Reliability Analysis of Tunnel Lining under Earthquake Loading Using Response Surface Method

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Abstract. The variation of rock mass mechanical parameters has a significant impact on the dynamic response of tunnel lining. Regarding the deformation modulus, Poisson's ratio, cohesion and frictional angle of rock mass as basic random variables, the effect of these parameters on the reliability index of tunnel ling was researched using finite element method. Response Surface Methodology was utilized to estimate the functional relationship between the load effects on tunnel lining and mechanical parameters based on numerical results. The results show that the reliability indices decrease with the means and variation coefficient of basic random variables increasing, and the mechanical parameters of rock mass have different sensitivities on the reliability index of tunnel lining in different situations.

Introduction

Generally, in aseismic design of tunnel, rock mass surrounding tunnel is considered as homogeneous material and the rock properties are assumed estimates from empirical characterization techniques or based on relevant data collected for other tunnel projects in similar area. More recently, probabilistic methods have been adopted to account for uncertainty in rock mass properties. Hang-Zhou Li and Bak Kong Low [1] regarded the cohesion, frictional angle and deformation modulus as normally distributed random variables and used the first-order reliability method (FORM) to calculated the reliability index of a circular tunnel. Qing Lü et al. [2] used the response surface method (RSM) to perform the reliability analysis of a circle tunnel and analyzed the failure probability with respect to different criteria. B.K. Low and H.H. Einstein [3] examined two existing definitions of factor of safety for a roof wedge in a tunnel and proposed a reliability-based design for tunnel supports. Liu Hui-jun and Yu Su [4] analyzed the stress and deformation of tunnel lining over time and determined the safety of the tunnel lining structure. However, these reliability analyses of tunnels have not considered the dynamic response under earthquake loading.

In this study, the reliability of tunnel lining suffering earthquake was investigated considering the deformation modulus, Poisson's ratio, cohesion and frictional angle of rock mass as basic random variables. The discussion was focused on the effect of variation coefficient and mean value of rock mass parameters on the reliability index of tunnel lining under earthquake. At same time the sensitivity of these parameters were also investigated.

Analysis of Loading Effect on Tunnel Lining

Performance Function. In tunnel engineering projects, numerical simulation results show that the shearing force carried by tunnel lining is much less than the shear strength of concrete. The eccentric compression is the main reason of tunnel lining failure [5]. Following the standard convention from the code for design of road tunnel (JTG D70-2004), the performance function for tunnel lining can be defined as:

$$Z=\begin{cases}\varphi\alpha\sigma_c bh-N & e_0\le 0.2h\\ \frac{1}{6}(1.75\varphi\sigma_t bh^2+hN)-M & e_0\ge 0.2h\end{cases}. \tag{1}$$

Where φ is longitudinal bending coefficient; α is eccentric effect factor; σ_c is compressive strength of concrete; σ_t is tensile strength of concrete; b is longitudinal dimension, and assumed to be equal to 1; h is the section thickness; $e_0=M/N$; N and M are the axial force and bending moment of tunnel lining, respectively.

Response Surface Function of Load Effects. RSM has a wide application in civil engineering reliability analysis [6]. In RSM, the actual function is approximated, usually by a polynomial function. The finite element analysis is used to evaluate the function in the region where the design point is expected to be located on the actual (but implicit) function [2]. The resulting approximate, fitted surface then becomes the explicit equivalent of the implicit function.

For the case that the reliability assessment problem under consideration is governed by n basic random variables x_1, x_2,…, x_n. Using a second-order polynomial response surface function to fit the relationship between internal forces and basic random variables:

$$Z=g(x_1,x_2,\ldots,x_n)=a+\sum_{i=1}^{n}b_i x_i+\sum_{i=1}^{n}c_i x_i, \tag{2}$$

where, a, b_i, c_i are indeterminate coefficients; x_1, x_2,…, x_n are basic random variables.

In this study, the deformation modulus E, Poisson's ratio μ, cohesion c and frictional angle φ of rock mass were regarded as basic random variables, other parameters are regarded as constant values. The rock mass mechanical properties and statistical characteristics were summarized in Table 1.

