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# Simulation and Implementation of Airborne Radar Ground Clutter Based on Weibull Distribution

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**Abstract.** The simulation and hardware realization of ground clutter in airborne radar are discussed in this paper. We use the statistical approach to model the clutter and utilize ZMNL method to achieve the simulation of coherent Weibull distribution sequences. This paper presents an efficient method of ground unit division and gives a mapping conversion process of coordinate transformation from ground area to radar system. Ultimately ground clutter time-domain echo signal is attained through coherent superposition and the results of clutter signal are verified on a hardware platform.

## Introduction

Clutter signal has a significant impact on the performance of airborne radar and the simulation of the real electromagnetic environment. Radar clutter simulation method is mainly divided into two types, one is the radar clutter power equation-based model[1], and the other, clutter coherent model[2], contains the amplitude and phase of the signal, which describes all the information about environment of radar. Since ground clutter is a kind of distributed surface clutter, traditional assembly point scattering model is no longer applicable. Grid mapping method[2] is proposed to partition the ground area which is irradiated by antenna beam. The radar cross-sectional area of clutter unit contains a non-uniform characteristic, whereas mathematical models are generally used for modeling the fluctuation characteristic. The statistical distribution and correlation characteristics of clutter cross-sectional area reflection characteristic are discussed in literature[3]. In engineering applications, the random sequence of clutter with specific amplitude distribution and relevant characteristics are commonly generated by the method of Monte Carlo, which is achieved by Zero Memory Non-Linearity (ZMNL)[4] and Spherically Invariant Random Process (SIRP)[5] according to statistical model based on theoretical and experimental data. The ground clutter signal within the range of which beam irradiate can be obtained by coherent summation of clutter echoes which comes from the ground scattering units.

## Modeling and Simulation of the Clutter

In order to reflect the actual coherent clutter echo signal which includes amplitude and phase, and indicates the impact of external environmental on signal, the method of coherent video signal model is adopted in this paper. The signal model of point scatter echo is defined as:

$$\varphi_R(t) = \varphi_T\left[t-\tau(t)\right]\left[\frac{\lambda^2}{(4\pi)^3 r^4}\right]^{1/2} G(t)\gamma(t) \tag{1}$$

Here, $\varphi_T(t)$ is a transmit signal, $\tau(t)$ is signal delay and $G(t)$ is antenna gain, where $\gamma(t)$ is the complex item which describes the amplitude and phase of the target reflection characteristics, it is defined as $\gamma = \sqrt{\sigma}\exp(j\phi)$. Seen from the above equation, the antenna pattern model $G_t$, the complex scattering cross section $\gamma$, the transmitted signal and relative distance will have an impact on the clutter echo.

**Statistical Modeling.** The motion of the scattering unit can lead to random fluctuation characteristic of the target reflection characteristic $\gamma$, which can be reflected by variation of the amplitude and phase of the clutter echo. Accordingly, $\gamma$ can be expressed as a random variable that satisfies the specific amplitude distribution and correlation characteristics. The simulation scenario of the article is set as follows: grazing angle is $10^0$, signal bandwidth is 40MHz, pulse width is 20us and terrain undulating is relatively uniform. By literature[6], it can be known that Weibull distribution is more reasonable. The probability density function of Weibull distribution is defined as:

$$p(x)=\begin{cases}\frac{p}{q}\left(\frac{x}{q}\right)^{p-1}\exp\left[-\left(\frac{x}{q}\right)^{p}\right] & ,x\geq 0,p>0,q>0\\ 0 & ,x<0\end{cases} \tag{2}$$

As for airborne radar ground clutter, the statistical model of clutter also contains the time correlation characteristics, which are commonly described by power spectrum density. The PSD of ground clutter is generally characterized by the Gauss model[7] , N spectrum model[8]错误！未找到引用源。 and other models.

