Chaotic characteristics analysis of simulation signal of second harmonic generation effect

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Abstract. In order to deepen the understanding of nonlinear properties of specimen in the process of fatigue crack extension, the chaos and fractal theory are presented to analyze the second harmonic generation (SHG) signal of receiver in nonlinear ultrasonic system. Firstly the usefulness of chaos and fractal theory to analyze the SHG signal should be verified theoretically. By constructed the simulation signal of SHG effect, simulation signals with the character of chaotic was found, and it showed that the Lyapunov exponent was sensitive to weaker nonlinearity and unsusceptible to noise, and chaotic characteristic values could make up for inadequacies of the relative nonlinear coefficient.

Keywords: nonlinear ultrasonic; nondestructive testing; fatigue crack extension; Lyapunov exponent.

1 Introduction

In the process of fatigue crack extension, crack size, quantity, the complexity of the crack extension path, the internal structure of specimen was changing [1], the SHG signal of receiver was unstable which related to the cumulative damage of specimens [2]. The frequency-domain method wasn't suitable to analyze the signal because it was adaptive for smooth linear signal [3,4]. Therefore how to effectively extract the SHG signal and reasonably characterize the nonlinearity of specimen were key issues in nonlinear ultrasonic testing.

The development of chaos theory for in-depth analysis the inner mechanism of nonlinear time series provided theoretical basis [5], and this method was widely applied in the fault diagnosis and feature extraction [6]. Due to the SHG signal was related to the cumulative damage of specimen, by analyzing it relevant features of system could be obtained. From physical view, the metal specimen with cracks was a nonlinear dissipative system, the propagation path of fatigue crack could be viewed as fractal curve, so the fractal dimension can be used to characterize the irregularity of the crack propagation path [7, 8]. Therefore the chaos and fractal theory were firstly presented to analyze the SHG signal in nonlinear ultrasonic testing.

Because of the fatigue crack extension experiment of metal was time-consuming and expensive, in order to avoid waste of resource and time, the effectiveness of chaos and fractal theory to analyze the SHG signal should be firstly verified. Therefore the simulation signal of SHG effect was firstly built,

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characteristic values of chaos and fractal were calculated to evaluate the nonlinearity of specimen, and verify the usefulness of the new analyzing method.

2 Chaos theory and characteristic values

Due to the chaos and fractal theory were used to analyze SHG signal, the Lyapunov exponent, correlation dimension and K entropy were calculated to evaluate the nonlinearity of specimen, therefore chaos theory and characteristic values were firstly introduced in the paper[5].

2.1 The Lyapunov exponent

The basic characteristic of chaos is extremely sensitive to initial condition, tracks generated by two close initial values separate exponentially with time, and the lyapunov exponent studies this feature. The biggest Lyapunov exponent can be estimated by the average separation rate of each pair of initial neighboring points [6].

2.2 Kolmogorov entropy

Kolmogorov entropy (K entropy) is the physical quantity which describes the degree of how quickly information produces in nonlinear system. When K entropy is larger, the system is more complex. The maximum likelihood method is usually used which has been proposed by Schouten to calculate K entropy from time series [5], the average separating time which dot pairs initial distance less than r_0 separate to the distance more than r_0 is used to estimate K entropy.

2.3 Correlation dimension

The fractal dimension is an important parameter of attractor and widely used in quantitative description nonlinear behaviors of system, and correlation dimension is one of them which measures the complexity of attractors[9]. The GP algorithm is always used to calculate the value.

3 The SHG simulation signal

Based on characteristics of the SHG signal which was non-sinusoidal time series with changed amplitude, phase and frequency [10], nonlinear time series was used to simulate the SHG signal. Depending on whether consideration ultrasonic attenuation, two cases of nonlinear time series were constructed.

3.1 Without ultrasonic attenuation

In nonlinear ultrasonic system, one 10 cycles sinusoidal signal of 5MHz from the function generator was fed into attenuator and low-pass filter, propagated in metal and produced the SHG effect due to the nonlinearity of material. The excitation signal was $x_0 = A_1 sin(2\pi f_1 t)$. For ease of calculating the relative nonlinear coefficient, the amplitude was set as 100V. Due to the SHG signal contained the second and third harmonic, so the simulation signal was constructed as $x = A_1 sin(2\pi f_1 t) + A_2 sin(2\pi(2f_1)t) + A_3 sin(2\pi(3f_1)t)$. Because any complicated signal could be decomposed into a series of sine signals, the signals x could simply simulate the SHG signal.

Because the value of A_2 could indirectly indicate the nonlinearity of specimen. According to Wu Bin's research-when the fatigue life of specimen was 5% and 60% [3,11], A_2 was calculated as

0.043V and 4.78V, and approximately equaled to 0.05V and 5V. In order to represent the nonlinearity of specimen strengthened, x was constructed with increasing A_2 from 0.05V, 0.1V, 0.15V, 0.2V, 0.25V, 0.5V, 0.75V, in turn to 5V. Due to the third harmonic was weaker than the second, A_3 was set as 0.01V.

