3 types of vehicle-bridge coupling vibration analysis of model comparison

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Abstract: Compares two 1/4 car models, two 1/4 model (changes the spring stiffness) and 1/2 car models three vehicles at the same speed in the displacement effect. Comparison: when the vehicle speed is less than 130km/h, and displacement response curves in the three models are calculated when a highway bridge coupling vibration analysis of vehicle is two one-fourth model can be used to simulate.

Introduction

Vehicle-bridge coupled vibration attracts many researchers' attention as early as 100 years ago. The current study has simplified the vehicle and bridge model in it to make for a simpler solution.

In recent years, many Chinese researchers are trying to use numerical method to solve coupling problems. The document[4] builds overall 3D model to make the interaction between vehicle and bridge as a whole, thus establishing differential equations, with the kelp of numerical methods, the dynamic response is obtained with respect to simple-supported girder and continuous beam in highway. The document[5-6] makes a model of vehicle bridge coupling of simple-supported beam bridge employing d'alembert's principle and uses numerical methods to solve the problem. Step-by-step integral is used to calculate the coupling system consisting of mass with uniform variable speed and a simple supported beam. The document[8] acquires the power series solution to Willis equation. The document[9] derives the vibration equation by ignoring moving constant loads of mass vehicle and thus gets the exact solution. The document[10] uses concentrated mass as the simplified bridge model and adds 2D multi-axis trailer load.

To investigate the influence of vehicle model on vehicle-bridge coupling, three kinds of models:two quarter-car models(spring stiffness changed) and half-car models with different velocity, are used to analyze displacement response. The overall laws are analyze in the first place which followed the analysis of difference of models.

Model

one-fourth car models

Figure 1 is vehicle model, the spring-damper-mass system is for simulating vehicles. Where m_b is vehicle mass. m_t is the frame and wheel mass; k_a is vertical stiffness; c_a is vertical damping; k_b is vertical stiffness; c_b is vertical damping; v is the speed when vehicles go cross the bridge.

The vibration equation of vehicle and bridge are as follows:

$$m_t \frac{d^2 y_t}{dt^2} + m_b \frac{d^2 y_b}{dt^2} + c_a\left(\frac{dy_t}{dt} + \frac{dy_w}{dt}\right) + k_a(y_t + w) = 0 \ ,$$

$$m_b \frac{d^2 y_b}{dt^2} + c_b\left(\frac{dy_b}{dt} + \frac{dy_t}{dt}\right) + k_b(y_b - y_t) = 0 \cdot$$

Supposing $w(x,t) = \sum_n X_n(x) T_n(t)$ by using variable separation method. Using simple supported beam as boundary condition and presuming that $X_n(x) = \sin(npx/l)$. According the orthogonality of principal mode, the vibration differential equation under loads is as follows:

$$\ddot{T}_n(t) + w_n^2 T_n = \frac{2(m_t + m_b)g}{ml} \sin\frac{pvt}{l} +$$

$$\ddot{T}_n(t) + \frac{2m_t}{ml}\frac{dy_t}{dt^2}\sin\frac{npvt}{l} + \frac{2m_b}{ml}\frac{d^2 y_b}{dt^2}\sin\frac{npvt}{l}$$

With the help of variable separation method, suppose $w(x,t) = \sum_n X_n(x) T_n(t)$ .By simply-supported beams with boundary condition,let $X_n(x) = \sin(npx/l)$,Again according to the principal mode of orthogonality,get the vibration differential equation of the bridge under the action of load as follows:

$$\ddot{T}_n(t) + w_n^2 T_n = \frac{2(m_t + m_b)g}{ml} \sin\frac{pvt}{l} + \frac{2m_t}{ml}\frac{dy_t}{dt^2}\sin\frac{npvt}{l} + \frac{2m_b}{ml}\frac{d^2 y_b}{dt^2}\sin\frac{npvt}{l} \ ,$$

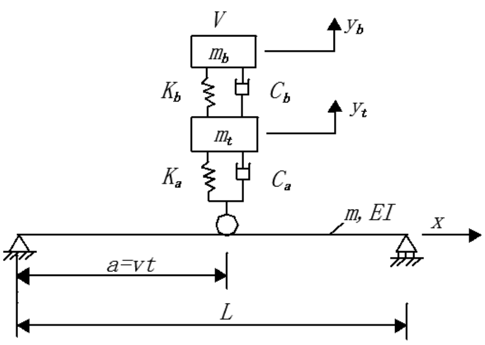

Fig. 1. One-fourth model under the effect of simply supported beam

Two quarter-car models will be used. The vibration equations for vehicle and bridge are taken into consideration separately, he the results are in superposition. The other one is the same.