**Generation of correlated Weibull distribution sequences.** This paper presents the process of generating coherent correlation Weibull distribution clutter through ZMNL transform, and the transformation diagram is as follows:

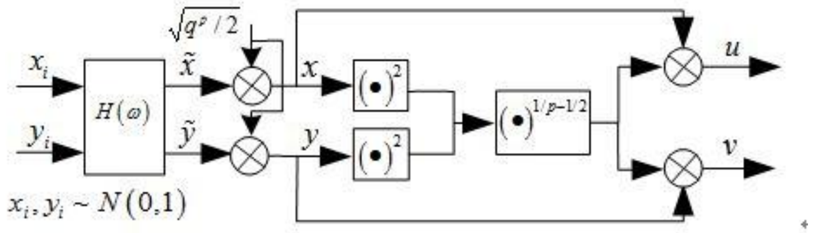

Figure.1 Correlated Weibull distribution clutter generator diagram

As illustrated in Fig.1, $\gamma=u+jv$ is the resulting complex correlated Weibull random variables, and it can be expressed as $u=x\left(x^2+y^2\right)^{1/p-1/2}$, $v=y\left(x^2+y^2\right)^{1/p-1/2}$.Where $x,y$ is the correlated Gauss sequence which respectively obtained by $x_i$ and $y_i$ which passed through the linear filter and modulated by amplitude correction. Since the ZMNL method is a nonlinear transformation, it's necessary to obtain the autocorrelation function mapping of correlation Gaussian sequences between $w=x+jy$ and $\gamma=u+jv$. The ACF of the resulting sequence $\gamma=u+jv$ is:

$$R_\gamma(k)=E\left[\left(u(n)+jv(n)\right)\left(u(n+k)+jv(n+k)\right)\right]=R_{uu}(k)+R_{vv}(k)+j\left[R_{vu}(k)-R_{uv}(k)\right] \tag{5}$$

Suppose that $\gamma$ is a generalized stationary stochastic process, then $R_{vu}(k)=-R_{uv}(k), R_{uu}(k)=R_{vv}(k)$, so $R_\gamma(k)=2\left[R_{uu}(k)-jR_{uv}(k)\right]$, and the normalization result of $R_\gamma(k)$ is obtained:

$$r_\gamma(k)=R_\gamma(k)/R_\gamma(0)=\left[r_{uu}(k)-jr_{uv}(k)\right] \tag{6}$$

The method of variable mapping for solving nonlinear equations is given:

$$\mu(k)=\frac{R_{uv}(k)}{R_{uu}(k)}=\frac{r_{uv}(k)}{r_{uu}(k)}=\frac{r_{xy}(k)}{r_{xx}(k)} \tag{7}$$

The mapping relationship between $r_{uu}$ and $r_{uv}$ can be obtained by the ZMNL transform equation of the correlated Weibull distribution:

$$r_{uu}(k)=\frac{pr_{xx}(k)}{2\Gamma(2/p)}\left[1-\left(1+\mu^2(k)\right){r_{xx}}^2(k)\right]^{2/p+1}\Gamma^2\left(\frac{1}{p}+\frac{3}{2}\right)\times{}_2F_1\left(\frac{1}{p}+\frac{3}{2},\frac{1}{p}+\frac{3}{2};2;\left(1+\mu^2(k)\right){r_{xx}}^2(k)\right) \tag{8}$$

The hyper geometric function is defined:

$$ {}_2F_1(a,b;c;z)=\frac{\Gamma(c)}{\Gamma(a)\Gamma(b)}\sum_{n=0}^{\infty}\frac{\Gamma(a+n)+\Gamma(b+n)}{\Gamma(c+n)}\frac{z^n}{n!},c\neq 0,-1,2,\cdots \tag{9} $$

The convergence condition is: $\left(1+\mu^2(k)\right)r_{xx}^2(k)<1\ |k|>0$ similarity, we can obtain the relationship between $r_{uv}(k)$ and $r_{xv}(k)$. Figures 2 gives the diagram of corresponding mapping relation:

