

An algorithm for analyzing complex slabs of buildings

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ABSTRACT: An algorithm especially for analyzing complex slabs of buildings in finite element method is proposed in this paper, which can improve the stress precision and obtain accurate and reliable analysis results. First, a general thin-thick plate element with the quadrilateral area coordinate is employed, which avoids shear locking and distortion; Second, based on paving method, an algorithm for quadrilateral mesh is designed, which can achieve high-quality results and improve the stress precision significantly. This algorithm has been already implemented and integrated in Paco, a software for structural design, independently developed by Beijing Institute of Architectural Design. With this improvement, smooth stress contour of complex slabs can be obtained and the precision is guaranteed.

INTRODUCTION

In recent years, with the diversification of building forms, the shape of the building slabs is becoming increasingly complex. The traditional approximate methods are not suitable for analyzing complex slabs, so the finite element method is necessary. Currently, the following issues are the main topics of slab analysis using finite element method:

a) Element type.

Element type is the core part of the analysis of a complex slab. The selection of element type directly affects the accuracy and speed of analysis. There are several issues needs to consider about the type of elements, including thin or thick plate, shear locking problem, accuracy, convergence, adaptability of distortion, and so on. Therefore, appropriate element type should be selected in order to obtain satisfactory analysis results.

b) Meshing.

The meshing quality of slabs is another important part of the complex slab analysis. The distortion of elements can be significantly reduced with a good meshing procedure, so the meshing method is very important for the accuracy of the slab analysis.

c) Post-analysis process.

Compare to the traditional approximate methods, the results of finite element analysis are quite complicated. Therefore, well-designed post-analysis tools are required to display the results of slab analysis intuitively.

In this paper, an algorithm especially for analyzing complex slabs of buildings in finite element method is proposed, and solutions for the above three issues are also introduced.

ELEMENT TYPE

A general thin-thick plate element with the quadrilateral area coordinate TACQ (Cen & Long 1999) is employed in this algorithm for complex slab analysis, which avoids shear locking and distortion.

Quadrilateral area coordinates

The area coordinates (L_1, L_2, L_3, L_4) of a point P in one quadrilateral is defined as

$$L_i = A_i / A \quad (i = 1, 2, 3, 4) \quad (1)$$

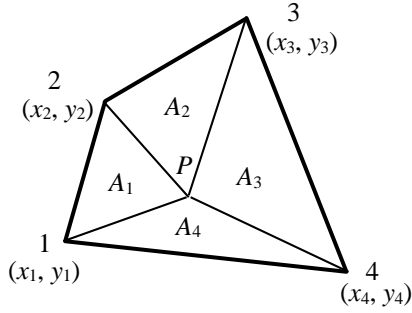


Figure 1. Definition of the quadrilateral area coordinate.

where A_1, A_2, A_3, A_4 are the areas of the 4 triangles composed by P and each side of the quadrilateral element, as shown in Figure 1.

The area coordinates can be represented by Cartesian coordinates in Equation 2:

$$L_i = \frac{1}{2A}(a_i + b_i x + c_i y) \quad (i = 1, 2, 3, 4) \quad (2)$$

where

$$\left. \begin{aligned} a_1 &= x_2 y_3 - x_3 y_2, & b_1 &= y_2 - y_3, & c_1 &= x_3 - x_2 \\ a_2 &= x_3 y_4 - x_4 y_3, & b_2 &= y_3 - y_4, & c_2 &= x_4 - x_3 \\ a_3 &= x_4 y_1 - x_1 y_4, & b_3 &= y_4 - y_1, & c_3 &= x_1 - x_4 \\ a_4 &= x_1 y_2 - x_2 y_1, & b_4 &= y_1 - y_2, & c_4 &= x_2 - x_1 \end{aligned} \right\} \quad (3)$$

Since it is a linear transform, the area coordinates and the Cartesian coordinates of points in a quadrilateral could be transformed to each other easily.

See Long et al. (1999) for detailed formula deductions and features of quadrilateral area coordinates.

TACQ

TACQ is a general thin-thick plate element with quadrilateral area coordinates. It is based on the reasonable interpolation of shear strain field and the generalized conforming theory (Long et al. 2009), with only 12 degrees of freedom.

