Nonlinear analysis of under-deck cable-stayed bridges

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ABSTRACT: Under-deck cable-stayed bridges have recently emerged as one of promising bridge forms due to their remarkable advantages such as high structural efficiency, easy construction, economy and elegant appearance. The stayed cables provide supports on the prestressed concrete deck. The cables are located under the deck and deviated through struts, which introduces cable deviation forces on th

e deck. In the design of this type of bridges, both the ultimate load and ductility should be examined, which requires the estimation of full-range behaviour. An analytical beam model and its corresponding beam finite element model for geometric and material nonlinear analysis are developed for this type of bridges. The model accounts for the interaction between the axial and flexural deformations of the deck, and uses the actual stress-strain curves of materials considering their stress path-dependence. In the structural system, the deck interacts with the stayed cables. With a nonlinear kinematical theory, complete description of the nonlinear interaction between the stay cables and the deck is obtained.

INTRODUCTION

Under-deck cable-stayed bridges are a special and innovative type of cable-stayed bridges in which the stay cables are located under the slender prestressed concrete deck and deviated through strut(s) as shown in Figure 1. The cable deviation forces, applied on the deck, cause the reduction of bending moment and shear force in the deck. The stay cables are self-anchored in the deck in the support sections over the abutments and piers (if present). Some bridges of this type have been built in the last forty years. Leonhardt (1982), Virlogeux *et al*. (1994), Holgate (1997), Schlaichand & Werwigk (2001), Forno & Cremer (2001), etc. illustrated design of some bridges of this structural type. Here after in this paper, this type of bridges is assumed unless otherwise stated.

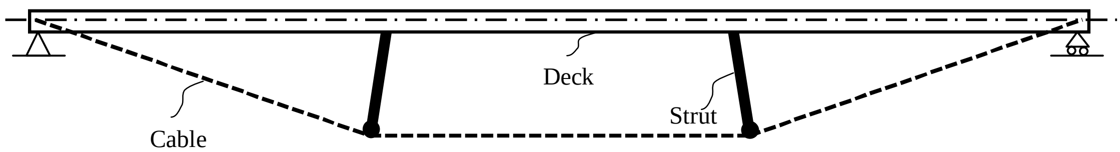


Figure 1. An under-deck cable-stayed bridge

Ruiz-Teran & Aparicio (2007) have identified the parameters governing the structural response of this type of bridges. Later Ruiz-Teran & Aparicio (2008a, b) studied its structural behaviour and proposed design criteria for bridges of this type. Some advantages, *e.g.* high structural efficiency, low self-weight, easy construction, economy and elegant appearance, have been identified by comparison with conventional bridges without stay cables.

Provided that there is sufficient clearance beneath the stay-cables, this bridge type has been shown to be very appropriate for use in highway overpasses (Ruiz-Teran & Aparicio 2008a). In 1987, Schlaich proposed an under-deck cable-stayed bridge for the Kirchheim overpass (Holgate 1997). However, this proposal was not adopted as the state authority concerned the possibility of the bridge collapsing due to the sudden breakage of cables in the case that a lorry with an extremely high load collides the under-deck cables. Ruiz-Teran & Aparicio (2009) investigated the response of under-deck cable-stayed bridges to the accidental breakage of stay cables, and found that this type of bridges can resist the accidental action with a higher degree of safety than that required by codes. In addition, a set of design criteria closely related with this issue were established.

However, little work has been done on the nonlinear behaviour of this type of bridges. In this study, a numerical model is proposed to predict the full-range behaviour of the bridges taking into account geometric and material nonlinearities, and the interactions between deck and under-deck cables.

THEORETICAL FORMULATION

Some assumptions are made for the theoretical formulation: (a) The shear deformation of the deck is negligible; (b) The stress-strain curves of concrete, non-prestressed and prestressing steel are as given by the constitutive material models; (c) The shear lag effect is disregarded in the present study; (d) Deflections may be large but rotations are small to moderate (Reddy 2004); (e) The relative interface slip between cables and struts are allowed; and (f) The failure of deck and cables occurs before failure of struts.

