Fractal study on salinity distribution in the North Branch of Yangtze Estuary

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KEYWORD: North Branch of Yangtze Estuary; Fractal theory; Salinity intrusion; Salt-fresh water mixing.

Abstract: In this paper, salinity characteristics in North Branch of Yangtze Estuary is studied based on Fractal Theory by analyzing the observed saline data. The inner rules of salt water and fresh water mixing system can be expressed by fractal characteristic values (fractal dimension *D* and fractal proportionality constant A). Larger fractal dimension *D* means more homogeneous saline vertical distribution and salt-fresh mixing degree is higher. Fractal proportionality constant *A* and salinity intrusion intensity have positive correlation. The following conclusions can be drawn. The salt-fresh water mixing degree and salinity intrusion intensity decrease from outside to inside in North Branch of Yangtze Estuary. The curve graph of daily salinity is similar with the tidal level. The maximum of salinity occurs near tidal flood and the minimum occurs near the tidal ebb. The mixture degree during the spring tide is larger than that during the neap tide. The salinity outside the estuary doesn't change much during one spring-neap tide cycle. The saltwater intrusion variation during spring-neap tide cycle is increasingly obvious from outside to inside in North Branch of Yangtze River.

GENERAL INSTRUCTIONS

Yangtze Estuary is the biggest estuary in China. There are three bifurcations and four outlets into the sea. Chongming Island divides the Yangtze Estuary into North Branch and So

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th Branch. Changxing Island and Hengsha Island divide South Branch into North Channel and South Channel. Jiuduan Shoal divides South Channel into North Passage and South Passage (Figure 1).

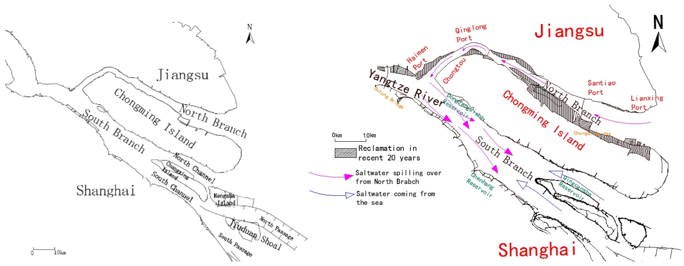


Figure 1. Map of Yangtze Estuary and salinity intrusion route

The amount of water in Yangtze River is sufficient and the running water has great self-purification ability. However salinity intrusion often happens in dry seasons. The water source is affected by both saltwater coming directly from sea and spilling over from North Branch (Figure 1). The condition of salinity intrusion in North Branch is most severe in four passages. Severe salinity intrusion will happen during the spring tide in dry season and North Branch will be fully occupied by highly salty water. The saltwater may even spill over and running into South Branch. As a consequence, recognizing the characteristic of salinity intrusion in North Branch and mechanism of saltwater spilling is significant for maintaining the security of water source in Yangtze Estuary. A simple way uses the ratio of runoff volume and tidal volume to judge the type of salinity intrusion. By this way, North Branch is a typical well-mixed estuary for strong tidal power and weak runoff power. Stratification Coefficient Method is another way by analyzing the vertical distribution of salinity and it can show the salt-fresh water mixture condition more effective. Mao (2004) found out well-mixed type is the main salt-fresh water mixture type in North Branch by using Stratification Coefficient Method. The first method is widely used because it is very simple and the data of runoff and tide is easy to achieve. However it is too macroscopic and do not take saline vertical distribution into consideration. The ratio of runoff volume and tidal volume cannot express the salt-fresh water mixture type completely. Stratification Coefficient Method do consider the saline vertical distribution, however, it only compare the salinity on the surface and at the bottom of water. The minimum of salinity normally appears on the surface but the maximum doesn't always appear at the bottom. Thus Stratification Coefficient Method also has its own limitations. Therefore, it is necessary to take a new method to analyze the saline vertical distribution and confirm the salt-fresh water mixture type in North Branch in Yangtze Estuary. More researches of characteristic of salinity intrusion should also be done.