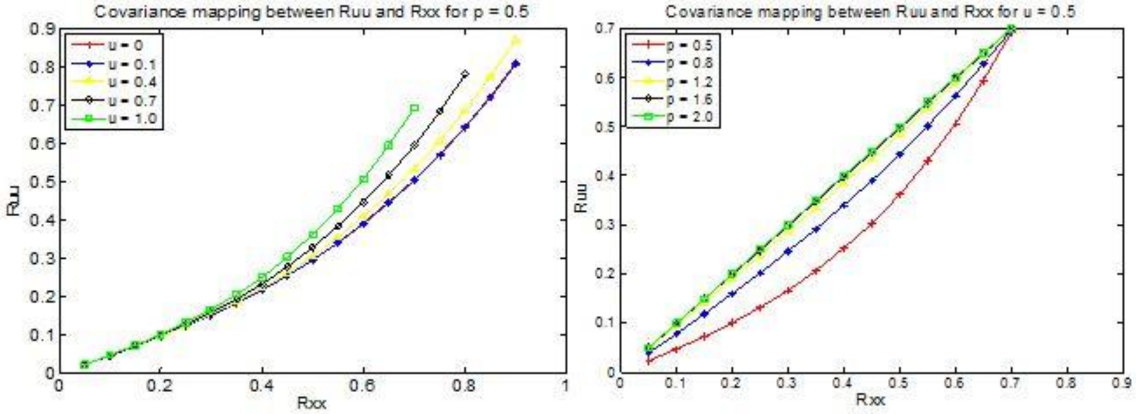

Figure.2 Nonlinear relations between $R_{uu}(k)$ and $R_{xx}(k)$

**Clutter simulation.** Process of generating the coefficients of the shaping filter can be expressed as the following flowchart(Fig.3). In this paper, we select a coherent pulse radar clutter power spectrum model, which is developed by Hawkes and Haykin[9], and the normalized autocorrelation function is given as follows:

$$ r_Z(k)=\frac{2}{3}\left[1+\frac{1}{2}\cos\left(4\pi kV_{or}\right)\exp\left(-8\pi^2k^2\sigma_d^2\right)\right]\exp\left(-j2\pi kV_{od}-2\pi^2k^2\sigma_d^2-\frac{k^2}{2T_0^2}\right) \tag{10} $$

Where $\sigma_d, V_{od}, V_{or}, T_0$ are the model parameters, We use the typical values for these parameters as in reference, i.e. $\sigma_d=0.025, \sigma_r=0.02, V_{od}=-0.05, V_{or}=0.30, T_0=24$ ,and the parameters of Weibull distribution is $p=1.5$ , $q=1.85$.

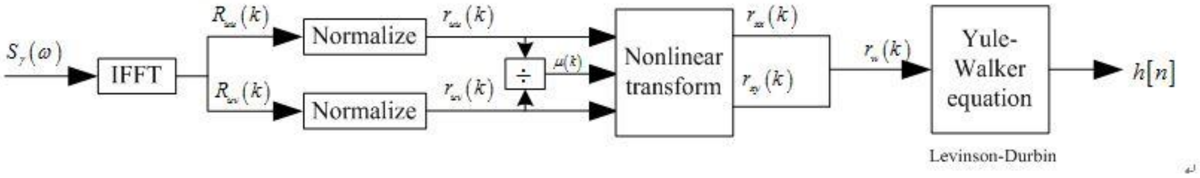

Figure.3 shaping filter coefficients generator diagram

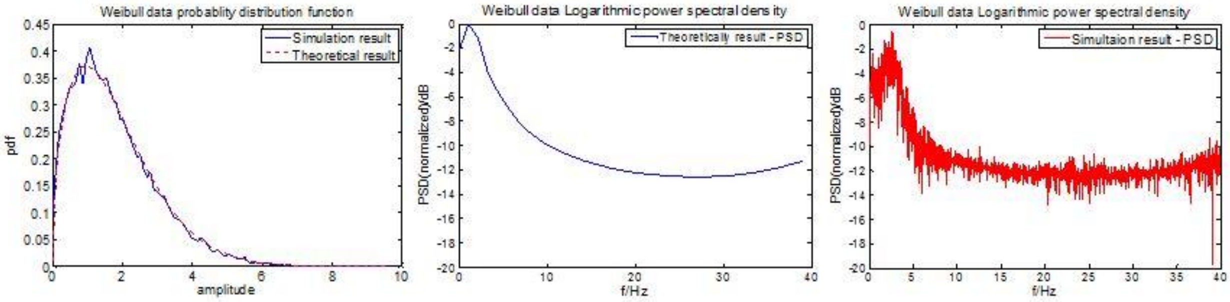

Figure.4 The PDF and PSD of the simulation and theoretical result

Fig.4 illustrates the PDF and PSD of the corresponding theoretical and generated process and the generated process which has experienced nonlinear mapping satisfies the specification very well.