Deck model

Referring to the deck segment as shown in Figure 2, the displacement u_f in the x-direction is given by

$$u_f = u - zv' \qquad (1)$$

where u is the longitudinal displacement of the axis of deck; and v is the total deflection of deck. By introducing geometric nonlinearity, the strain of a generic fiber of the deck in the deformed configuration can be obtained by

$$e_f = \frac{\partial u_f}{\partial x} + \frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2 = e_0 - z\frac{\partial^2 v}{\partial x^2} \qquad (2)$$

where $e_0 = u' + (1/2)(v')^2$ is the strain of deck axis, and the term $(1/2)(v')^2$ is due to geometric nonlinearity.

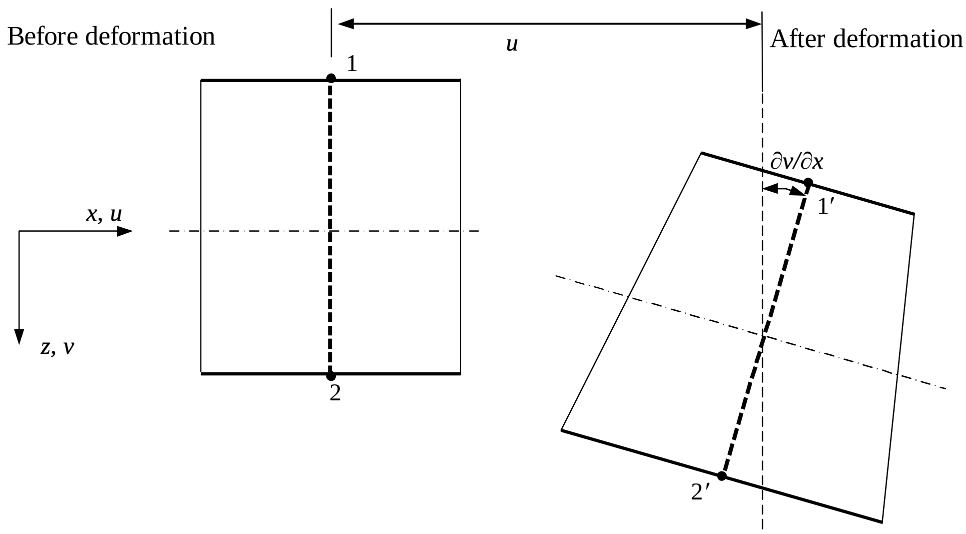


Figure 2. Deformation of a deck segment

The virtual strain energy δU_b in the deck is given by

$$dU_b = \int_L \int_A s_f de_f \mathrm{d}A\mathrm{d}x = \int_L \int_A s_f \left\{ de_0 - z\frac{\partial^2 dv}{\partial x^2} \right\} \mathrm{d}A\mathrm{d}x$$
$$= \int_L N de_0 \mathrm{d}x + \int_L M\left(-\frac{\partial^2 dv}{\partial x^2}\right)\mathrm{d}x \qquad (3)$$

where σ_f is the normal stress of the deck; A is area of the deck; and the axial force N, the bending moment M of the deck are given respectively by

$$N = \int_A s_f \mathrm{d}A, \quad M = \int_A s_f z\mathrm{d}A \tag{4a-b}$$

Modelling of cables

The cables interact with the deck through anchorages and struts, as the cable elongation depends on the global deformation of the whole structural system. In this study, only the case of free slip between cables and struts is considered. Dall'Asta *et al.* (2007) developed a model for the cables with a nonlinear kinematical theory and provided a complete description of the nonlinear interaction between external prestressing tendons and girder. This model is adopted for this type of bridges.