TACQ have several significant advantages. Firstly, because the point and side-average generalized compatibility conditions are both satisfied, the convergence of this element is guaranteed. Secondly, thickness of plate is a divisor of the shear strain formula, so the shear strain will approach zero with the thickness is reduced, which means no shear locking problems. Furthermore, numerical results show that TACQ has satisfactory accuracy and good resistance of distortion, and it is suitable for a wide range from very thin to thick plate.

See Cen & Long (1999) for detailed formula deductions and numerical results of TACQ.

MESHING

Generally speaking, quadrilateral elements are more precise than triangles so that quadrilateral mesh is more popular in finite element analysis. Paving is a traditional and effective quadrilateral mesh generation technique, which is accomplished by layering or paving the geometry with rows of quadrilateral elements from the boundaries toward the interior. However, the quality of the elements generated by the original paving method is not very satisfying. Thus, in this paper, modifications and new techniques are employed to improve the mesh quality and reliability.

The original procedure of paving can be found in Blacker & Stephenson (1991) and the improvements are outlined as follows.

- Triangular elements are introduced, so the number of nodes on boundaries is not restricted to even. For example, if the fixed boundary has a sharp corner, one triangular element will be generated directly.
- Closure check is simplified. Especially for six-node boundary, various cases are classified as only three categories. Besides, odd-node boundaries can also be handled.

- c) The starting node of rows for paving is selected more intelligently to improve the quality. For example, if two adjacent nodes are row-end nodes, and the distance is much smaller than the average element size, the first node should be selected as the starting node.
- d) The tolerance of seaming is adjusted according to the optimal number of elements around the node. For example, there should be four quadrilateral elements for each node inside the mesh region and two elements for the node on the boundary line. If the number of elements after seaming is smaller than the standard value, the tolerance should be larger, and vice versa.
- e) Intersection check is redefined. Two edges close and parallel to each other but not actually crossing are also considered as intersection. Every two edges are marked and the ones with highest score will be merged. In general, the closer the two edges are, and the smaller the intersection angle is, the higher the score will be. Additionally, if the score of all edge pairs are low and there are two nodes on the interior boundary very close to each other, these two nodes can be merged instead of edges.
- f) The criteria of row adjustment is relaxed. If the size of boundary edges are much different from the expectation, row size should also be adjusted, no matter whether the rows are expanding or contracting.
- g) Several small changes are made to improve the mesh quality. For example, if seaming occurs on the fixed boundary and the fixed edge is much longer than the other one, a quadrilateral element can be generated directly instead of seaming.

Besides the modifications for paving method, another improvement is the topological cleanup after meshing. The main idea is to reduce the number of irregular nodes (the number of adjacent quadrilateral elements are not four). a) Redefine lots of standard patterns and corresponding cleanup procedures which reduce the irregular nodes; b) search the mesh and find the actual pattern matching one of the standard pattern; c) execute the predefined cleanup procedure. The key part of this method is to set up the library of standard patterns empirically. In this paper, about 30 standard patterns are found and can cover most cases. This procedure can improve the mesh quality significantly, as shown in Figure 2.

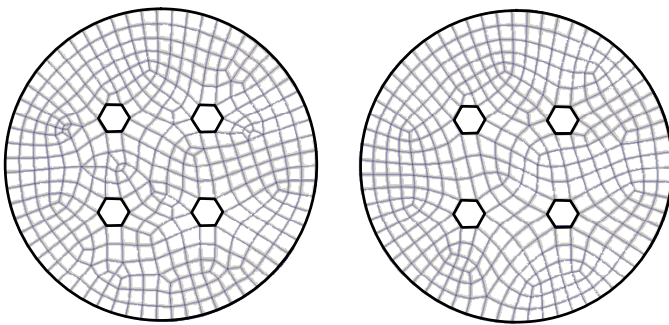


Figure 2. Paving mesh and topological cleanup

GETTING CONTOUR MAPS

Because finite element method is employed, analysis results of complex slabs are usually huge and unintuitive. There are two common approaches to display the results.

- a) Display the results of each mesh point nearby. This approach is suitable when the results of a specific mesh point or element are required.
- b) Create contour maps. Contour maps are very intuitive if global results are needed, such as positions of maximum and minimum value, regions of stress, etc.