Fractal theory is one of achievements in the development of nonlinear mathematical method and it is also a new worldview and methodology. In fractal theory, many objects in nature has the character that parts are similar with the whole in certain ways. We call it self-similarity. Fractal theory is applied in the research of fluid mechanism and material transportation. Pan (2015) used fractal interpolation to study the back-silting in 12.5-m deep channel in Yangtze Estuary. Ni etc. (2008, 2011) studied the fractal characteristic of velocity and sediment concentration distribution in Yangtze River and Yellow River. Study of saline distribution in North Branch also belongs to the research of fluid mechanism and material transportation. In this paper, fractal theory is used to study the saline distribution and salt-fresh water mixture in North Branch of Yangtze Estuary. We are trying to study the laws of salinity intrusion in North Branch from inner fluid structure in fractal way so that can provide service for water source security in Yangtze Estuary

FRACTAL MODEL

The self-similarity of fractal object can be expressed by the next equation:

$$u = e^{A} y^{-D} \qquad (1)$$

In this equation, u is the Euclidean length; y is the measuring size; D is fractal dimension which is a constant value; A is fractal proportionality constant. Fractal dimension D is the measure of self-similarity of fractal object. The bigger the fractal dimension D is, the better the self-similarity is. The self-similarity of fractal object can be exactly the same or similar statistically. We cannot find objects in nature strictly meet Equation (1), thus we need to use a method which named Transformed Dimension Fractal to transform scale of the research object. There are many ways to transform scale. For example, the Cumulative Transformation Method, the One-dimensional Dynamic Hausdorff Fractal Dimension Method, the Extended Reversed S-shaped Fractal Dimension Model Method and so on.

When the research object is saline vertical distribution, the physical meaning of fractal dimension D is the dimension of salinity length in vertical line and the self-similarity is the homogeneous degree of saline vertical distribution. The Cumulative Transformation Method is used to transform scale and saline vertical distribution and can be expressed by the next equations:

$$S^{(n)} = \{s_1^{(n)}, s_2^{(n)}, \ldots\ldots s_N^{(n)}\} \qquad (2)$$

$$S^{(n)} = e^{A} H^{-D} \qquad (3)$$

In Equation (2) and Equation (3), n is Scale transformation time; m is the number of measuring point location and the number from surface to bottom is 1,2,3……N. $S^{(n)}$ expresses the post-transformed dimension of salinity length in vertical line and is the set of measuring points' salinity. $s_m^{(n)}$ is the measuring point(No.m) salinity after n-time-transformation. H is the measuring size,varying from 0 to the maximum depth; h_m is the depth of measuring points. D is the fractal dimension of saline distribution. *A* is fractal proportionality constant and different vertical line has different values. Logarithmic transformation is done to Equation (3):

$$\ln(S^{(n)}) = -D\ln(H) + A \qquad (4)$$

According to Equation (4), *D* is the curve slope of $ln(H)$ and $\ln(S^{(n)})$. The relation of $S^{(n+1)}$ and $S^{(n)}$ can be expressed by following equation:

$$S^{(n+1)} = \{s_1^{(n)} + s_2^{(n)} + \cdots s_N^{(n)}, s_2^{(n)} + \cdots s_N^{(n)}, \ldots\ldots s_N^{(n)}\} \qquad (5)$$

An optimun fitting straight line can be got with a proper *n*. The opposite number of slope of this straight line is fractal dimension *D*, and intercept is fractal proportionality constant *A*. Fractal dimension *D* and fractal proportionality constant *A* are called the characteristic value of saline vertical distribution.

HYDROLOGICAL SURVEY AND OTHER DATA SOURCE

Ten vertical lines was set in the hydrological survey in North Branch of Yangtze Estuary during spring tide and neap tide (8th ~12th, December 2012). The location of lines are shown in Figure 2. All the vertical lines were synchronous observed during a daily tide cycle completely. The river discharge of Datong Hydrological Station which is the nearest hydrological station of Yangtze Estuary was 19800m^3/s. Water samples were got by "Six-point Method" and saline data was tested in the laboratory.