The profile of cable is defined by the locations of end anchorages and struts. Each location point, $\mathbf{Loc}_{0,j}$, can be expressed as

$$\mathbf{Loc}_{0,j} = x_j\mathbf{i} + e_j\mathbf{k} \tag{5}$$

where **i** and **k** are unit vectors parallel to *x*- and *z*-axis respectively; and x_j and e_j are the *x*-value and eccentricity, respectively, of the *j*-th location point with a strut or anchorage (j=0, 1, 2…n). The initial total length of cable L_{t0} can be given by

$$L_{t0} = \sum_{j=1}^{n} \left|\mathbf{c}_{0,j} - \mathbf{c}_{0,j-1}\right| \tag{6}$$

After deformation, the location point $\mathbf{Loc}_{0,j}$ moves to a new position $\mathbf{Loc}_j$ of

$$\mathbf{Loc}_j = \left[x_j + u_j - e_j v_j'\right]\mathbf{i} + \left[e_j + v_j\right]\mathbf{k} \tag{7}$$

where u_j and v_j are the displacements at x_j. The total length of cable after deformation L_t is given by

$$L_t = \sum_{j=1}^{n} \left|\mathbf{Loc}_j - \mathbf{Loc}_{j-1}\right| \tag{8}$$

The additional elongation ΔL_t of the cable is then obtained as $\Delta L_t = L_t - L_{t0}$.

Assuming that the strain ε_p of the cable remains small if the inclination of the cable profile is moderate, the cable deformation measured in its deformed configuration can be simplified as

$$e_p = \frac{\Delta L_t}{L_{t0}}$$
$$\approx \frac{1}{L_{t0}} \sum_{j=1}^{n} \left\{ \frac{1}{2} \frac{\left(\Delta_j e + \Delta_j v\right)^2}{l_{p,j}} + \frac{\Delta_j x}{l_{p,j}} \left[\Delta_j x + \Delta_j u - \Delta_j \left(ev'\right)\right] \right\} - 1 \tag{9}$$

where $\Delta_j \bullet = \bullet_j - \bullet_{j-1}$; and $l_{p,j} = \sqrt{\left(\Delta_j x\right)^2 + \left(\Delta_j e\right)^2}$.

The assumption made here is acceptable if the cable profile meets the condition that the ratio of maximum vertical length of strut to bridge span is less than or equal to 1/10. Ruiz-Teranand & Aparicio (2007) recommended the ratio to be 1/10 to obtain layout that is both satisfactory from an aesthetic point of view and efficient from a structural point of view. Consequently, the virtual strain of the cable is given by

$$de_p = \frac{1}{L_{t0}} \sum_{j=1}^{n} \left\{ \frac{\left(\Delta_j e + \Delta_j v\right)}{l_{p,j}} \Delta_j dv + \frac{\Delta_j x}{l_{p,j}} \left[\Delta_j du - \Delta_j \left(ed v'\right)\right] \right\} \tag{10}$$

The virtual strain energy δU_p associated with the cable can be obtained by

$$dU_p = L_{t0} A_p s_p de_p \tag{11}$$

For simplification, only one resultant cable is considered hereafter. For the cases with more cables, straightforward generalization can be achieved by superposition.

Global structural system

There are interactions between the deck and cables in the structural system. The global behaviour of the whole structural system can be obtained by the principle of virtual work as

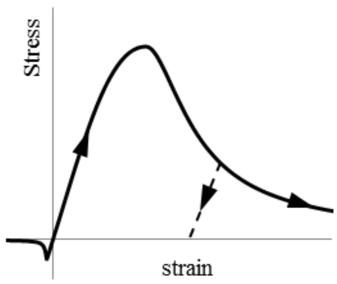
$$dU_b + dU_p = dW_E \tag{12}$$

where δW_E is the virtual work associated with the external load vector **f** as given by

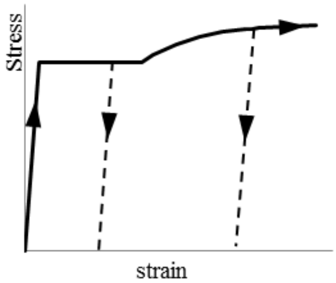
$$dW_E = \int_0^L d\,\mathbf{d}^T \mathbf{f} \mathrm{d}x \tag{13}$$

Material laws

The material laws are adopted to consider the material nonlinearity. Figure 3(a) shows the model for concrete comprising the stress-strain curve in compression proposed by Attard & Setunge (1996) and that in tension proposed by Carreira & Chu (1986), and Guo & Zhang (1987). The stress-strain curve recommended by Mander *et al.* (1984) is used for steel bars as shown in Figure 3(b). The stress-strain curve for prestressing steel proposed by Menegotto & Pinto (1973) is shown in Figure 3(c) and is adopted for the under-deck cables.



