Simulation and Implementation of Two Stage Rotary Inverted Pendulum

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Abstract. Inverted pendulum control system is a complex, nonlinear, unstable system. This design on the basis of studying the law of motion of the inverted pendulum, build its trajectory mathematical model, using MATLAB simulation analysis. Based on mathematical modeling and simulation, according this build the system hardware circuit, after several times debug, can realize stable rotary and stand. Using K60 micro controller combined with PID algorithm to drive hollow cup motor rotate, according to the feedback information of the sensor, adjusting stand pendulum by PD algorithm, adjusting motor speed by PI algorithm. Using double circuit PD/PI control scheme realizes the rotating arm swinging angle and position closed loop control at the same time.

The Main Components of the System

The system block diagram is shown in Fig. 1, including the main control unit, motor control, sensor unit, and power supply and communication part. The sensor adopts two photoelectric rotary encoders, which are used to measure the angle of the swing rod and the speed of the motor. According to the sensor feedback information, the control signal is transmitted to the motor drive motor by using PID algorithm, which drives the rotation arm to rotate, and then control the swing rod to form a double closed loop control network[1-3].

The main control unit uses K60 micro controller. The chip is a 32 bit high performance microcontroller, on-chip resource rich, built-in PIT, FTM timer, 50MHZ clock frequency, with high calculation capacity. Sensor unit is photoelectric encoder. It can improve the measurement accuracy, ease of installation. Hollow cup motor is a DC permanent magnet servo motor, and. Hollow cup motor has excellent energy saving characteristics, sensitive and convenient control characteristics and stable operation characteristics. Hollow cup motor with gear box and photoelectric encoder, at the same time through the transformation of the reducer can reduce the impact of the virtual machine; can completely meet the design requirements [4-5].

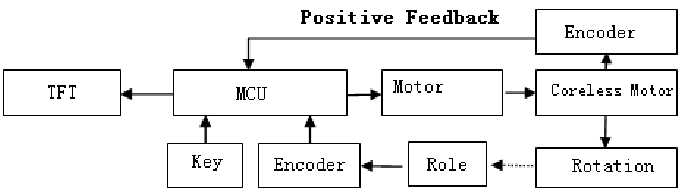

Figure 1. System chart

Theoretical Analysis and Calculation

Basic Model of the Inverted Pendulum. The rotating arm S1 is driven by a hollow cup motor at the rotating shaft, which drives the swing rod to rotate at the same time as shown in Fig. 2.

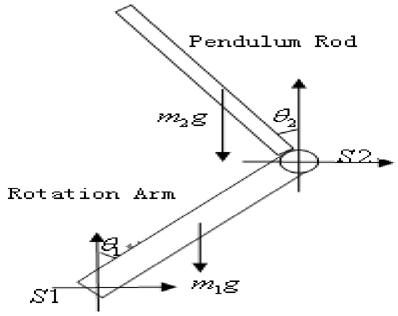

Figure 2. Model structure of inverted pendulum

A more accurate mathematical model is the basis of the design of the control system, as shown in Fig. 2, in the non-inertial frame S2[6]:

$$J_2 \ddot{\theta}_2 + f_2 \dot{\theta}_2 = M_{12} + m_2 g L_2 \sin\theta_2 \quad (1)$$

J_2 is pendulum inertia of rotating shaft, f_2 is friction coefficient, m2 corresponding to the quality of the pendulum, L2 as the pendulum shaft to centre distance, θ_2 is the angle between the swing rod and the vertical line, $\ddot{\theta}_2$ for the θ_2 two countdown, $\dot{\theta}_2$ is a derivative of θ_2, g for the acceleration of gravity, M_{12} rotating arm the torque of the swing rod is rotated, R is the length of rotation.

$$M_{12} = m_2 L_2 R[\dot{\theta}_1^2 \sin(\theta_1 - \theta_2) - \ddot{\theta}_1 \cos(\theta_1 - \theta_2)] \quad (2)$$

In the inertial system of S1, the rotating arm is

$$J_1\theta_1 + f_1 \dot{\theta}_1 = M_0 + M_{21} + m_1 g L_1 \sin\theta_1 \quad (3)$$

Among them, J1 is the moment of inertia of the rotary arm shaft (including the inertia of the motor shaft, F1 friction coefficient corresponding to the M1 is the quality of the rotating arm, rotating arm to L1 is the centre of the shaft distance[7-8]. θ1 rotating arm and the vertical line of the M0 motor output torque angle:

$$M_0 = K_m(u - K_e \dot{\theta}) \quad (4)$$

Km, Ke respectively for the motor torque coefficient and the counter electromotive force coefficient, to add the control voltage on the motor, M12 for the swing rod on the rotating arm of the torque. Simultaneous (1) - (4), eliminating the intermediate variable M12, and bringing the M0 into the system, the nonlinear model is obtained:

$$\begin{bmatrix} J_1 + m_2R^2 & m_2RL_2\cos(\theta_1-\theta_2) \\ m_2RL\cos(\theta_1-\theta_2) & J_2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} +$$

$$\begin{bmatrix} f_1 + K_mK_e & m_2RL_2\sin(\theta_2-\theta_1)\dot{\theta}_2 \\ m_2RL_2\sin(\theta_2-\theta_1)\dot{\theta}_1 & f_2 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} K_m \\ 0 \end{bmatrix} u + \begin{bmatrix} (m_1gL_1 + m_2gR)\sin\theta_1 \\ m_2gL_2\sin\theta_2 \end{bmatrix} \quad (5)$$

By using the method combined, can get all the mechanical parameter values into equation (5), available model.