Half-car models

Figure 2 is vehicle model, the spring-damper-mass system is for simulation of vehicles. Where m_b is vehicle mass. I_b is the rigidity of vehicle body; m_t is the mass of wheels and structures; k_a is vertical stiffness; c_a is vertical damping; k_b is vertical stiffness; c_b is vertical damping; v is the speed when vehicles go cross the bridge(constant value). The vibration equations are as follows[6]:

$$\frac{dT_n(t)}{dt^2}+w_n^2T_n(t)=\frac{2p_1(t)}{ml}\sin\frac{np(vt-a)}{l}d_1(t)+\frac{2p_2(t)}{ml}\sin\frac{npvt}{l}d_2(t)$$

Which:

$$d_1(t)=\begin{cases}1, & \frac{a}{v}\le t\le\frac{l+a}{v};\\ 0, & \text{else}\end{cases},\qquad d_2(t)=\begin{cases}1, & 0\le t\le\frac{l}{v};\\ 0, & \text{else}\end{cases},$$

Analysis of load of the vehicle system:
$$\left.\begin{aligned}p_1(t)&=m_{a1}g+\frac{m_b}{2}g+m_{a1}y''_{t1}+\frac{m_s}{2}y''_b-\frac{I_b}{a}q''\\ p_2(t)&=m_{t2}g+\frac{m_b}{2}g+m_{t2}y''_{t2}+\frac{m_b}{2}y''_b-\frac{I_b}{a}q''\end{aligned}\right\}$$

Analysis of load of the bogie:

$$m_{t1}y''_{t1}+c_{a1}(y'_{t1}+w'_1)+k_{a1}(y_{t1}+w_1)+c_{b1}(y'_{t1}-y'_b+\frac{a}{2}q')+k_{b1}(y_{t1}-y_b+\frac{a}{2}q)=0$$

$$m_{t2}y''_{t2}+c_{a2}(y'_{t2}+w'_2)+k_{a2}(y_{t2}+w_2)+c_{b2}(y'_{t2}-y'_b-\frac{a}{2}q')+k_{b2}(y_{t2}-y_b-\frac{a}{2}q)=0$$

Which:

$$w_1=\sum_n X_n(vt-a)T_n(t);\qquad w_2=\sum_n X_n(vt)T_n(t)$$

Analysis of load of the body:

$$m_b y''_b+c_{b1}(y'_b-y'_{t1}-\frac{a}{2}q')+k_{b1}(y_b-y_{t1}-\frac{a}{2}q)-c_{b2}(y'_{t2}-y'_b-\frac{a}{2}q')-k_{b2}(y_{t2}-y_b-\frac{a}{2}q)=0$$

$$I_b q''-\frac{a}{2}\left[c_{b1}\left(y'_b-y'_{t1}-\frac{a}{2}q'\right)+k_{b1}\left(y_b-y_{t1}-\frac{a}{2}q\right)\right]+\frac{a}{2}\left[c_{b2}\left(y'_b-y'_{t2}-\frac{a}{2}q'\right)+k_{b2}\left(y_b-y_{t2}-\frac{a}{2}q\right)\right]=0\cdot$$

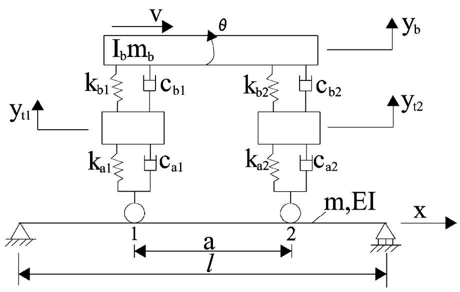

Fig. 2.Half-car model under the effect of simply supported beam

Results

Bridge data in document[11] and vehicle data in document[12] are used, the length of simple supported beam is L=32m. Mass per unit length $m=5.41\times10^3 kg/m$. flexural rigidity $EI=3.5\times10^{10}N\bullet m^2$. Following are relevant data of quarter-car model: $m_{t1}=m_{t2}=4330kg$; $m_b=19250kg$; $a=8.4m$; $k_{a1}=k_{a2}=4.28\times10^6 N/m$; $k_{b1}=k_{b2}=2.535\times10^6 N/m$; $c_{a1}=c_{a2}=9.8\times10^4 kg/s$; $c_{b1}=c_{b2}=1.96\times10^5 kg/s$ 。1/2 car models: $c_{a1}=c_{a2}=9.8\times10^4 kg/s$; $m_b=38500kg$; $m_{t1}=m_{t2}=4330kg$; $I_b=2.446\times10^6 kg\bullet m^2$; $a=8.4m$;

$k_{a1} = k_{a2} = 4.28\times10^{6}\, N/m$； $k_{b1} = k_{b2} = 2.535\times10^{6}\, N/m$； $c_{b1} = c_{b2} = 1.96\times10^{5}\, kg/s$。

Figure 3～Figure 6 compare 3 kinds of vehicles vertical displacement at mid-point of beam at same speed. Figure 7 shows the largest displacement of the 3 kinds of models on the bridge under different speed. The comparison of curves in the figure shows that the 3 kinds of models could embody coupling response.