(a)



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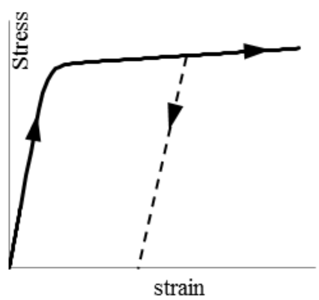


(c)

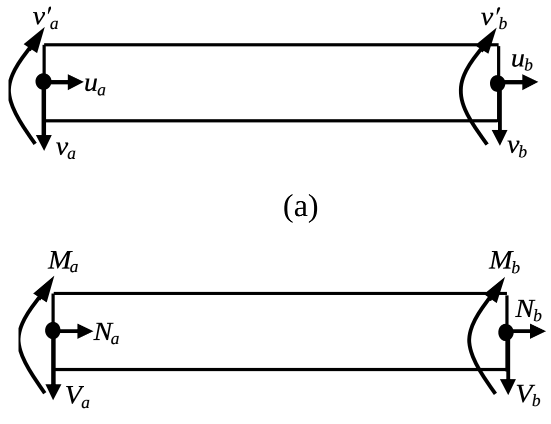
Figure 3. Stress-strain curves of materials: (a) concrete; (b) steel bars; and (c) prestressing steel (for cables)

FINITE ELEMENT FORMULATION

A finite element model is formulated based on the theory proposed.

Finite element formulation of deck

A C^1 two-node beam element is formulated as shown in Figure 4.



(b)

Figure 4. A C^1 two-node beam element: (a) degrees of freedom; and (b) internal nodal forces

In the beam element, the displacement vector **d** comprising the longitudinal displacement u of the beam axis and deflection v, which are interpolated by linear and Hermite cubic polynomials respectively, namely

$$\mathbf{d}=\begin{Bmatrix} u \\ v \end{Bmatrix}=\mathbf{N}\mathbf{d}^e=\begin{Bmatrix} \mathbf{N}_u \\ \mathbf{N}_v \end{Bmatrix}\mathbf{d}^e \tag{14}$$

where the nodal displacement vector is $\mathbf{d}^e=\{u_a \quad v_a \quad v'_a \quad u_b \quad v_b \quad v'_b\}^{\mathrm{T}}$ with the element nodes numbered locally as a and b; the shape function matrix **N** contains sub-matrices $\mathbf{N}_u$ and $\mathbf{N}_v$; and the superscript e refers to the beam element.

From Equation (2), the strain vector can be obtained as

$$\boldsymbol{\varepsilon}=\begin{bmatrix} e_0 \\ -\dfrac{\partial^2 v}{\partial x^2} \end{bmatrix}=\begin{bmatrix} \dfrac{\partial u}{\partial x}+\dfrac{1}{2}\left(\dfrac{\partial v}{\partial x}\right)^2 \\ -\dfrac{\partial^2 v}{\partial x^2} \end{bmatrix}=\left\{\mathbf{B}_l+\frac{1}{2}\mathbf{B}_{nl}\right\}\mathbf{d}^e \tag{15}$$

where the linear and nonlinear strain-displacement matrices $\mathbf{B}_l$ and $\mathbf{B}_{nl}$ are given respectively as

$$\mathbf{B}_l=\begin{bmatrix} \dfrac{\partial}{\partial x} & 0 \\ 0 & -\dfrac{\partial^2}{\partial x^2} \end{bmatrix}\begin{bmatrix} \mathbf{N}_u \\ \mathbf{N}_v \end{bmatrix}, \text{ and}$$

$$\mathbf{B}_{nl}=\begin{bmatrix} 1 \\ 0 \end{bmatrix}\left(\frac{\partial \mathbf{N}_v}{\partial x}\mathbf{d}^e\right)^{\mathrm{T}}\frac{\partial \mathbf{N}_v}{\partial x} \tag{16a-b}$$

The variation form of Equation (15) is

$$d\boldsymbol{\varepsilon}=[\mathbf{B}_l+\mathbf{B}_{nl}]d\,\mathbf{d}^e \tag{17}$$

Atypical cross section of bridge deck is shown in Figure 5. The cross section is divided into a number of layers for analysis. The concrete strain in each layer is assumed to be uniformly distributed and equal to the strain at the centre of layer (Lou & Xiang 2006).

