InSAS Interferogram Filtering Via A New Coupled PDE Method

HUANG Pan[1, a], TANG JinSong[1,b *] and ZHONG HePing[1,c]

[1]Naval institute of underwater acoustic technology, Naval university of engineering Wuhan, P.R.China

[a]conan1986@126.com, [b]jinsongtangwh@163.com, [c]zheping525@sohu.com

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Abstract. Interferogram filtering is one of the most important and widely studied problems in InSAS image processing. Traditional approaches, such as median filter, mean filter and Gaussian filter, can remove noise, but they also smooth the edges. This paper proposed a new interferogram filtering algorithm via a coupled partial differential equation(PDE) method. The processing results including both simulated and trial data shows the noise can be removed effectively and the image edge and details can be preserved.

Introduction

Interferometric Synthetic Aperture Sonar (InSAS) provides a means of obtaining high resolution three-dimensional images of targets on the sea floor[1,2]. The interferogram obtained from InSAS always has much noise that disturbs the quality of the interferogram. Therefore, how to remove the phase noise and preserve the edge feature effectively is an important aspect in InSAS data processing. Traditional filtering methods, such as media filter and mean filter, always produce the problems of over-smoothing. Recently, many different PDE methods have been developed and widely used to remove the noise and preserve the edge[3-5]. Since it is not feasible to discuss all the methods here, we concentrate on a few methods which are related to our proposed method.

In 1990, Perona and Malik [7] proposed the following nonlinear diffusion that had great influence in this field:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \nabla \cdot \left(g\left(|\nabla u|\right) \nabla u \right), & on \ (0,\mathrm{T}) \times \Omega, \\ \left.\dfrac{\partial u}{\partial n}\right|_{\partial\Omega} = 0 & on \ (0,\mathrm{T}) \times \partial\Omega, \\ u(x,y,0) = u_0(x,y), & on \ \Omega. \end{cases} \tag{1}$$

where ∇u denotes the gradient of image, $\partial\Omega$ denotes the boundary of image domain Ω , n represents the direction normal to $\partial\Omega$, and $u_0(x,y)$ is the original image. The diffusion coefficient function $g\left(|\nabla u|\right)$ is a nonincreasing function with $g(0)=1$, $g(s)\ge 0$,and $g(s)\to 0$ when $s\to\infty$. The nonlinear diffusion can remove the noise and preserve the image edge effectively by choosing the appropriate diffusion function. However, it does not perform well for large noisy images because this will result in false edges and this model is ill-posed[8] if $|\nabla u| \cdot g\left(|\nabla u|\right)$ is nonincreasing since it cannot ensure the existence and uniqueness of the solutions. To avoid this problem, Catte et al.[8] established the following regularized model:

$$\frac{\partial u}{\partial t} = \nabla \cdot \left(g\left(|\nabla G_\sigma * u|^2 \right) \nabla u \right), \quad u(x,y,0) = u_0(x,y). \tag{2}$$

where $G_\sigma * u$ denotes a convolution of the image with a Guassian kernel, $G_\sigma(x,y) = \frac{1}{\sqrt{2\pi\sigma}} \exp(-\frac{x^2+y^2}{4\sigma})$. Namely, it uses Gauss pre-filtering before nonlinear diffusion.

Although this change improves the original de-noising procedure, Guassian convolution is an isotropic diffusion, this leads not to preserve important edges, especially corners and junctions. Moreover, the variance of Gauss function must be set manually before iterations, which cannot satisfy the requirement of adaptability. To solve the problem mentioned above, Chen [9] gave a coupled PDE model based on the variational method.

