

# Study on the Application of the Pure Angle Measurement of the Photoelectric Detector in the Aerial Maneuvering Target

Take the photoelectric radar as an example

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**Abstract**—Photoelectric detector is widely used in the military; it is mainly used to measure the azimuth of the air target. This paper mainly studies the requirements of the aircraft maneuvering when the photoelectric detector is pure angle measurement of air maneuvering targets. According to the principle of the pure angle detection of the photoelectric detector, using the maximum likelihood estimation method, the calculation formula is deduced, and a calculation method is presented.

**Keywords**—Maximum likelihood estimation method; Pure angle measurement; Solving formula; Photoelectric detecting device

## I. INTRODUCTION

In the photoelectric detection, the red of the target is accepted and measured. External radiation to determine the location of the target is referred to as the infrared passive location [1]. along with The rapid development of electronic technology, as well as its wide range in the modern war Pan application, the two sides of the operational requirements of the hidden weapons and equipment system is also More and more high, therefore, the development of passive location technology is the trend of the times[2].

The method of target motion analysis based on pure angle measurement is an effective method to measure the parameters of the target, and it has been widely used in civil and military fields. Because the military aircraft can use the photoelectric detection device to complete the aerial target azimuth measurement, so it has its specific advantages. Military aircraft can use the photoelectric detection device, according to the target hot spot to capture the target, they do not emit electromagnetic waves, so it has a strong anti-interference ability, and can play a very good stealth role; Secondly, due to the high speed flight of the aircraft, the heat radiation generated by the skin and air friction is very strong, so that the target can be measured at a distance, to ensure that there is enough time to intercept the target[3]. In this paper, based on the principle of pure angle measurement, the calculation formula of the aircraft to the air target parameters is obtained, and the specific calculation process is given according to various known conditions.

## II. HYPOTHETICAL CONDITION

In this paper, we study the condition of pure angle measurement in two-dimensional plane, and extend it to three dimensional spaces, which have no significant influence on the condition of this paper.

The relationship between air target and own aircraft motion is shown in Fig. 1:

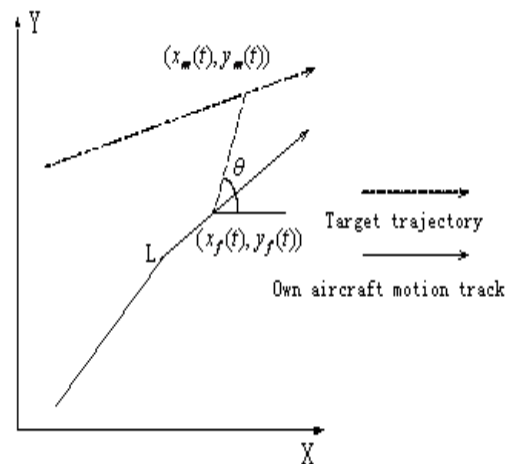


Fig. 1. The relationship between air target and own aircraft motion

Assume the target for uniform linear motion and the machine for a uniform motion to L points, and then change the direction to continue to make uniform linear motion. The measurement equation is:

$$\theta(t) = \arctan\left[\frac{y_m(t) - y_f(t)}{x_m(t) - x_f(t)}\right] + e(t) \quad (1)$$

$\theta(t)$  : The angle of measurement error;

$(x_m(t), y_m(t))$  : Target location for t time;

$(x_f(t), y_f(t))$  :Position for the t moment of its own aircraft;

$e(t)$ : Measurement error for t time.

Assuming that  $e(t)$  follows normal distribution,

$e(t) \sim N(0, \sigma^2)$ , and  $E[e(i) \cdot e(j)] = \begin{cases} \sigma^2, & i = j \\ 0, & i \neq j \end{cases}$ , two random measurement errors at any time are not correlated.

$P = [x_{m0}, y_{m0}, \dot{x}_m, \dot{y}_m]$ : Target motion parameters;

$Q = [x_{f0}, y_{f0}, \dot{x}_f, \dot{y}_f]$ : Own aircraft motion parameters.

