-Wing Fractional-Order Chaotic System

A four-wing fractional-order system is described as

$$\begin{cases} \dfrac{d^{\alpha} x_1}{dt^{\alpha}} = a x_1 + k y_1 - y_1 z_1 \\ \dfrac{d^{\beta} y_1}{dt^{\beta}} = -b y_1 - z_1 + x_1 z_1 \\ \dfrac{d^{\gamma} z_1}{dt^{\gamma}} = -x_1 - c z_1 + x_1 y_1 \end{cases} \tag{1}$$

Where $a \in R$, $b \in R$, $c \in R$, and $k \in R$ are called as system parameters; $0 < \alpha < 1$, $0 < \beta < 1$ and $0 < \gamma < 1$ refer to the fractional order. In this paper, we will mainly discuss the condition of the fractional order $\alpha = \beta = \gamma = 0.9$. It is found that the system has a four-wing chaotic attractor when $a = 5$, $b = 12$, $c = 5$, and $k = 1$, as shown in Fig.1. Given this, the system (1) can be used to realize the synchronization of two four-wing fractional-order chaotic systems with identical structure, and the four-wing chaotic attractors will be adopted to investigate its applications in secure communication.



(a) (b)

Fig.1 Chaotic attractors (a) $x_1 - z_1$ plane (b) $y_1 - z_1$ plane

Synchronization design and Circuit implementation

It is well known that chaos synchronization is the foundation of the application of chaotic system in secure communication. So the synchronization of four-wing fractional-order chaotic system is firstly investigated and will be adopted to implement its application in secure communication. Here, the system (1) can be used as a driving system. And according to the observer idea, the corresponding response system is described as follows

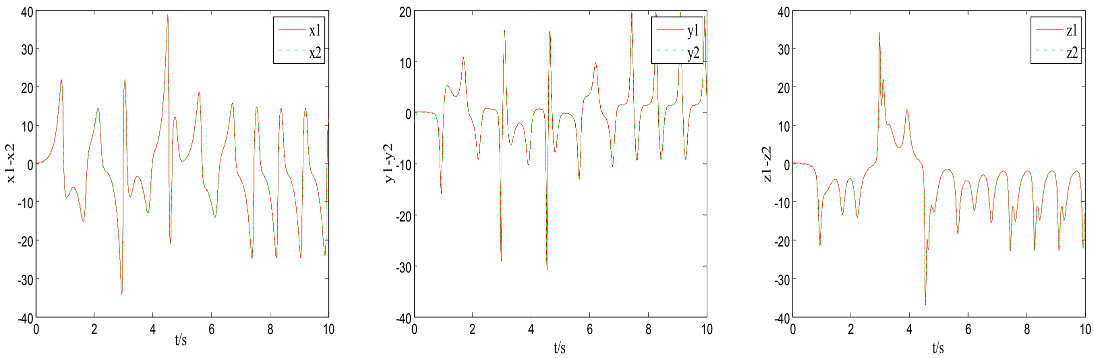
$$\begin{cases} \dfrac{d^{\alpha} x_2}{dt^{\alpha}} = a x_2 + k y_2 - y_2 z_2 + u_1 \\ \dfrac{d^{\beta} y_2}{dt^{\beta}} = -b y_2 - z_2 + x_2 z_2 + u_2 \\ \dfrac{d^{\gamma} z_2}{dt^{\gamma}} = -x_2 - c z_2 + x_2 y_2 + u_3 \end{cases} \tag{2}$$

Where u_1 , u_2 and u_3 are the control input to realize the synchronization of system (1) and system (2). According to the observer theory and the stability theory of fractional order linear systems, selecting

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} -y_1 z_1 \\ x_1 z_1 \\ x_1 y_1 \end{bmatrix} - \begin{bmatrix} -y_2 z_2 \\ x_2 z_2 \\ x_2 y_2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{bmatrix} \tag{3}$$

the system (2) will be synchronized with the system (1). The numerical simulation has been carried out by MATLAB, with the initial values of the state variables being $x_1(0)=0.1$, $y_1(0)=0.2$, $z_1(0)=0.3$, $x_2(0)=-1$, $y_2(0)=-2$, $z_2(0)=-3$, and time steps of 0.001. The simulation results are indicated in Fig.2 (a), (b), and (c), which are the time series of different state variables, respectively. The results from numerical analysis show that this synchronization method is practical.