## Ground Scattering Unit Division

**Grid mapping method.** The classical and efficient processing method – grid mapping is adopted in the paper for modeling and simulating the ground clutter echo with a large number of scattered bodies. Whereby the ground area will be divided into grid cell and the size of grid unit is $\Delta r\times\Delta f$ ( $\Delta r$ is range resolution and $\Delta f$ is frequency resolution).

**Range ring and azimuth.** The actual space of ground clutter needs to be converted to radar space, i.e. scatter unit as previously mentioned needs to be mapped to the radar coordinate system composed of Range-Doppler. This article uses equi-distance and equi –Doppler method to partition the clutter grid under radar coordinate. Range resolution $\Delta r$ and azimuth frequency resolution $\Delta\theta$ are determined by the grid mapping method, where $\Delta r = C/2B$ and horizontal range ring width is $\Delta\rho = r\Delta r/\rho$. The Doppler resolution is $\Delta f = f_r / N$ ($N$ is the length of coherent pulse train), where $f_r$ denotes the pulse repetition period. Assuming that the azimuth and elevation angle of the center of the ground clutter is $(\theta_0, \phi_0)$, accordingly, the Doppler frequency of the unit is defined as: $f_d = 2V_r \cos\theta\cos\phi/\lambda$, and the angular resolution of $\Delta\theta$ can be determined by using the differential method presented by following :

$$\begin{aligned} df_d &= -2V_r \sin(\theta)\cos(\phi)\,d\theta/\lambda \\ d\theta &= -\lambda df_d / 2V_r \sin(\theta)\cos(\phi) \end{aligned} \tag{11}$$

The angle resolution is satisfied:

$$\Delta\theta \le \frac{\lambda}{2V_r \sin\theta_{\max}\cos\phi}\Delta f \tag{12}$$

Suppose that beam width in the pitch and azimuth direction of main lobe is respectively $(\theta_e, \theta_a)$. Then the number of range ring and the angle ring can be expressed as:

$$\begin{aligned} N_r &= \frac{r\theta_a}{\Delta\rho} = \frac{r_0\theta_a}{r_0\Delta r/\rho_0} \\ N_\theta &= \frac{\theta_e}{\Delta\theta} = \frac{2V_r \sin\theta_{\max}\cos\phi}{\lambda f_r} N\theta_a \end{aligned} \tag{13}$$

We can obtain the azimuth and grazing angle $(\theta_n, \varphi_m)$ of ground clutter scattering unit relative to the airborne radar after the determining the number of horizontal range ring and azimuth angle ring. Where $\varphi_m$ represents the beam grazing angle of the clutter unit in $m-th$ range ring, accordingly, $\theta_n$ denotes the center azimuth angle of the clutter cell in $n-th$ azimuth ring. The function of $(\theta_n, \varphi_m)$ is given by:

$$\varphi_m = \arctan\left(H/\left(\rho_{\min} + (m-1/2)\Delta\rho\right)\right),\ \theta_n = \theta_0 + (n-1/2)\Delta\theta \tag{14}$$

The clutter scattering unit area can be defined as $\Delta A_{m,n} = \rho_m \Delta\rho_m \Delta\theta$, and radar cross-sectional area of clutter unit in the position $(\theta_n, \varphi_m)$ can be expressed as $\sigma_{m,n} = \sigma_0 \Delta A_{m,n}$.