Table 1: Statistical characteristics of variables

Random Variables	E(GPa)	μ	c (MPa)	tanφ
Mean value	12	0.26	2	0.7
Standard deviation	1.8	0.15	0.2	0.1
Coefficient of variation	0.15	0.058	0.1	0.143
Distribution	Normal	Normal	Normal	Normal
Variable x_i	x_1	x_2	x_3	x_4

It was assumed that the axial force N and bending moment M of tunnel lining were the functions of E, μ, c and tanφ. The functions can be approximated by a second-order model polynomial model according equation (2):

$$M=a_0+a_1x_1+a_2x_2+a_3x_3+a_4x_4+a_5x_1^2+a_6x_2^2+a_7x_3^2+a_8x_4^2, \tag{3}$$

$$N=b_0+b_1x_1+b_2x_2+b_3x_3+b_4x_4+b_5x_1^2+b_6x_2^2+b_7x_3^2+b_8x_4^2, \tag{4}$$

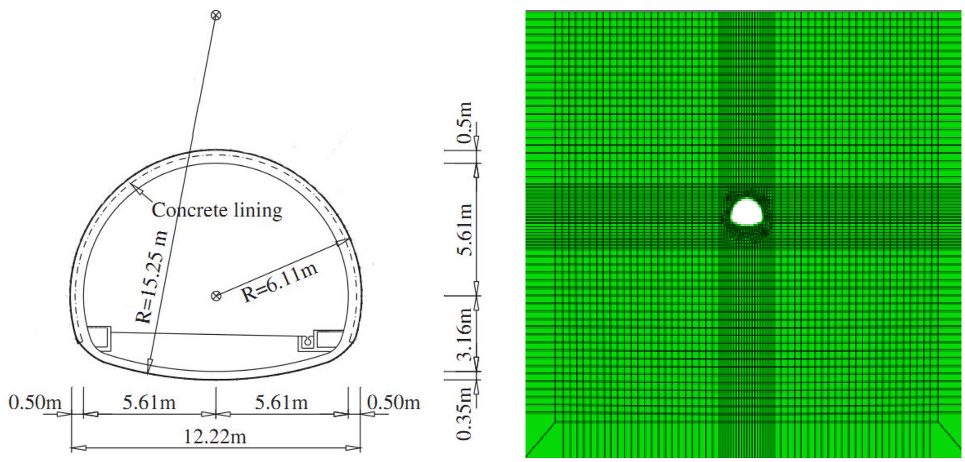
where，x_1, x_2, x_3 and x_4 are the deformation modulus E, Poisson's ratio μ, cohesion c, and frictional angle φ of rock mass respectively. a_0, a_1,…, a_8 and b_0, b_1,…, b_8 are regression coefficients. The regression coefficients are obtained from discrete evaluations of the implicit function.

A general method proposed by Gong Jinxin [7] for computing reliability index was used to calculate the reliability index of tunnel lining under seismic loading.

Reliability Analysis of a Circle Tunnel Lining

A Supported Tunnel and Its Parameters. The geometric characterized of the current tunnel was shown in Fig. 1(a). It is a horseshoe tunnel with height and span of 9.62m and 12.22m, respectively. The lining is assumed to be 0.5m thick. It is buried with the depth of 64m from the top of the tunnel to ground surface.

As illustrated in Fig. 1(b), the rock mass around the tunnel was modeled with a 2D finite element model consisting of four-node quadrilateral elements. The tunnel lining was simulated using beam elements. In order to reduce or eliminate reflections of earthquake waves for boundaries of the finite element model, the infinite elements were added to the finite element model on the three sides of the model. The Mohr-Coulomb failure criterion was taken to simulate the plasticity of the rock mass and the lining is assumed as elastic. The statistical characteristics of rock parameters were summarized in Table 1. For tunnel lining, E=24GPa, μ=0.17 and ρ=2400kg/m^3. The El Centro wave was selected as input excitation. The Peak Ground Acceleration (PGA) is 100gal. The attenuation coefficient is assumed to be 5%.



(a) cross-section of tunnel in geometry (b) finite element model

Fig. 1: The tunnel lining characteristics and finite element model

Reliability Index of Tunnel Lining Affected by Rock Parameters Variable Coefficient. For rock mass mechanical parameters, E, μ, c and φ, reasonable ranges are full of uncertainty. In order to clear the effect of these parameters on the reliability of tunnel lining, the reliability and sensitivity indices under earthquake loading were calculated to quantify the impact of variation coefficients and mean values of rock mass mechanical parameters on the reliability of tunnel lining.