Figure 2. The location of vertical lines in the hydrological survey

RESULT AND ANALYSIS

The meaning of fractal characteristic values

The characteristic of saline vertical distribution is expressed by fractal characteristic values in fractal theory and is analyzed by Cumulative Transformation Method. All the saline vertical distributions are corresponding with the 1-order transformed dimension fractal relations. The fractal dimension *D* is between -0.6~-0.5 and the fractal proportionality constant *A* is between -3 and 4.

In fractal theory, larger fractal dimension means better self-similarity, more inhomogeneous saline vertical distribution and larger salt-fresh water mixture degree. Therefore, the fractal dimension *D* can represent the salt-fresh water mixture degree.

Fractal proportionality constant *A* and average vertical salinity *s* in the same vertical line at the same time showed positive correlation. The relation between *A* and *s* can be fitted through the scatter diagram (Figure 3) as follows:

$$A = \ln(s) + 0.6 \qquad (6)$$

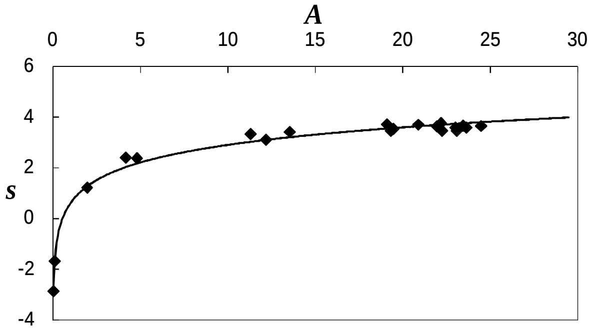


Figure 3. Relation between fractal proportionality constant *A* and average vertical salinity *s*

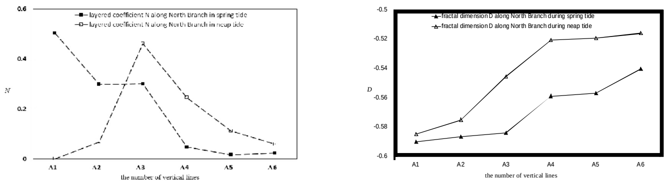
Salt-fresh water mixture degree and time-space distribution of salinity intrusion

Longitudinal distribution law

Using the vertical lines A1，A2，A3，A4，A5，A6 to analyze the salt-fresh water mixture degree and the longitudinal distribution law of the salinity intrusion intensity from outer estuary to inner estuary.

Judging the salt-fresh water mixture degree by ratio of runoff volume and tidal volume, it is well-mixed type in this survey. Stratification Coefficient Method is used to make the longitudinal variation along the river of stratification coefficient *N* (Figure 4(a)) and compared with the longitudinal variation along the river of fractal dimension value *D* (Figure 4(b)).

From Figure 4(a), judged by Stratification Coefficient Method, salt-fresh water mixture degree shows a larger trend from longitudinal outside to inside. During neap tide, from outside of the estuary to Hongyang Harbor, the mixture degree first decreases along the way and increase afterwards. Thus, the salt-fresh water mixture degree of North Branch calculated by Stratification Coefficient Method does not show synchronous law in spring-neap tide but change along with the variation of hydrological conditions. The research shows that surface salinity are greatly influenced by wind but the wind impact is getting weaker when reaching the bed. If only consider the difference between surface and bottom salinity, the wind effect will be overestimated. So the longitudinal salt-fresh water mixture distribution law by Stratification Coefficient Method is not got in this survey may due to the limitation of this method. As shown in Figure 4(b), the longitudinal law of salt-fresh water mixture under fractal theory is significant. During both spring and neap tide, fractal dimension *D* is increasing from outside to inside of the estuary. It means inhomogeneous degree of the saline vertical distribution is getting weaker as well as the mixture degree of salt-fresh water. In fractal theory, the mixture of fresh and salt water is a dynamic system. It generates in the outside of the estuary and energy dispersion exists during the extension to inside, which leads to the decrease of the salt-fresh water mixture degree. This is consistent with the survey result in Figure 4(b).



(a) (b)

Figure 4. Longitudinal changes of stratification coefficient *N* and fractal dimension *D*

In fractal theory, the fractal proportionality constant *A* represents the intensity of the salinity intrusion. Discussing the vertical distribution law of salinity intrusion intensity, a graph (Figure 5) of the variation for the fractal proportionality constant *A* from outside of the estuary to the inside can be drawn. In Figure 5, fractal dimension *D* decreases from outside to inside of the estuary.