Then, based on the frequency-domain approximate, we design an analog circuit for synchronization control to verify the correctness of the theoretical analysis, as shown in Fig.3. In the circuit, LF347N is selected as the amplifier, AD633(output gain is 0.1) is selected as the multiplier, and $R_1 = R_{15} = R_{19} = 2K\Omega$, $R_2 = R_8 = R_{14} = R_{20} = R_{26} = R_{32} = 200\Omega$, $R_3 = R_9 = R_{13} = R_{21} = R_{27} = R_{31} = R_{38} = 10K\Omega$, $R_4 = R_{10} = R_{16} = R_{22} = R_{28} = R_{34} = 1.55M\Omega$, $R_5 = R_{11} = R_{17} = R_{23} = R_{29} = R_{35} = 62M\Omega$, $R_6 = R_{12} = R_{18} = R_{24} = R_{30} = R_{36} = 2.5K\Omega$, $R_7 = R_{25} = 833.3\Omega$, $R_{33} = 1.67K\Omega$, $R_{37} = 1K\Omega$, $C_1 = C_4 = C_7 = C_{10} = C_{13} = C_{16} = 0.7\mu F$, $C_2 = C_5 = C_8 = C_{11} = C_{14} = C_{17} = 0.52\mu F$, $C_3 = C_6 = C_9 = C_{12} = C_{15} = C_{18} = 1.1\mu F$. The experimental results are only given the synchronous phase trajectory and timing sequence diagram on the $x_1 - x_2$, as shown in Fig.4, the rest is no longer listed. From the result, we can observe that the two chaotic systems are synchronized.



(a) Time series of x_1 and x_2 (b) Time series of y_1 and y_2 (c) Time series of z_1 and z_2

Fig.2 Time series of different state variables

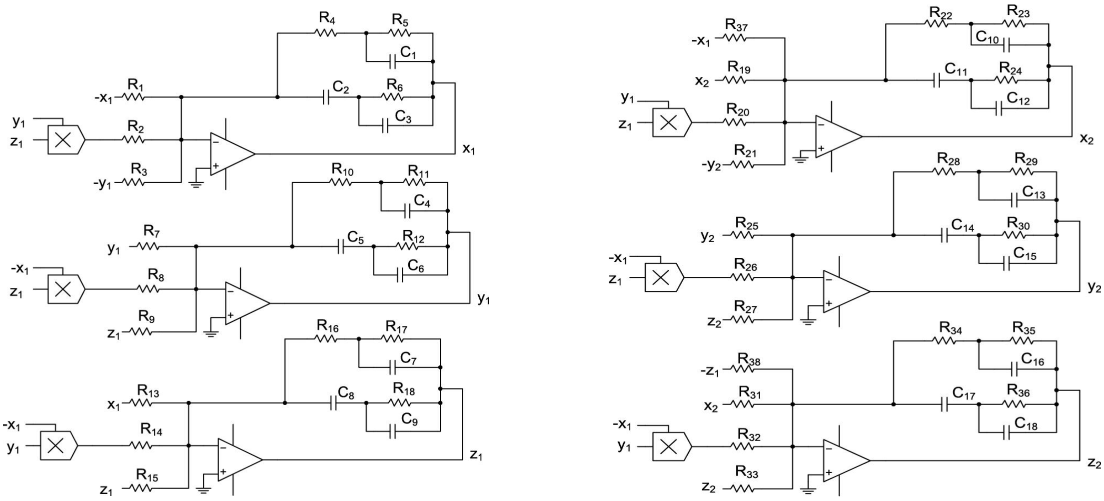
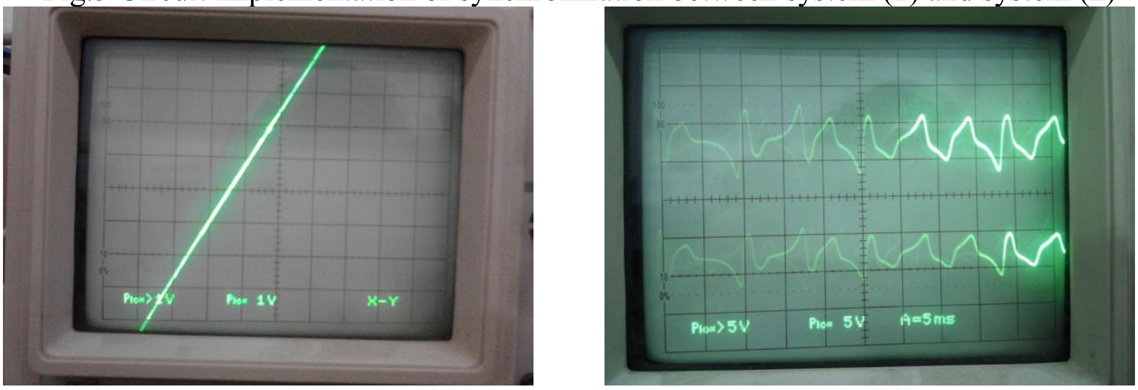


Fig.3 Circuit implementation of synchronization between system (1) and system (2)