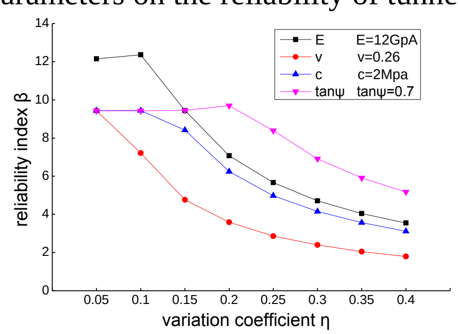
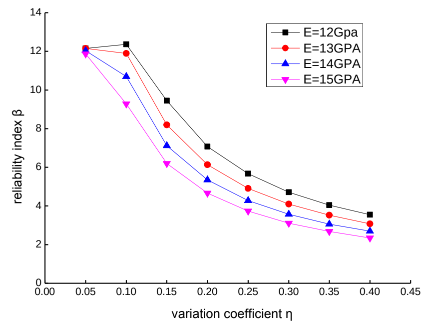


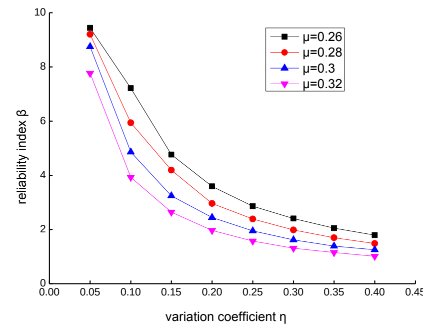
Fig. 2: Changing curves of reliability index with varying of variation coefficient

Eight cases with different coefficients of every variation were calculated to investigate the reliability index variation. Coefficient varies from 0.05 to 0.4. In these analyses, when one input variable regarded as random variables, other input variables are treated as deterministic inputs. Fig. 2 shows the trend of the effect of variation coefficient on the reliability index. The reliability indices decrease significantly with the increase of variation coefficients when the mean values of E, μ and c are determined. This indicate that the reliability index has a great change with a small change of the variation coefficients. When the variation coefficient is 0.05, the reliability index is 12.15. It is dramatically reduced to 3.55 when the variation coefficient is 0.4, decreasing 68.5%. When the variation coefficients are greater than

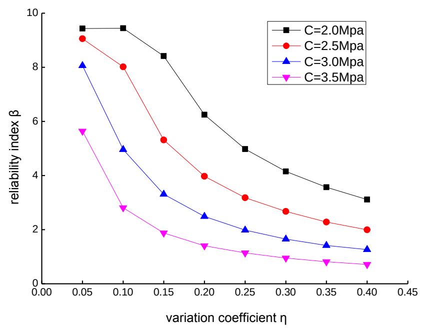
0.05, the declining tendency of reliability indices becomes slower. For the parameter tanφ, the reliability index is almost constant when the variation coefficient ranges from 0.05 to 0.2. However, the reliability index presents drastically decreasing tendency when it is greater than 0.2.



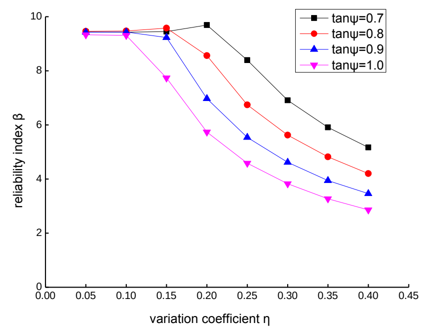
(a) effect of E mean value



(b) effect of μ mean value



(c) effect of c mean value



(d) effect of $\tan\varphi$ mean value

Fig. 3: Changing curves of reliability index with varying of mean value

Fig. 3 shows the change curves of reliability index with the increasing of variation coefficient when s the mean value of rock mass mechanical parameters increases. The reliability index increase with the same coefficient of variation. When the mean value of E increases from 12GPa to 15GPa, the reliability index decreases except the case that the variation coefficient equals to 0.05. When the mean value of deformation modulus E is 15GPa, the index decreases about 74.8% with the variation increasing to 0.4 from 0.1. As for parameter μ and c, the reliability indices of tunnel lining also decline with the mean value increasing in different values of variation coefficient. The reliability index is almost stability by changing the mean value of tanφ while the variation coefficient increases to 0.1 from 0.05.

Sensitivity analysis. An approximation of the sensitivity of reliability index can be determined for each parameter x using the following equation [8]:

$$S_\beta = \frac{\Delta\beta}{\Delta x} \bullet \frac{x_0}{\beta_0}\,. \tag{7}$$

Where S_β is the sensitivity of reliability index, x_0 is a constant of the variable x in basic case, β_0 is the corresponding reliability index when x takes x_0.