$$\begin{cases} \frac{\partial u}{\partial t} = \alpha g_k(|\nabla v|)|\nabla u|\nabla \cdot (\frac{\nabla u}{|\nabla u|}) + \alpha \nabla(g_k(|\nabla v|)) \cdot \nabla u - \beta(u-I)|\nabla u|, & (3) \\ \frac{\partial v}{\partial t} = a(t)\nabla \cdot (\frac{\nabla v}{|\nabla v|}) - b(v-u), & (4) \\ \frac{\partial u}{\partial n}|_{\partial\Omega} = 0, \frac{\partial v}{\partial n}|_{\partial\Omega} = 0, u(x,y,0) = u_0(x,y), v(x,y,0) = u_0(x,y). \end{cases}$$

where α, β are constant, I represents the magnitude of original image gradient, v is an edge-strength function, which plays a very important role in the quality of the recovered images. The Eq.4 is obtained by minimizing the following energy function which ensures that v is not too far from u,

$$E_u(v) = \int a(t)|\nabla v| + \frac{b}{2}|v-u|^2 \, dx,$$

where $a(t)$ is a monotonically decreasing function of time t, and b is a constant. By reducing $a(t)$ as time increases, and by keeping b fixed, we are effectively increasing the agreement between u and v as time increases.

This paper is organized as follow, In Section 2, a modified coupled PDE method is introduced based on the mentioned above, and we presents a numerical implementation of the proposed coupled PDE method. In Section 3, results of numerical simulations and real experimental data are presented respectively, which demonstrate the superiority of the proposed method.

Formula Development And Numerical Implementation

The main problem with Chen method is that too many parameters should be adjusted manually and it is computationally expensive. Based on the above statements, this paper utilizes a special coupled PDE model

$$\begin{cases} \frac{\partial u}{\partial t} = \nabla \cdot \left(c\left(|\nabla v|^2\right) \nabla u \right) & (5) \\ \frac{\partial v}{\partial t} = a(t) p \nabla \cdot (\frac{\nabla v}{|\nabla v|^{2-p}}) + bq(u-v)^{q-1} & (6) \\ \frac{\partial u}{\partial n}|_{\partial\Omega} = 0, \frac{\partial v}{\partial n}|_{\partial\Omega} = 0, u(x,y,0) = u_0(x,y), v(x,y,0) = u_0(x,y). \end{cases}$$

where b is a constant, $a(t)$ is a monotonically decreasing function of time t, as the same as in Eq.4. In this paper, we chose the diffusivity function $c\left(|\nabla v|^2\right) = 1 \Big/ \left(1 + |\nabla v|^2 / K^2\right)$, where K is constant and is fixed by histogram of absolute value of gradient throughout the image[10]. The Eq.6 can be obtained by minimizing the energy function

$$E(v) = \int a|\nabla v|^p + b|u-v|^q \, dx,$$

where p , q are constants and $0 < p < 2$. Note that in the image edges, the value of $|\nabla v|$ is very large, the value of $1 \Big/ |\nabla v|^{2-p}$ in Eq.6 is very small, that is, the diffusivity is very small. Thus, the image edge can be efficiently preserved. On the contrary, in the flat area of the image, $|\nabla v|$ is very small, the diffusivity is very large, so this area is smoothed well. In fact, if $p > 2$, namely $p - 2 > 0$, since $\frac{1}{|\nabla v|^{2-p}} = |\nabla v|^{p-2}$, at the boundary of the image, $|\nabla v|$ is very large, so $|\nabla v|^{p-2}$ is also very large, this will result in the diffusivity is large at the edges and the edge features can not be preserved, which is not wanted. Additionally, the image is more smoothing as the value of p increasing.

Let $u_{i,j}$ denote $u(i,j)$, the images at times $t_n = n\Delta t$ is $u(i,j,t_n)$,denoted by $u_{i,j}^n$. The time derivative u_t at (i,j,t_n) is approximated by the forward difference $u_t = u_{i,j}^{n+1} - u_{i,j}^n / \Delta t$. The diffusion term in this model can be discretized by forward and backward finite difference

$$\begin{aligned}\nabla\cdot(c\cdot\nabla u) &= \frac{\partial}{\partial x}(c\cdot u_x) + \frac{\partial}{\partial y}(c\cdot u_y)\\ &= c_{i,j+1}(u_{i,j+1} - u_{i,j}) - c_{i,j}(u_{i,j} - u_{i,j-1})\\ &\quad + c_{i+1,j}(u_{i+1,j} - u_{i,j}) - c_{i,j}(u_{i,j} - u_{i-1,j})\end{aligned}$$