The first approach is easy to implement. For the second approach, there is an outstanding issue for quadrilateral elements. Since only the results of element corners are outputted, the results of a point in the element are obtained by linear interpolation as a rule phrase. Unlike triangle elements, there are 4 corners in each quadrilateral element, so the contour map is uncertain. Therefore, a contour map determination method for quadrilateral elements is necessary. In this paper, a simple divi-

sion method is applied. The main process of this method is as follows by taking the stress results as an example:

a) Find the point $M(x_M, y_M)$ use Equation 4:

$$x_M = \frac{x_1 + x_2 + x_3 + x_4}{4}, \quad y_M = \frac{y_1 + y_2 + y_3 + y_4}{4} \quad (4)$$

where (x_1, y_1) , (x_2, y_2) , (x_3, y_3) , and (x_4, y_4) are the Cartesian coordinates of the 4 corners of the quadrilateral. Because the quadrilateral is convex by using the paving method above, it is able to prove that the point M is inside the quadrilateral.

b) Assume the stress field is bilinear in the quadrilateral. Therefore, the stress of the point M σ_M can be calculated by using Equation 5:

$$s_M = \frac{s_1 + s_2 + s_3 + s_4}{4} \quad (5)$$

where $\sigma_1, \sigma_2, \sigma_3$, and σ_4 are the stresses of the 4 corners of the quadrilateral.

c) Use the point M to divide the quadrilateral to 4 triangles $(1, 2, M)$, $(2, 3, M)$, $(3, 4, M)$, and $(4, 1, M)$.

d) Draw the contour maps for each triangle since the contour maps of a triangle is certain under the linear hypothesis.

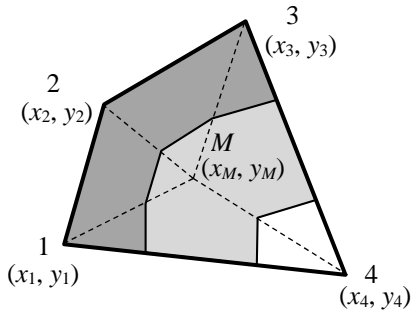


Figure 3. Method to divide a quadrilateral element for contour map.

TESTS AND EXAMPLES

To prove the accuracy and adaptability of the proposed algorithm, numerical tests are performed.

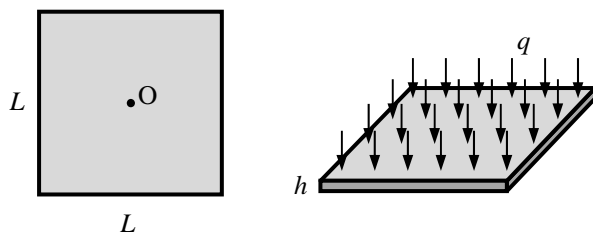


Figure 4. Definitions of the Square slabs for tests.

A set of simply-supported square slabs under a uniformly distributed load with different thickness span ratios are built, as shown in Figure 4. Denote L as the span of the slab, h as the thickness of the slab, q as the uniformly distributed load size, E as the elastic modulus of the material. The results of the center deflections and bending moment with different thickness-span ratios and mesh numbers are compared between numerical results of Paco and analytical solutions in Table 1 (All the results have been nondimensionalized).

Table 1a. Center deflections of the slabs
($\times 10^3 Eh^3/12(1-\mu^2) qL^4$).

Thickness	Mesh number				Analytical
span ratio	4×4	8×8	16×16	32×32	solution
0.001	4.045	4.060	4.062	4.062	4.062
0.01	4.047	4.062	4.063	4.064	4.064
0.10	4.228	4.255	4.267	4.271	4.273
0.15	4.484	4.518	4.531	4.535	4.536
0.20	4.857	4.889	4.900	4.903	4.906
0.25	5.346	5.367	5.375	5.377	5.379
0.30	5.949	5.953	5.956	5.956	5.956
0.35	6.666	6.646	6.642	6.641	6.641

Table 1b. Center bending moments of the slabs
($\times 10^2/qL^2$).

Thickness	Mesh number				Analytical
span ratio	4×4	8×8	16×16	32×32	solution
0.001	5.008	4.839	4.801	4.792	4.789
0.01	5.012	4.842	4.804	4.794	4.789
0.10	5.223	4.941	4.834	4.801	4.789
0.15	5.344	4.962	4.836	4.801	4.789
0.20	5.429	4.972	4.837	4.801	4.789
0.25	5.485	4.978	4.837	4.801	4.789
0.30	5.523	4.981	4.837	4.801	4.789
0.35	5.549	4.983	4.838	4.801	4.789

As shown in the table, whether for thick or thin slab, the accuracy of the proposed algorithm is satisfactory with proper mesh number, and there is no shear locking phenomenon.