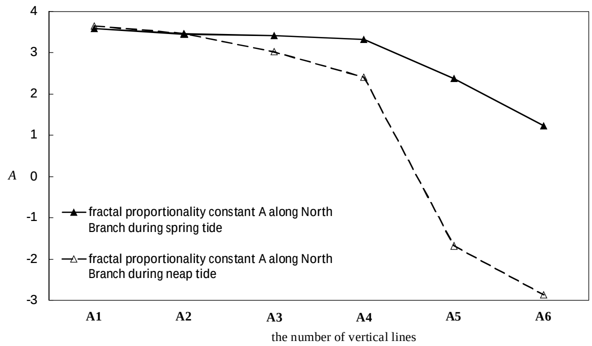


Figure 5. Longitudinal changes of fractal proportionality constant *A*

Temporal distribution law

Discuss the temporal distribution law from the perspective of daily variation and half-month's variation respectively. Daily variation is the change of mixture degree and salinity intrusion in a tidal cycle. The half-month's change is the different features of mixture degree and salinity intrusion intensity in the same half-month during both spring tide and neap tide.

The change of fractal dimension *D* in a tidal cycle is not much, with a characteristic of showing up rise and fall for four-time. Take Line A3 during neap tide for example. Figure 6 shows curves of fractal dimension *D* and velocity *v*, velocity of flow direction is positive to the ebb. The maximum of the fractal dimension *D* occurs when velocity comes to zero, while the minimum occurs near the time of extremum. The fractal dimension *D* corresponding to the maximum velocity is always less than that to the adjacent minimum velocity. The physical meaning in fractal theory is, near tidal flood and tidal ebb, comes to the most inhomogeneous saline vertical distribution and the lowest level of mixture. Near the maximum flood and maximum ebb, comes to the most inhomogeneous saline vertical distribution and the highest mixture degree, and t mixture degree for the maximum flood is larger than that for the maximum ebb. There is a positive relationship between the fractal proportionality constant *A* and the average salinity *s*. So the temporal distribution law for salinity intrusion by fractal theory is of the same recognition with the traditional theory. The recognition is that the daily variation of saline process is similar to the tidal process, there are twice rise and fall in one day, the maximum occurs near the tidal flood and the minimum occurs near the tidal ebb .

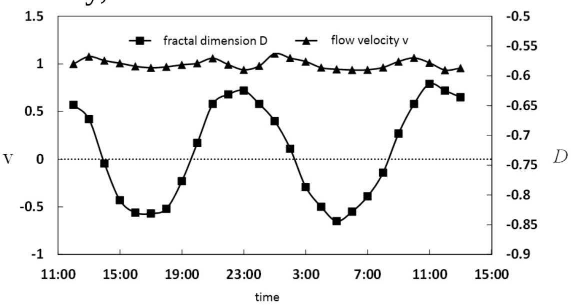


Figure 6. Curves of fractal dimension *D* and flow velocity *v* (Line A3, neap tide)

CONCLUSION

(1) Fractal characteristic values represent the inner laws of the salt-fresh water mixed system. Larger fractal dimension *D* means better self-similarity, more homogeneous saline vertical distribution and larger salt-fresh water mixture degree. There is a positive relationship between the fractal proportionality constant *A* and the average vertical salinity. The saline vertical distribution in North Branch of Yangtze Estuary corresponds with the 1-order transformed dimension fractal relations. Fractal dimension *D* is between -0.6~-0.5 and the fractal proportionality constant A is between -3 and 4.

(2) From outside to inside North Branch of Yangtze Estuary, fractal dimension *D* and fractal proportionality constant *A* increase. It means that the salt-fresh water mixture degree and salinity intrusion intensity decrease.

(3) The variation of fractal dimension *D* in a tidal cycle is not much, with a characteristic of showing rise and fall for fourth times. The curve of daily salinity is similar to that of tidal level. Half-month variation on saltwater intrusion is gradually more and more obvious from outside to inside in North Branch of Yangtze.

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