The term $|\nabla v|$ in Eq.5 is approximated by the upwind scheme developed in [6],

$$|\nabla v| = \sqrt{(\max(\Delta_i^- v_{i,j},0))^2 + (\min(\Delta_i^+ v_{i,j},0))^2 + (\max(\Delta_j^+ v_{i,j},0))^2 + (\min(\Delta_j^- v_{i,j},0))^2}$$

where

$$\begin{aligned}&\Delta_i^+ v_{i,j} = v(i,j+1) - v(i,j),\ \ \Delta_i^- v_{i,j} = v(i,j) - v(i,j-1),\\ &\Delta_j^+ v_{i,j} = v(i+1,j) - v(i,j),\ \ \Delta_j^- v_{i,j} = v(i,j) - v(i-1,j)\end{aligned}.$$

The first term in the right side of Eq.6 can be approximated by

$$\nabla\cdot\left(\frac{\nabla v}{|\nabla v|^{2-p}}\right) = \frac{v_{xx}(v_x^2+v_y^2) - (2-p)(2v_x v_y v_{xy} + v_x^2 v_{xx} + v_y^2 v_{yy}) + v_{yy}(v_x^2+v_y^2)}{(v_x^2+v_y^2)^{2-p/2}},$$

Since the proposed model consists of a coupled system of PDEs, the numerical solution $u_{i,j}^{n+1}$, $v_{i,j}^{n+1}$ is obtained from $u_{i,j}^n$, $v_{i,j}^n$ by first using the discretization of Eq. (6) to obtain $v_{i,j}^{n+1}$, then using the discretization of Eq. (5) to obtain $u_{i,j}^{n+1}$.

Experimental results and analysis

Using this new coupled PDE model , we process simulated and real experimental data, respectively. The original noisy interferogram simulated by the computer is shown in Fig.1. The Chen method was run with $\alpha=0.5, \beta=0.0005, b=0.1$, $\Delta t=0.1$, $p=1$, $q=2$, $a(0)=1$ for 30 iterations and the result is shown in Fig.2. The proposed method was run with the parameter values for $b=0.1$, $\Delta t=0.1$, $p=1$, $q=2$, $a(0)=1$. Fig.3 and Fig.4 are the results images after 5 and 10 iterations, respectively. In this paper, residue numbers and phase mean square deviation (MSD) are chosen as the evaluation criterions, which is given in Table 1. From table 1, it is shown that comparing with Chen method, the proposed method can reach satisfied result with little time elapse.

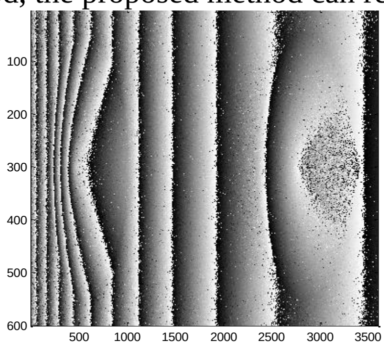

Fig. 1 Original noise image

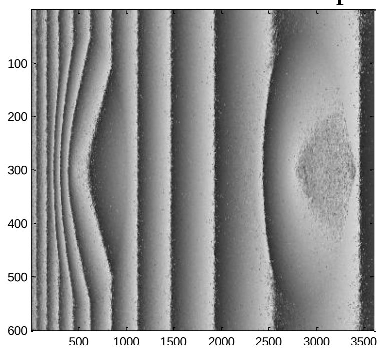

Fig. 2 Chen method (30 iterations)