**Model of backscattering coefficient.**The commonly used Morchin model[10] of ground clutter is chosen to describe the clutter backscattering coefficient $\sigma_0$ in this paper, and the model takes into account the effects of topography on backscattering coefficient. The $\sigma_0$ can be expressed as:

$$\begin{aligned} \sigma^0 &= \frac{A\sigma_c^0 \sin\theta_g}{\lambda} + u\cot^2\beta_0 \exp\left(-\frac{\tan\left(B-\theta_g\right)}{\tan^2(\beta_0)}\right) \\ \sigma_c^0 &= \begin{cases} \left(\theta_g/\theta_c\right)^k & \theta_g < \theta_c \\ 1 & \theta_g \ge \theta_c \end{cases} \end{aligned} \tag{15}$$

Here, $u = \sqrt{f_0}/4.7$ , $f_0$ is the carrier frequency of airborne radar, $\theta_c = \arcsin\left(\lambda/36.2\pi h_e \beta_0^{2.2}\right)$ , other parameters of the model in the article are set as: $\sigma_c^0 = 1, A = 0.0126, B = \pi/2, \beta_0 = 0.4$.

**Antenna pattern modulation model.** The gain of the echo signal is modulated by the antenna pattern according to the model of coherent clutter echo signal.In this paper, the approximate Gauss pattern function is used to model the main lobe and side lobe of the antenna beam:

$$
\begin{aligned}
&G_{MB}(\theta)=\exp\left(-2\ln 2\frac{\theta^2}{\theta_B^2}\right) && |\theta|\le\mu\\
&G_{B1}(\theta)=g_1\times\exp\left(-2\ln 2\frac{(\theta\pm1.5\mu)^2}{\theta_{B1}^2}\right) && \mu\le|\theta|\le2\mu\\
&G_{B2}(\theta)=g_2\times\exp\left(-2\ln 2\frac{(\theta\pm2.5\mu)^2}{\theta_{B2}^2}\right) && 2\mu\le|\theta|\le3\mu
\end{aligned}
\tag{16}
$$

Here, $\theta_B$ is 3dB main lobe width, and $g_1, g_2$ and $g_3$ denotes the gain peak of first, second and the rest of the side lobe respectively. Where $\mu$ indicate the angle $\theta$ when the gain value equals to $g_3$, where the value of $\theta_{B1},\theta_{B2}$ can be determined by $\theta_{Bj}=0.5\mu_1\sqrt{(-2\ln 2)/\ln(g_3/g_j)}$.

## Implementation of Hardware Platform

The proposed method is verified and achieved on the hardware platform based on FPGA. Gaussian white noise sequence needs to generate first before producing Weibull distribution clutter random sequence. This paper adopts the Combined Tausworthe uniform random number generating algorithm and Box-Muller Gaussian distribution transformed algorithm in achieving real-time WGN and Weibull sequences on FPGA(as seen in Fig.5 and Fig.6).

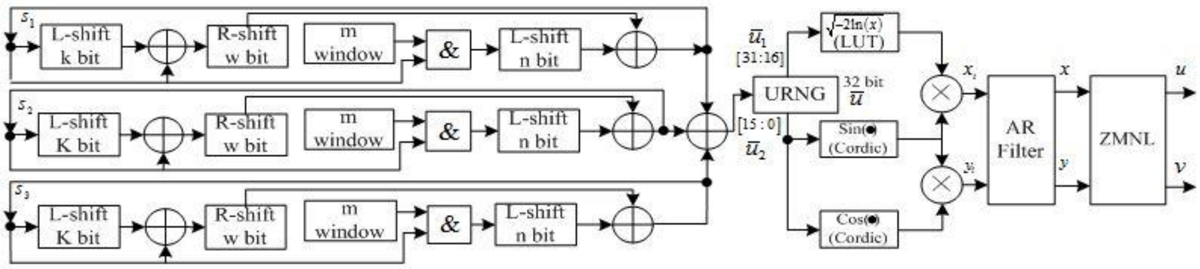

Figure.5 Generation process of WGN and Weibull sequences on FPGA

Combined Tausworthe random number generator can be expressed as:

$$
\bar{u}_n=\sum_{i=1}^{L}\left(x_{1,ns_1+i-1}\oplus x_{2,ns_2+i-1}\oplus\cdots\oplus x_{J,ns_J+i-1}\right)2^{-i}
\tag{19}
$$

Where $x_{J,ns_J+i-1}$ is a output sequence of $J-th$ Tausworthe random number generator. Then, generated number will be modulated with a Gauss function by Box-Muller means. The process can be expressed as: $\alpha=\sqrt{-2\ln(\bar{u}_1)}\sin(2\pi\bar{u}_2)$, $\beta=\sqrt{-2\ln(\bar{u}_1)}\cos(2\pi\bar{u}_2)$.