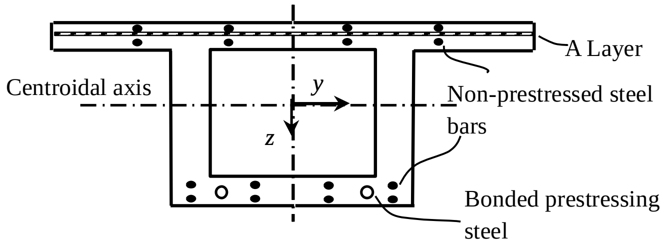


Figure 5. Typical cross sections and simplified arrangement of steels of a bridge deck

According to section equilibrium conditions, the stress resultants N and M can be expressed as follows

$$N = \sum_i s_{ci} A_{ci} + \sum_j \left(s_{sj} - s_{csj} \right) A_{sj} + \sum_k \left(s_{bpk} - s_{cbpk} \right) A_{bpk} \tag{18a}$$

$$M = \sum_i s_{ci} A_{ci} z_{ci} + \sum_j \left(s_{sj} - s_{csj} \right) A_{sj} z_{sj} + \sum_k \left(s_{bpk} - s_{cbpk} \right) A_{bpk} z_{bpk} \tag{18b}$$

where σ, A and z correspond to stress, area and vertical coordinate respectively; subscripts ci, sj and bpk correspond to the ith concrete layer, jth ordinary reinforcing steel and kth bonded prestressing steel (if present) respectively; and subscripts csj and $cbpk$ correspond to concrete at the level of jth ordinary reinforcing steel and concrete at the level of kth bonded prestressing steel (if present) respectively.

The differentials of Equations (18a) to (18b) are

$$\begin{aligned} dN = &\sum_i E_{Tci} A_{ci} de_{ci} + \sum_j \left(E_{Tsj} - E_{Tcsj} \right) A_{sj} de_{sj} \\ &+ \sum_k \left(E_{Tbpk} - E_{Tcbpk} \right) A_{bpk} de_{bpk} \end{aligned} \tag{19a}$$

$$\begin{aligned} dM = &\sum_i E_{Tci} A_{ci} z_{ci} de_{ci} + \sum_j \left(E_{Tsj} - E_{Tcsj} \right) A_{sj} z_{sj} de_{sj} \\ &+ \sum_k \left(E_{Tbpk} - E_{Tcbpk} \right) A_{bpk} z_{bpk} de_{bpk} \end{aligned} \tag{19b}$$

where E_T is the tangent modulus of material.

Substituting Equation (2) into Equation (19a-b) yields the section tangent stiffness matrix $\mathbf{D}_T$ in terms of the internal force vector $\mathbf{F}_I$ as

$$\mathbf{D}_T = d\mathbf{F}_I / d\boldsymbol{\varepsilon} = \begin{bmatrix} d_{11} & d_{12} \\ Sym. & d_{22} \end{bmatrix} \tag{20}$$

where

$$\mathbf{F}_I = \begin{bmatrix} N & M \end{bmatrix}^{\mathrm{T}} \tag{21}$$

and the other elements are given by

$$d_{11} = \sum_i E_{Tci} A_{ci} + \sum_j \left(E_{Tsj} - E_{Tcsj} \right) A_{sj} + \sum_k \left(E_{Tbpk} - E_{Tcbpk} \right) A_{bpk}\,; \quad \begin{aligned} d_{12} = d_{21} = &\sum_i E_{Tci} A_{ci} z_{ci} + \sum_j \left(E_{Tsj} - E_{Tcsj} \right) A_{sj} z_{sj}\,; \\ &+ \sum_k \left(E_{Tbpk} - E_{Tcbpk} \right) A_{bpk} z_{bpk} \end{aligned}$$

and

$$\begin{aligned} d_{22} = &\sum_i E_{Tci} A_{ci} z_{ci}^2 + \sum_j \left(E_{Tsj} - E_{Tcsj} \right) A_{sj} z_{sj}^2 \\ &+ \sum_k \left(E_{Tbpk} - E_{Tcbpk} \right) A_{bpk} z_{bpk}^2 \end{aligned}.$$