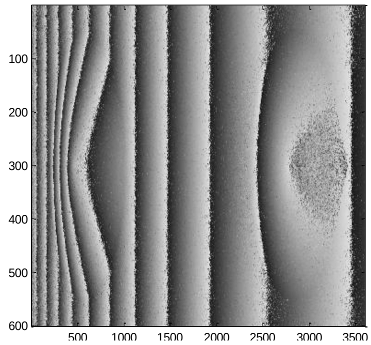

Fig. 3 the proposed method(5 iterations)

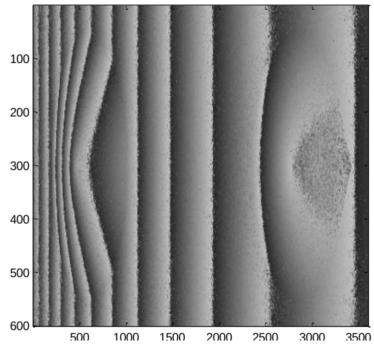

Fig. 4 the proposed method(10 iterations)

Table 1 Results of the simulated experiment

	Residue numbers	Phase MSD	Elapse time
Original image	21594	3.4772	—
Chen method(30 iterations)	8846	2.7854	131.3s
The proposed method(5 iterations)	8868	2.8387	23.5s
The proposed method (10 iterations)	888	2.7307	46.3s

The real experimental data is generated from InSAS images of Qiandao lake in Zhejiang Province. The original noisy interferogram with 1000×4000 pixels is shown in Fig.5. The result of Chen method with 15 iterations is shown in Fig.6, Fig.7 is the result of the proposed method in this paper after 10 iterations. Table 2 gives the results data of real experiment. From Table 2, it can be seen that the residue numbers reduced from 167581 to 4151, after 10 iterations.

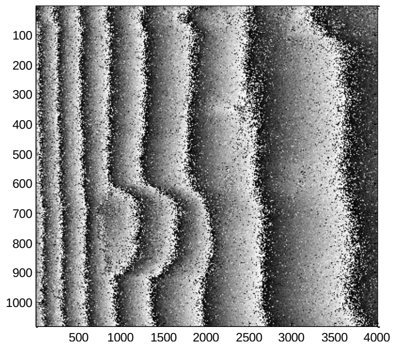

Fig. 5 Original noise image

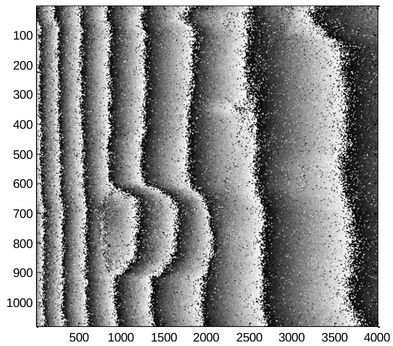

Fig.6 Chen method(15 iterations)

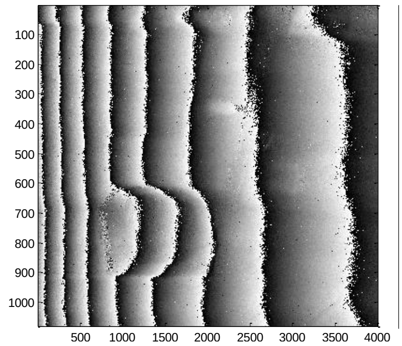

Fig.7 the proposed method(10 iterations)

Table 2 Results of the real experiment

	Residue numbers	Elapse time
Original image	167581	
Chen method(15 iterations)	21140	199.43s
The proposed method (10 iterations)	4151	107.7s

The simulation and real experimental results indicate that the image is smoothed as the iteration increasing. The data analysis of the simulation and real experimental results show that this new coupled PDE method can remove noise and preserve the edge details effectively. Compared with the Chen coupled PDE model, the processing time of this model and the number of residues are less.

Conclusions

In this paper, a new coupled PDE method for InSAS interferogram filtering was presented. This model can satisfy the requirement of the adaptability which avoids the Gauss pre-filtering. Numerical experiment results and analysis indicate that this method is able to remove the noise effectively and preserve the image edge details much better. This new coupled PDE model is very important for the InSAS real time processing.

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