The formula (1) can be written:

$$\theta(t) = a \tan \left[ \frac{(y_{m0} - y_{f0}) + (\dot{y}_m - \dot{y}_f)t}{(x_{m0} - x_{f0}) + (\dot{x}_m - \dot{x}_f)t} \right] + e(t) \quad (2)$$

Hypothesis (2) the result is  $[x_{m0}, y_{m0}, \dot{x}_m, \dot{y}_m]$ , the existence of any  $a \neq 0$ , so that the next set up:

$$\theta(t) = a \tan \left[ \frac{a(y_{m0} - y_{f0}) + a(\dot{y}_m - \dot{y}_f)t}{a(x_{m0} - x_{f0}) + a(\dot{x}_m - \dot{x}_f)t} \right] + e(t) \quad (3)$$

Through the (3) can have to solve the infinite number of:

$$P = [a(y_{m0} - y_{f0}) + y_{f0}, a(\dot{y}_m - \dot{y}_f) + \dot{y}_f, a(x_{m0} - x_{f0}) + x_{f0}, a(\dot{x}_m - \dot{x}_f) + \dot{x}_f]$$

So it is not able to determine the target motion parameters, and therefore need to maneuver the aircraft to come to the actual value of the target motion parameters. However, in some special circumstances, the aircraft can't be measured after maneuver, Here is not discussed in detail, please refer to the details of the reference [4], which has a detailed discussion of the three-dimensional situation. In this paper, we assume that the aircraft maneuver can be measured.

### III. MEASUREMENT PRINCIPLE AND METHOD

Now the condition is applied to the three-dimensional space, in the three-dimensional space of the target and the motion of the machine as shown in Fig. 2. In this paper, the problem is studied in the geographic coordinate system for the sake of observation[5].

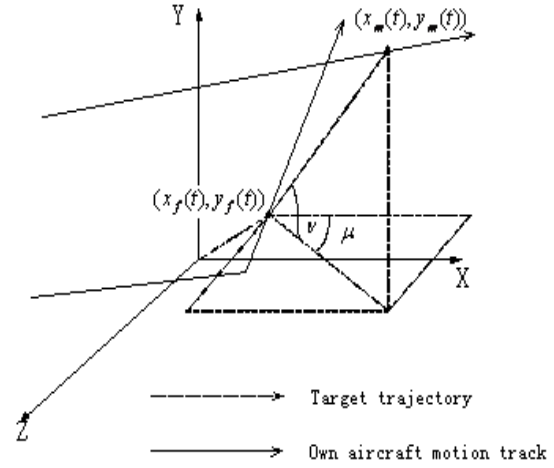


Fig. 2. In the three-dimensional space of the target and the motion of the machine

Assume the target for uniform linear motion, and their aircraft to make uniform linear motion, and then change the direction to continue to make uniform linear motion.

Target motion parameters are:

$$M = [x_{m0}, y_{m0}, z_{m0}, \dot{x}_m, \dot{y}_m, \dot{z}_m];$$

The motion parameters of own aircraft

$$F = [x_{f0}, y_{f0}, z_{f0}, \dot{x}_f, \dot{y}_f, \dot{z}_f];$$

the motion parameters of our aircraft are known.

Therefore can be obtained:

$$\begin{bmatrix} \mu(t) \\ \nu(t) \end{bmatrix} = \begin{bmatrix} a \tan \left[ \frac{z_m(t) - z_f(t)}{x_m(t) - x_f(t)} \right] \\ a \tan \left[ \frac{y_m(t) - y_f(t)}{\sqrt{[x_m(t) - x_f(t)]^2 + [z_m(t) - z_f(t)]^2}} \right] \end{bmatrix} + \begin{bmatrix} e_\mu(t) \\ e_\nu(t) \end{bmatrix} \quad (4)$$

Hypothesis:

$$\begin{cases} f(M, t) = a \tan \left[ \frac{z_m(t) - z_f(t)}{x_m(t) - x_f(t)} \right] \\ g(M, t) = a \tan \left[ \frac{y_m(t) - y_f(t)}{\sqrt{[x_m(t) - x_f(t)]^2 + [z_m(t) - z_f(t)]^2}} \right] \end{cases} \quad (5)$$

Then (4) can be written:

$$\begin{bmatrix} \mu(t) \\ \nu(t) \end{bmatrix} = \begin{bmatrix} f(M, t) \\ g(M, t) \end{bmatrix} + \begin{bmatrix} e_\mu(t) \\ e_\nu(t) \end{bmatrix} \quad (6)$$

That is:

$$\mu(t) = f(M, t) + e_\mu(t) \quad (7)$$

$$\nu(t) = g(M, t) + e_\nu(t) \quad (8)$$

Hypothesis:  $e_\mu(t) \sim N(0, \sigma_\mu^2)$ ,  $e_\nu(t) \sim N(0, \sigma_\nu^2)$

In this paper, the maximum likelihood estimation method is used to calculate the maximum likelihood estimation method. The so-called maximum likelihood estimation method is to select the parameters value causes the sample maximum possibility and use this value as the value estimation of unknown parameters. The first solution is formula (7) and (8) the calculation method is basically the same[6].