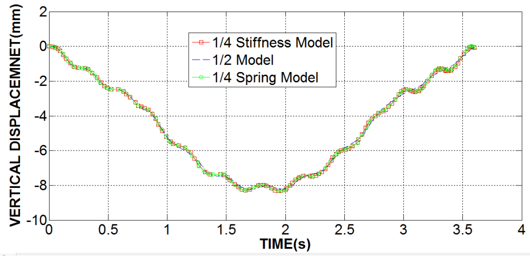

Fig. 3. Vertical displacement at mid-point of beam at speed 40km/h

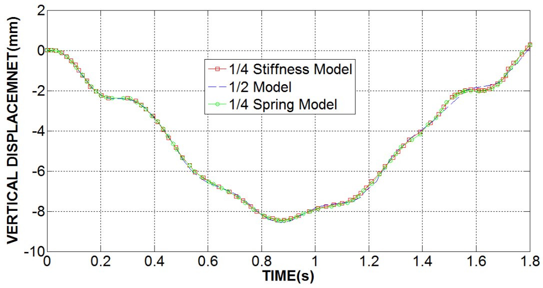

Fig. 4. Vertical displacement at mid-point of beam at speed 80km/h

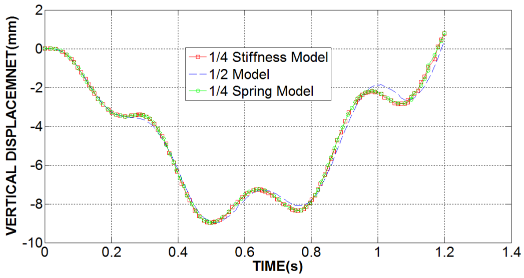

Fig. 5. Vertical displacement at mid-point of beam at speed 120km/h

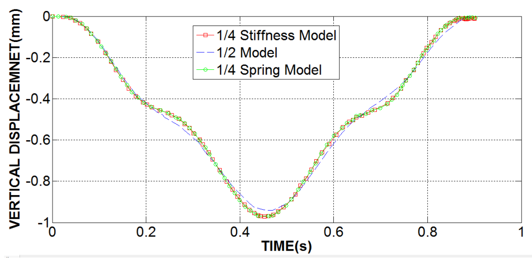

Fig. 6. Vertical displacement at mid-point of beam at speed 160km/h

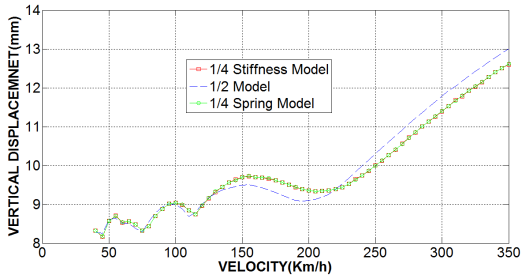

Fig. 7. The largest displacement on the bridge under different speed

General laws could be arrived at based on above figures:

(1) They all give good expression to the vibration response.

(2) Higher speed will make a slower displacement response of mid-span.

(3) Response Maximum displacement in bridge when vehicle appear in mid-span.

(4) When speed is under 200km/h, response of maximum displacement bridge do not take on a linear relationship.

The difference between vehicle models are as follows:

(1) The difference between 1/4 spring model of rigidity and 1/4 spring model is the change of rigidity. The displacement response in mid-span of bridge fits well with each other. Spring stiffness has no impact on vehicle-bridge coupling.

(2) When speed is under 130km/h, the discrepancy of maximum displacement in 1/4 vehicle model and 1/2 vehicle model is small.

(3) When speed is under 220km/h, the maximum displacement in 1/2 vehicle model is smaller than that 1/2 vehicle model. The vehicle-bridge coupling is more sensitive to moment of inertia.

(4) When speed is more than 220km/h, the maximum displacement in 1/2 vehicle model is greater than that 1/2 vehicle model.

(5) Generally speaking, the vehicle speed is no more than 120/h, The vehicle-bridge coupling can be realized using 1/4 model in highway bridge.

Conclusions

To study the response of factor of vehicle model to the vehicle-bridge coupling, the displacement of mid-span of 3 kinds of models are caculated. Comparison shows that the 3 kinds of models are in good agreement with vibration response of vehicle-bridge coupling. With the add of rotational inertia, the agreement will be better yet make the analysis complicated. When velosity is under 130km/h, the 3 kinds of models are in good agreement with each other. Quarter-car models can be used to analyse vibration of vehicle-bridge coupling in highway bridge.

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