(a) Time series of $x_1 - x_2$ (b) Phase synchronization of $x_1 - x_2$

Fig.4 Experimental result of synchronization from the oscilloscope

Secure Communication based on Chaotic Synchronization

The basic principle of secure communication system in this paper is basing on the synchronization between two four-wing fractional-order chaotic systems. That is, the signal to be transmitted is firstly superposed to the chaotic signal generated by the fractional-order chaotic system (1), subsequently send them to the receiver, the original signal is finally taken out by removing the corresponding synchronized signal of chaotic system (2). Therefore, the synchronization of the system is the key to the reliable and effective transmission of the information in the chaotic secure communication system [13]. In this paper, on the basis of the above analog circuit of synchronization, an analog circuit for secure communication is designed, as shown in Fig.5. Here, we select the state variable x_1 as the driving carrier signal, and a sine signal as original signal to be encrypted. Meanwhile the corresponding response signal x_2 is selected as the decrypt signal. The experimental results are shown in Fig.6, and the decryption signal can be observed to be exactly like the original signal. That is to say, the chaotic synchronization proposed in this paper can be used in secure communication.

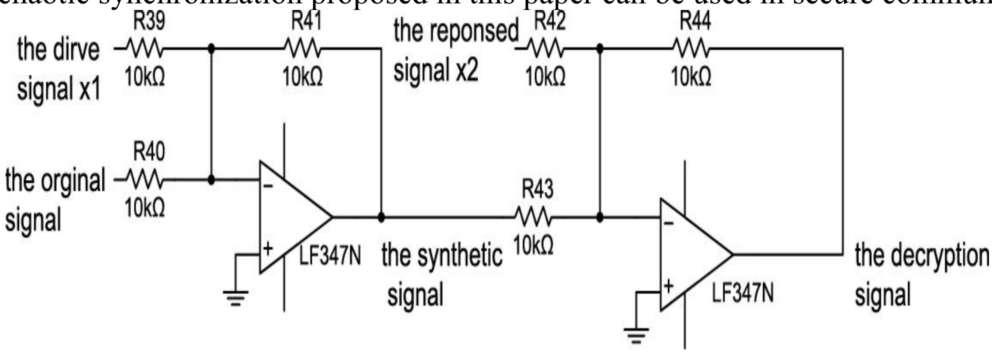
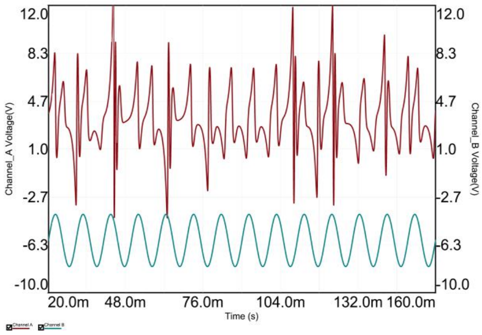
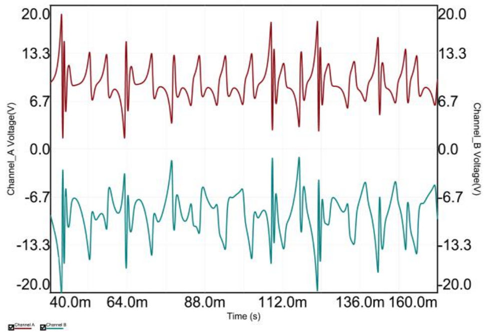


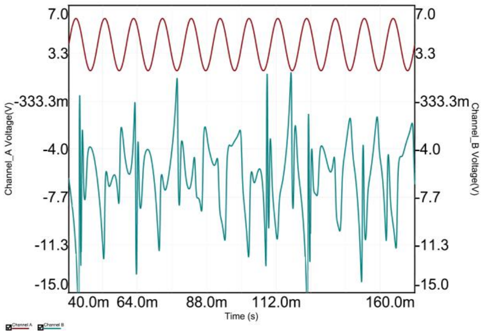
Fig.5 Analog circuit on secure communication



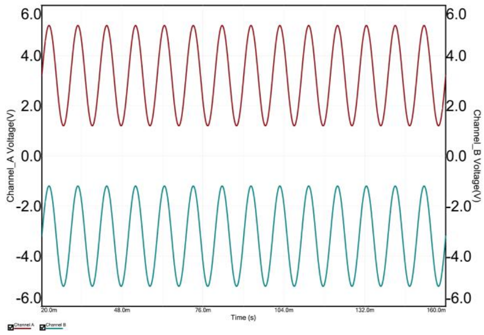
(a) The drive signal-The original signal



(b) The drive signal-The synthetic signal



(c) The original signal-The synthetic signal



(d) The original signal-The decryption signal

Fig.6 The experimental results of secure communication

Conclusions

In this paper, based on the control theory of observer, a chaotic synchronization of two four-wing fractional-order chaotic systems with identical structure is realized. The numerical simulation results are shown to verify the feasibility of this method. Meanwhile, an analog circuit is also designed to implement the synchronization control. At last, a kind of chaotic masking technology is used to investigate the chaotic secure communication based on the synchronization control, and the results from the circuit experiments show that the encryption method in this paper is also feasible. All of these provide a new model and technology support for the study on chaotic encryption.

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