Figure.6 Results of generated WGN and Weibull sequences on FPGA

The Weibull distribution sequence of clutter amplitude modulation has been achieved, Eventually, the entire clutter echo signal is calculated in FPGA and the actual signal is exported by DA module.

## Realization of Ground Clutter Echo Signal

The clutter echo signal of ground scattering unit within the position of $(\theta_n,\varphi_m)$ can be obtained according to the point target echo model of coherent video signal:

$$
s(\theta_n,\varphi_m)=s_t\left(t-\tau_{n,m}\right)\left[\frac{\lambda^2}{(4\pi)^3 {r_{n,m}}^4}\right]^{1/2}G(\theta_n,\varphi_m)\sqrt{\sigma_0 S_{n,m}}\cdot n_{n,m}\cdot\exp\left(j2\pi f_{di}\left(t-\tau_{n,m}\right)\right)
\tag{17}
$$

Where $\tau_{n,m}$ denotes the delay of the echo signal, $S_{n,m}$ stands for the area of clutter scattering unit. Here, $n_{n,m}$ is complex sequence which satisfy the Weibull amplitude distribution and Gauss type correlation characteristics. The echo signal (as shown in Fig.7) of all clutter units within the range ring and azimuth angle ring can be expressed as:

$$s_r(t)=\sum_{m=1}^{N_r}\sum_{n}^{N_\theta}s(\theta_n,\varphi_m) \tag{18}$$

The paper takes the situation of airborne PD radar in the condition of target tracking to model the ground clutter, and the important parameters in the model are set as follows: the transmit waveform is LFM signal, pulse width is $20\mu s$, where coherent processing pulse number $N=128$, $\lambda=0.03$, $B=40MHz$, $V_r=1000m/s$, airborne radar height is $H=5000$, the beam direction pitching angle is 10 °, azimuth angle is 30 °.

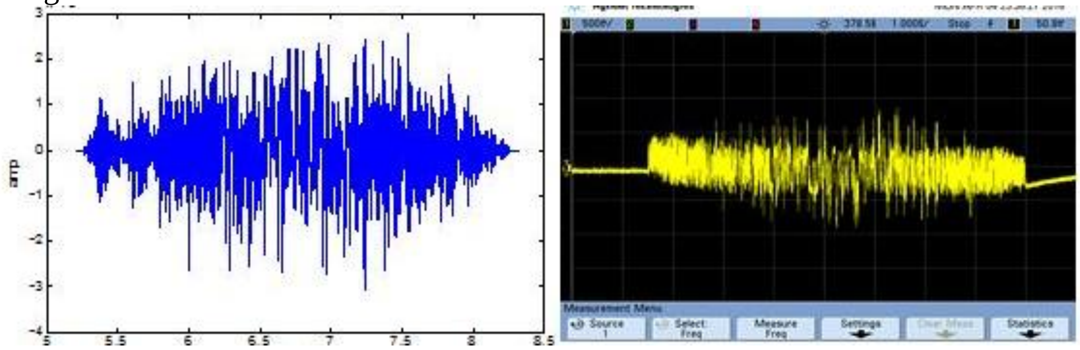

Figure7. Simulation and Implementation results of clutter echo signal

## Summary

The modeling and simulation of ground clutter for airborne radar is discussed in this paper. Specifically, the realization process of complex correlated Weibull distribution clutter is presented and the nonlinear effect of ZMNL variation on the autocorrelation function of the sequence has been discussed. The mapping method has been used to form a rectangular grid for this distributed ground clutter, and the clutter scattering region is partitioned by the method of equi-distance – equi-Doppler. The coherent superposition of the scattering body is finally carried out based on the point target echo formula of coherent video signal. The method proposed in this paper is achieved and verified on the hardware platform.

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