Introducing Equations (17) and (21) to the virtual strain energy of the deck in Equation (3) yields

$$dU_b^e = \int_{L^e} (d\boldsymbol{\varepsilon})^{\mathrm{T}} \mathbf{F}_I \mathrm{d}x = \int_{L^e} \left(d\mathbf{d}^e\right)^{\mathrm{T}} \left([\mathbf{B}_l + \mathbf{B}_{nl}]^{\mathrm{T}}\right) \mathbf{F}_I \mathrm{d}x$$
$$= \left(d\mathbf{d}^e\right)^{\mathrm{T}} \mathbf{f}_b^e \qquad (22)$$

where L^e is the element length; and the equivalent element nodal load vector $\mathbf{f}_b^e$ is given by

$$\mathbf{f}_b^e = \int_{L^e} \left([\mathbf{B}_l + \mathbf{B}_{nl}]^{\mathrm{T}}\right) \mathbf{F}_I \mathrm{d}x \qquad (23)$$

The differential $\mathrm{d}\mathbf{f}_b^e$ of Equation (23) is

$$\mathrm{d}\mathbf{f}_b^e = \int_{L^e} (\mathbf{B}_l + \mathbf{B}_{nl})^{\mathrm{T}} \mathrm{d}\mathbf{F}_I \mathrm{d}x + \int_{L^e} (\mathrm{d}\mathbf{B}_{nl})^{\mathrm{T}} \mathbf{F}_I \mathrm{d}x \qquad (24)$$

Substituting Equations (20)-(21) and (16b) into Equation (24) yields the element tangent stiffness matrix as

$$\mathrm{d}\mathbf{f}_b^e = \mathbf{k}_b^e \mathrm{d}\mathbf{d}^e \qquad (25)$$

where

$$\mathbf{k}_b^e = \int_{L^e} \left(\mathbf{B}_l^{\mathrm{T}} \mathbf{D}_T \mathbf{B}_l\right) \mathrm{d}x$$
$$+ \int_{L^e} \left(\mathbf{B}_{nl}^{\mathrm{T}} \mathbf{D}_T \mathbf{B}_l + \mathbf{B}_l^{\mathrm{T}} \mathbf{D}_T \mathbf{B}_{nl} + \mathbf{B}_{nl}^{\mathrm{T}} \mathbf{D}_T \mathbf{B}_{nl}\right) \mathrm{d}x$$
$$+ \int_{L^e} \left(\frac{\partial \mathbf{N}_v}{\partial x}\right)^{\mathrm{T}} \frac{\partial \mathbf{N}_v}{\partial x} N \mathrm{d}x$$

is the element tangent stiffness of deck.

Finite element formulation of cables

The deformation of cable given by Equation (10) depends on the displacements of anchorages and struts. It is assumed that the positions of the (n+1) struts or anchorages coincide with some of the m nodal points of the mesh. Consequently, the virtual strain $\delta\varepsilon_p$ of the cable is given by

$$de_p = \frac{1}{L_{t0}} \sum_{j=1}^{n} \left\{ \frac{(\Delta_j e + \Delta_j v)}{l_{p,j}} \Delta_j dv + \frac{\Delta_j x}{l_{p,j}} \left[\Delta_j du - \Delta_j (edv')\right] \right\}$$
$$= \frac{1}{L_{t0}} \chi d\mathbf{D} \qquad (26)$$

where $\mathbf{D}$ is the global nodal displacement vector of the deck; and the kinematic compatibility vector of the cable for expanding to cope with the global displacement vector is

$$\boldsymbol{\chi} = [\chi_1 \quad \chi_2 \quad \mathbf{L} \quad \chi_{m-1} \quad \chi_m]$$ with