Constructing maximum likelihood function:

$$L_1(M) = \prod_{k=1}^n \frac{1}{\sqrt{2\pi}\sigma_\mu} \exp\left\{-\frac{1}{2\sigma_\mu^2} [\mu(t) - f(M, t)]^2\right\} \quad (9)$$

Hypothesis:  $t = kT$ ;  $T$ : Measurement period,  $k = 1, 2, \dots, n$

$$\max_M L_1(M) = \sum_M \text{Max} \ln L_1(M) \text{ is } \min_M [-\ln L_1(M)]$$

in this way, we can transform to solve the unconstrained extremum problem, which can be used in the method of damped Newton method[7].

The expression of  $-\ln L_1(M)$  is as follows:

$$-\ln L_1(M) = \sum_{k=1}^n \ln \sigma_\mu + \frac{1}{2} \sum_{k=1}^n \frac{1}{\sigma_\mu^2} [\mu(kT) - f(M, kT)]^2 \quad (10)$$

#### IV. SOLUTION OF EQUATION

Hypothesis:  $p(M) = -\ln L_1(M)$ ,  $p(M)$  according to the Taylor formula and spent more than two times of the items available:

$$p(M) \approx q^{(k)}(M) = p(M^{(k)}) + \nabla p(M^{(k)})(M - M^{(k)})^T + \frac{1}{2}(M - M^{(k)})\nabla^2 p(M^{(k)})(M - M^{(k)})^T \quad (11)$$

Among:

$$\nabla p(M) = \begin{bmatrix} \frac{\partial p}{\partial x_{m0}} & \frac{\partial p}{\partial y_{m0}} & \frac{\partial p}{\partial z_{m0}} & \frac{\partial p}{\partial \dot{x}_m} & \frac{\partial p}{\partial \dot{y}_m} & \frac{\partial p}{\partial \dot{z}_m} \end{bmatrix}$$

$$\nabla^2 p(M) = \begin{bmatrix} \frac{\partial^2 p}{\partial x_{m0}^2} & \frac{\partial^2 p}{\partial x_{m0}\partial y_{m0}} & \frac{\partial^2 p}{\partial x_{m0}\partial z_{m0}} & \frac{\partial^2 p}{\partial x_{m0}\partial \dot{x}_m} & \frac{\partial^2 p}{\partial x_{m0}\partial \dot{y}_m} & \frac{\partial^2 p}{\partial x_{m0}\partial \dot{z}_m} \\ \frac{\partial^2 p}{\partial y_{m0}\partial x_{m0}} & \frac{\partial^2 p}{\partial y_{m0}^2} & \frac{\partial^2 p}{\partial y_{m0}\partial z_{m0}} & \frac{\partial^2 p}{\partial y_{m0}\partial \dot{x}_m} & \frac{\partial^2 p}{\partial y_{m0}\partial \dot{y}_m} & \frac{\partial^2 p}{\partial y_{m0}\partial \dot{z}_m} \\ \frac{\partial^2 p}{\partial z_{m0}\partial x_{m0}} & \frac{\partial^2 p}{\partial z_{m0}\partial y_{m0}} & \frac{\partial^2 p}{\partial z_{m0}^2} & \frac{\partial^2 p}{\partial z_{m0}\partial \dot{x}_m} & \frac{\partial^2 p}{\partial z_{m0}\partial \dot{y}_m} & \frac{\partial^2 p}{\partial z_{m0}\partial \dot{z}_m} \\ \frac{\partial^2 p}{\partial \dot{x}_m\partial x_{m0}} & \frac{\partial^2 p}{\partial \dot{x}_m\partial y_{m0}} & \frac{\partial^2 p}{\partial \dot{x}_m\partial z_{m0}} & \frac{\partial^2 p}{\partial \dot{x}_m^2} & \frac{\partial^2 p}{\partial \dot{x}_m\partial \dot{y}_m} & \frac{\partial^2 p}{\partial \dot{x}_m\partial \dot{z}_m} \\ \frac{\partial^2 p}{\partial \dot{y}_m\partial x_{m0}} & \frac{\partial^2 p}{\partial \dot{y}_m\partial y_{m0}} & \frac{\partial^2 p}{\partial \dot{y}_m\partial z_{m0}} & \frac{\partial^2 p}{\partial \dot{y}_m\partial \dot{x}_m} & \frac{\partial^2 p}{\partial \dot{y}_m^2} & \frac{\partial^2 p}{\partial \dot{y}_m\partial \dot{z}_m} \\ \frac{\partial^2 p}{\partial \dot{z}_m\partial x_{m0}} & \frac{\partial^2 p}{\partial \dot{z}_m\partial y_{m0}} & \frac{\partial^2 p}{\partial \dot{z}_m\partial z_{m0}} & \frac{\partial^2 p}{\partial \dot{z}_m\partial \dot{x}_m} & \frac{\partial^2 p}{\partial \dot{z}_m\partial \dot{y}_m} & \frac{\partial^2 p}{\partial \dot{z}_m^2} \end{bmatrix}$$