The basic case, with the properties of rock mass listed in Table 1, is applied to calculate the sensitivity of reliability to changes in random variables for cases with different variation coefficient and mean value. Corresponding, the reliability index β_0 at the crown of liner equal to 9.443 in the basic case.

In the sensitivity analysis of the effect of variation coefficient, variation coefficient of E, μ, c and tanφ were treated as a random variable to reflect its uncertainty, and the mean values of these parameters were regarded as constants. As seen in Fig. 4, the sensitivity of rock mass mechanical parameters on reliability index increases with the variation coefficient η. The variability of μ has a larger impact on the sensitivity of reliability index than other rock mass mechanical parameters. When the coefficient of variation is 0.4, the sensitivity of Poisson's ratio μ increases to 5 while the sensitivities of the cohesion c and deformation modulus E are about 2.7. The sensitivity of friction coefficient tanφ increases slowly with the variation coefficient changing. When the coefficient of variation is 0.4, the sensitivity of friction coefficient tanφ is just 1.286.

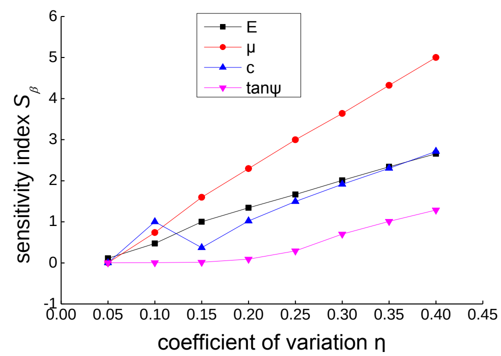


Fig. 4: The sensitivity indexes of reliability under surrounding rock parameter variation

Table 2: The sensitivity indexes S_E under the mean change of E

E (Gpa)	11.5	12	12.5	13	13.5	14	14.5	15
β_E	7.59	7.07	6.60	6.14	5.74	5.35	4.98	4.66
S_E	1.74	1.77	1.80	1.94	1.89	2.04	2.11	2.07

Table 3: The sensitivity indexes S_μ under the mean change of μ

μ	0.25	0.26	0.27	0.28	0.29	0.3	0.31	0.32
β_μ	3.92	3.59	3.26	2.96	2.68	2.45	2.20	1.97
S_μ	2.44	2.38	2.71	2.84	3.00	2.91	3.49	3.74

Table 4: The sensitivity indexes S_c under the mean change of c

c (MPa)	2.0	2.2	2.4	2.6	2.8	3	3.2	3.4
β_c	6.25	5.21	4.36	3.64	3.01	2.49	2.01	1.59
S_c	2.06	2.19	2.35	2.55	2.93	3.13	3.81	4.45

Table 5: The sensitivity indexes $S_{\tan\varphi}$ under the mean change of $\tan\varphi$

$\tan\varphi$	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2
$\beta_{\tan\varphi}$	8.96	9.16	9.69	8.56	6.97	5.74	4.75	3.94
$S_{\tan\varphi}$	0.09	0.13	0.38	1.05	2.05	2.15	2.28	2.46

Table 2 to Table 5 show the sensitivity of rock mass mechanical parameters on reliability index changing with the mean value of parameter. For deformation modulus E of rock mass, the fluctuation of sensitivity index is stability, it means that the reliability index has less affected by the mean value of E. For μ and c, the sensitivity indexes obviously increase with the changing of their means, hence the mean values of μ and c have an essential impact on reliability index. However, the sensitivity index is small and changes smoothly when the mean of tanφ ranges from 0.5 to 0.7, which means the reliability index is stability. The sensitivity index gradually increases as the mean value of tanφ changes from 0.7 to 1.2. The reliability index decreases rapidly when the mean value of tanφ is greater than 0.7. So it has a giant effect on reliability index.

Summary and Conclusions

Comparing with results, the contents and conclusions can be summarized as follows:

(1) With the increasing of variation coefficient, the reliability index of tunnel lining is trending downward. But for the parameter tanφ, the reliability index is almost constant when the variation coefficient ranges from 0.05 to 0.2.

(2) When the mean value of E, μ, c and tanφ changes, the variation tendencies of reliability indices are almost the same. With the increasing of mean value, the reliability index is on the decrease. But the decreasing extents are different when the mean values increase with a certain step length.

(3) The variability of μ has a larger impact on the sensitivity of reliability index than other rock mass mechanical parameters. The reliability index has less affected by the mean value of E. For μ and c, the sensitivity indexes obviously increase with the changing of their means.

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