Then the slabs of a real 5-story reinforced concrete frame structure model are taken as an example, as shown in Figure 5. There are 1091 beams, 347 columns, and 441 elastic slabs in the whole models. All the modeling and computing work was performed by Paco software.

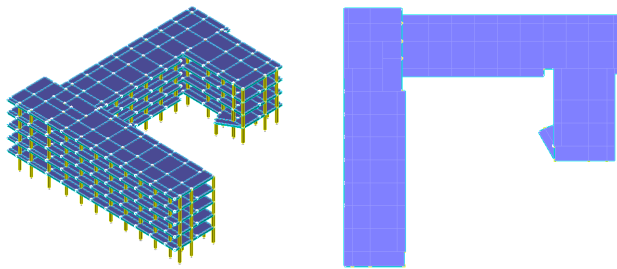


Figure 5. Model for example

The mesh results of the 4th story slabs are shown in Figure 6. The mesh size is 1000mm. It is observed that the quadrilateral elements are quite uniform and approximate rectangle-shaped with the proposed meshing method.

The contour map of the bending moments of the 4th story slabs is shown in Figure 7. With the proposed post-analysis method, the smooth contour map can be obtained.

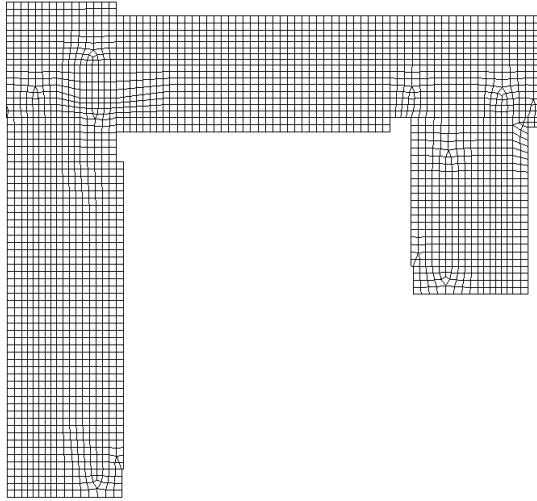


Figure 6. Mesh results of the 4th story slabs of the example model

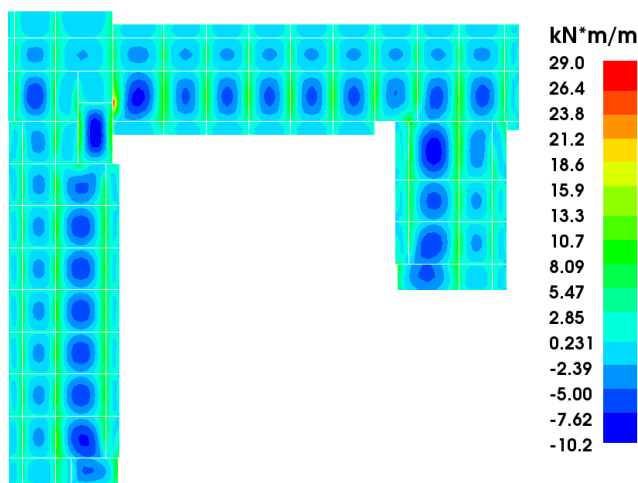


Figure 7. Contour map of the X-Axis bending moments of the 4th story slabs of the example model under dead load.

The deviation between the maximum center bending moment with mesh size 1000mm and the finite element convergence solution (with element size 250mm) is less than 1%, which means that the adaptability of the proposed algorithm is satisfactory for practical application.

CONCLUSION

In this paper, an algorithm especially for analyzing complex slabs of buildings in finite element method is proposed. The advantages of this method are shown as follow:

- a) This method has good resistance of distortion, and shear locking phenomenon can be avoided.
- b) The quality of the generated quadrilateral mesh of the slabs is quite high.
- c) Contour maps of quadrilateral elements can be displayed smoothly.

This algorithm has been already implemented and integrated in Paco, a software for structural design, independently developed by Beijing Institute of Architectural Design, and satisfactory results has been obtained with this improvement.

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