Here we take  $\min_M q^{(k)}(M)$ , is  $\nabla q^{(k)}(M) = 0$  solution as  $\min_M p(M)$  next approximation  $M^{(k+1)}$ . By the formula (11) can be obtained:  

$$\nabla q^{(k)}(M) = \nabla p(M^{(k)}) + \nabla^2 p(M^{(k)})(M - M^{(k)}) \quad (12)$$

By the formula (12) can get the root of  $\nabla q^{(k)}(M) = 0$  as:  

$$M = M^{(k)} - [\nabla^2 p(M^{(k)})]^{-1} \nabla p(M^{(k)})$$

Iterative formula:

$$M^{(k+1)} = M^{(k)} - [\nabla^2 p(M^{(k)})]^{-1} \nabla p(M^{(k)}) \quad (13)$$

In order to ensure the convergence of the formula and the choice of the initial point no special requirements, the formula (13) into the following form[8]:

$$M^{(k+1)} = M^{(k)} - t_k [\nabla^2 p(M^{(k)})]^{-1} \nabla p(M^{(k)}) \quad (14)$$

Among them tk make:

$$p[M^{(k)} - t_k \nabla p(M^{(k)})] = \min_t p[M - t_k \nabla p(M)] \quad (15)$$

For formula (7), because it contains only 4 unknowns, that is:  $[x_{m0}, z_{m0}, \dot{x}_m, \dot{z}_m]$ , So after getting the results, should be introduced into the formula (8), in order to obtain  $[y_{m0}, \dot{y}_m]$ , of course, the equation is relatively simple compared to the above all kinds of.

#### V. SUMMARY

For a long time, the mathematical methods of passive measurement of target parameters at home and abroad can be divided into two categories: geometric positioning method and filtering method. Among them, the first one is the geometric location method, but the passive estimation problem is nonlinear, is not conducive to the use of geometric positioning method, and thereafter, with the development of modern signal processing technology, proposed a series of filtering algorithms. As a result of the maneuverability of the single measurement, we have made the research on the measurement of the target parameters.

To meet only the observability requirements of the passive measurement of the only carrier, only the passive detection and location of the radiation is realized by the accumulation of time. At present, there are two approaches to the passive measurement of target parameters, which are mainly used in foreign countries. At present, there are two approaches to the passive measurement of target parameters in foreign countries. One is the machine's own maneuver, and the parameters of the moving target are measured to achieve the state estimation; two is to try to increase the observation information, to achieve the purpose of rapid and accurate positioning.

As the photoelectric detector is a passive measurement device, the distance between the target and the target is unknown, so there is a problem that can be measured. In this

paper, we study the condition of pure angle measurement in two-dimensional space under the condition that the target is given, and then in three dimensional spaces, the calculation formula of the air target is derived, an efficient solution method is presented, known as damped Newton method. This method is fast, and has no special requirements for the selection of initial points. On the whole, the idea of the theoretical method is relatively new, it has its application value in the measurement of the air target in their aircraft, and it can be used in the future to carry out further research on it.

When the machine is used for measuring and locating, the system is not observable when the carrier does not carry out the maneuver. This requires the machine to carry out its maneuver, however, the machine is only a necessary condition for the observation of the system, therefore, the performance of the passive measurement method of the single machine is affected by the measurement noise, also affected by the impact of this machine's motor program, and the estimated time is relatively long. Therefore, considering the use of multi machine joint estimation method, the orientation of the same target is fused to estimate the target motion factors. This method avoids the machine's maneuver and shortens the time of estimation. We can use the space position distribution to realize the instantaneous detection and location of the target radiation source. According to the different measurement information, we can form a variety of positioning system. There are some methods to estimate the parameters of the target, such as cross

position, time difference, and combined location and so on. At present, the world has been deployed or in the research system basically sampling two kinds of systems: the time difference positioning system and the angle of the cross positioning system.

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