$$\chi_i = \begin{cases} \mathbf{0}_{1\times 3} \quad when\ x_i \neq x_j \ (j = 0, 1, 2, \mathbf{K}, n) \\ \left[-\frac{\Delta_1 x}{l_{p,1}}, \quad -\frac{\Delta_1 e + \Delta_1 v}{l_{p,1}}, \quad e_0 \frac{\Delta_1 x}{l_{p,1}} \right] \quad when\ x_i = x_j \ (j = 0) \\ \left[\frac{\Delta_j x}{l_{p,j}} - \frac{\Delta_{j+1} x}{l_{p,j+1}}, \quad \frac{\Delta_j e + \Delta_j v}{l_{p,j}} - \frac{\Delta_{j+1} e + \Delta_{j+1} v}{l_{p,j+1}}, \quad -e_j \left(\frac{\Delta_j x}{l_{p,j}} - \frac{\Delta_{j+1} x}{l_{p,j+1}} \right) \right] \\ \qquad when\ x_i = x_j \ (j = 1, 2, \mathbf{L}, n-1) \\ \left[\frac{\Delta_n x}{l_{p,n}}, \quad \frac{\Delta_n e + \Delta_n v}{l_{p,n}}, \quad -e_n \frac{\Delta_n x}{l_{p,n}} \right] \quad when\ x_i = x_j \ (j = n) \end{cases}$$

In the kinematic compatibility vector $\boldsymbol{\chi}$, the sub-vector $\boldsymbol{\chi}_i$ corresponds to the i-th nodal point with abscissa x_i. Hence, the virtual work δU_p done by the cable can be written as

$$dU_p = L_{t0} A_p s_p de_p = A_p s_p \chi d\mathbf{D} = \mathbf{F}_p d\mathbf{D} \qquad (27)$$

where A_p is the cross sectional area of cable, σ_p is the cable stress, and the equivalent load vector $\mathbf{F}_p$ comprises the forces that the cable transmits to the deck. The tangent stiffness matrix $\mathbf{K}_p$ contributed by the cable is

$$\mathbf{K}_p = \frac{\partial \mathbf{F}_p}{\partial \mathbf{D}} = \frac{\partial \left(A_p s_p \chi \right)}{\partial \mathbf{D}} = A_p \frac{\partial s_p}{\partial e_p} \frac{1}{L_{t0}} \chi^{\mathrm{T}} \chi + A_p s_p \frac{\partial \chi}{\partial \mathbf{D}} \quad (28)$$

Global finite element modelling

In accordance with the conventional finite element formulation, the global stiffness equation is obtained as

$$\mathbf{K} d\mathbf{D} = d\mathbf{F} \quad (29)$$

where the global stiffness **K** comprises the global deck stiffness matrix $\mathbf{K}_b$, and the global stiffness matrix due to cable $\mathbf{K}_p$; and **F** is the global nodal force vector of the deck.

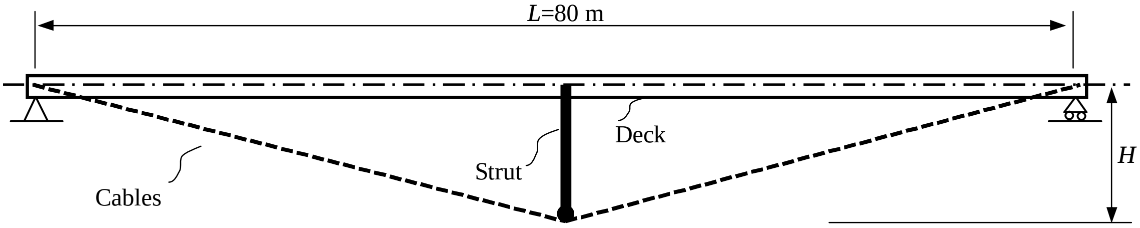
Solution algorithm

Since the structural system is nonlinear, an iterative method must be employed for the solution. The constant arc-length technique is employed in this study.