Inverted Control Based on State Feedback. Inverted control is to maintain the inverted state near the unstable equilibrium position by means of proper control, which is the most basic requirement of the control of the inverted pendulum. When $\theta_1 \to 0, \theta_2 \to 0,$ Local linearization of the above formula (5), order $X=\begin{bmatrix}\theta_1 & \theta_2 & \dot{\theta}_1 & \dot{\theta}_2\end{bmatrix}^T$, $Y=[\theta_1 \quad \theta_2]^T$, The state equation of the system is obtained:

$$\begin{cases} X = AX + BU \\ Y = CX \end{cases} \tag{6}$$

Among them, A, B, C are as follows

$$A=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & 11.2347 & -3.0924 & -0.5864\\ 0 & 110.7574 & -6.6851 & -5.7810\end{bmatrix},\ B=\begin{bmatrix}0\\ 0\\ 48.8263\\ 105.5510\end{bmatrix},C=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

The system is unstable, and is completely controllable and observable. So we can design a controller based on state feedback system of pole configuration, so that the system can be stable in a neighborhood of the origin. The inverted pendulum device without directly measuring the angular velocity of the device, we adopt the angle difference approximation. In view of measurement, hardware and software using the filtering method. After pole placement design, the state feedback matrix is obtained.

$K=\begin{bmatrix}-3.1623 & 22.1221 & -2.1456 & 2.1444\end{bmatrix}$;

This group of parameters in the system for real-time control, the control effect of Fig. 3, shown in Fig. 4.

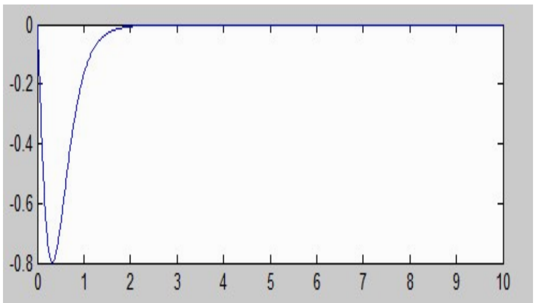

Figure 3. The control response curve after the disturbance

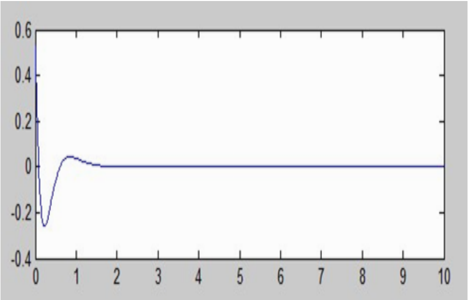

Figure 4. control response curve

From the above we can know that the inverted pendulum system is not stable but can control the system.

Hardware Circuit and Program Design

Hardware Design of the System. The hardware of the system is mainly composed of four parts, namely, the power part, the main control part, the motor drive part and the man-machine interactive part. One of the most important is the motor drive part, detailed features are as follows:

Motor Drive Part. Motor driven by double BTS7970, it is high power H bridge drive. We use two pieces of BTS7970B to form a complete full bridge drive current up to 70A, and the use of SCM and coupler isolation. It can be very good to achieve the motor forward, reverse, energy consumption brake.

System Software Design. PID algorithm:

After the construction of mathematical model, the model of the final use of the simplified model of the pendulum rod encoder and motor encoder of the double loop feedback system, using the classic PID control algorithm, the completion of the control of the inverted pendulum system[9].

PID control algorithm for the corresponding:

$$u = K_e e(t) + K_i \int_0^t e(t)dt + K_d \frac{de(t)}{d(t)} + u_0$$

$$= K_P[e(t) + \frac{1}{T_I}\int_0^t e(t)dt + T_d \frac{de(t)}{dt} + u_0] \qquad (7)$$

For the control of output in μ0 for initial deviation is zero mu, e (T) as the regulator input function, i.e. the deviation to quantitative and output, Kp is proportional gain, Ti is integral time constant, Td is differential time constant. The function can be divided into three parts, which are proportional control, integral control and differential system. The core controller is a discrete component, which needs to be approximated by the difference equation:

$$\int_0^t e(t)dt = \sum_{j=0}^{k} T_e(j) \text{，即} \quad \frac{de(t)}{de} = \frac{e(k) - e(k-1)}{T} \qquad (8)$$

Type T for the sampling period; K for sampling serial number; E (k) and E (k-1) were the first k-1 and K sub control cycle of the deviation, integrated above a few, can be a business point of the equation:

$$u(k) = K_P \left\{ e(k) + \frac{T}{T_I}\sum_{j=0}^{k} e(j) + \frac{T_d}{T}[e(k) - e(k-1)] \right\} + u_0 \qquad (9)$$

Test Results

Test method: when the pendulum hangs in the natural state, the external pull up the pendulum to the position of 165 DEG, the external force is withdrawn, the start control rotary arm, measuring pendulum keep the inverted state time, and the measurement of rotation angle of the rotating arm.

Table 1 The rotation angle of the rotating arm when the swing rod is kept on the inverted pendulum

times	1	2	3	4	5	6	7	8
Angle	12	12	10	8	11	9	11	13
Inverted time (s)	long	long	long	long	long	long	long	long

Summary

Through theoretical derivation and Simulation of MATLAB in the early completion of the possibility of inverted pendulum model in theory, through the hardware circuit design and software programming, to complete the physical production[10]. And after repeated testing and debugging, the final completion of the stability of the inverted pendulum system, the realization of the two level inverted pendulum system theory and real production.

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