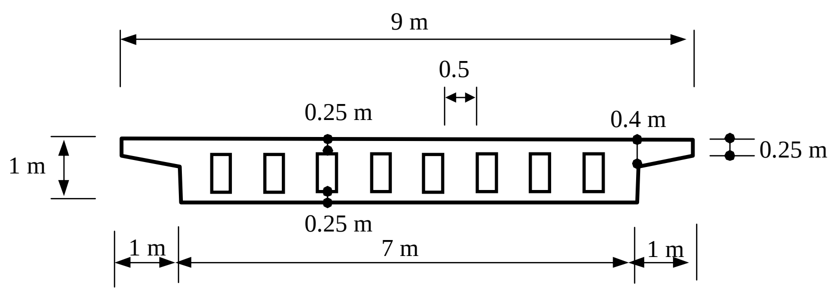
NUMERICAL EXAMPLE

Bridge properties

The nonlinear behaviour of an under-deck cable-stayed bridge as shown in Figure 6 is analyzed. The under-deck cable-staying system, made up of 130 strands, each with a cross section of 140 mm^2. The strands are self-anchored to the deck and deflected by struts. The prestressing ratio of cables is 65%. The reinforcement ratio of deck is assumed to be 3.5%. The concrete strength is assumed to be 60 MPa. The moduli of elasticity of steel bars and steel cables are assumed to be 200 GPa. The yield strength of steel bars is 400 MPa. The ultimate strength of prestressing steel for cables is 1860MPa. The material laws of concrete, steel bars, and prestressing steel for cables are as given in Section 2.4. The effect of arrangement of under-deck cables is addressed. The ratio of strut length to span, *H/L*, varies from 0.0375 to 0.1.



(a)



(b)

Figure 6. An under-deck cable-stayed bridges analyzed: (a) elevation; and (b) cross section of deck

Results

The load-deflection curves of bridge analyzed are shown in Figure 7. The ultimate loads of bridge with ratios *H/ L* of 0.0375, 0.05, 0.075 and 0.1 are 56656 kN, 66772 kN, 91099 kN, and 99086 kN respectively. The larger the ratio *H/ L* is, the higher the ultimate load is, while the earlier the bridge enters the plastic stage. The bridge with higher ratio *H/ L* has higher stiffness.

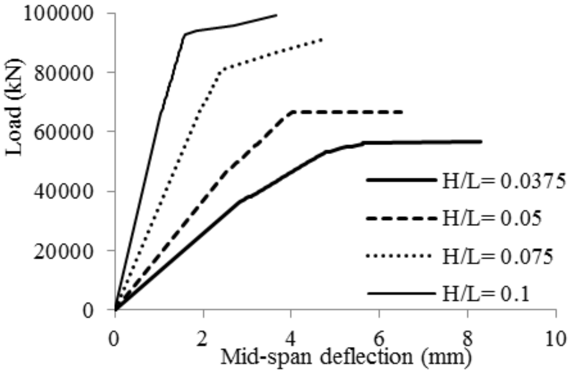


Figure 7. Load-deflection curves of bridge analyzed

The values of cable stress of bridge are shown in Figure 8. The ultimate cable stresses of bridge with ratios *H/ L* of 0.0375, 0.05, 0.075 and 0.1 are 1517 MPa, 1478 MPa, 1430 MPa, and 1355 MPa respectively. The larger the ratio *H/ L* is, the lower the ultimate cable stress is.

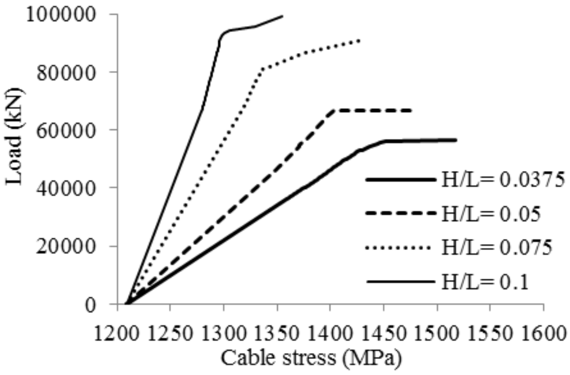


Figure 8. Variation of cable stress of bridge analyzed

The effect of other parameters such as strut number and load type are also needed to be further studied.

CONCLUSIONS

The present study proposed a numerical model to predict the nonlinear behaviour of under-deck cable-stayed bridges taking into account the interaction between deck and under-deck cables. The model considers the geometric nonlinearity and the actual stress-strain curves as well as strain reversals. A nonlinear finite element method is developed accordingly. A numerical example is given for illustration.

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