Therefore simulation signals were as follows:

$$f_1 = 5 \times 10^6, f_2 = 1.0 \times 10^7, f_3 = 1.5 \times 10^7, A_1 = 100, A_3 = 0.01;$$
$$x_0 = A_1 \sin(2\pi f_1 t);$$
$$x = A_1 \sin(2\pi f_1 t) + A_2 \sin(2\pi f_2 t) + A_3 \sin(2\pi f_3 t)$$

Because of the excitation signal was 10 cycles sinusoidal signal, according to the sampling frequency was 50MHz of the oscilloscope, 4096 function points of signal x were selected to ensure the SHG signal contained 10 cycles. In order to veritably simulate the signal, based on the signal x, the random white noise was added which the expectation was a and the standard deviation was b.

3.2 With ultrasonic attenuation

In the process of transmission, the energy of ultrasonic gradually weakened with propagation distance due to diffusing of wave. When the speed of ultrasonic was stable, the propagation distance was proportional to propagation time, hence ultrasonic attenuation increased with time. When constructed the simulation signal x_1 with ultrasonic attenuation, α_1, α_2 and α_3 were attenuation coefficient of frequency f_1, f_2, and f_3 in the solid medium, c was the transmission speed of ultrasonic. When analyzing the simulation signal x_1, 4096 function points were selected and the random white noise was added.

$$f_1 = 5 \times 10^6, f_2 = 10 \times 10^6, f_3 = 15 \times 10^6, A_1 = 100, A_3 = 0.01,$$
$$x_0 = 100 \sin(2\pi f_1 t);$$
$$x_1 = A_1 \sin(2\pi f_1 t) e^{-\alpha_1 c t} + A_2 \sin(2\pi f_2 t) e^{-\alpha_2 c t} + A_3 \sin(2\pi f_3 t) e^{-\alpha_3 c t}$$

4 Chaotic characteristics analysis of SHG simulation signals

Based on simulation signals, the relative nonlinear coefficient, the Lyapunov exponent, K entropy and correlation dimension were calculated, then variation curves of characteristic values were gotten.

4.1 Chaotic Characteristics Analysis of Signal x

Variation curves of the relative nonlinear coefficient, the Lyapunov exponent, K entropy and correlation dimension were showed in Figure1. The x-axis represented amplitudes of A_2 increasing from 0.05V to 5V. In each group, there were seven curves with different marking which represented the simulation signal x with the SNR of 50, 40, 30, 25, 20, 15, 10, this could analyze the influence of noise to characteristic values.
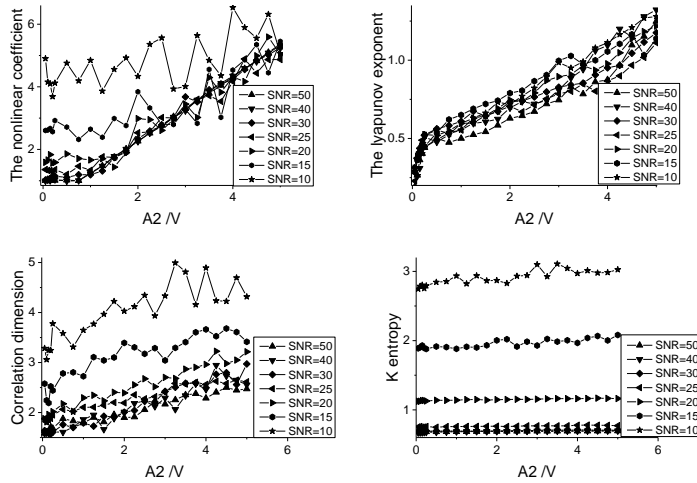
Figure 1. Characteristic values of simulation signal without attenuation

Variation curves of the relative nonlinear coefficient were firstly analyzed. When A_2 less than 1V, that curves were almost flat was meant the value couldn't well characterize the nonlinear effect. When A_2 was greater than 1V, that the value monotonically fortified. When the SNR was lower than 15, the volatility of curves was larger and the value no longer monotonically increased, this explained it was fairly sensitive to noise of time series. Therefore the relative nonlinear coefficient wasn't sensitive to weaker nonlinearity and was susceptible to noise of time series.

That the Lyapunov exponent were greater than zero indicated simulation signals had chaotic character. It monotonously increased with A_2, especially when A_2 less than 0.5V it increased faster, this indicated the lyapunov exponent was more sensitive to weaker nonlinearity. The variation trend of seven curves was consistent illustrated the value was insensitive to noise of time series.

The correlation dimension monotonically increased with A_2, this indicated correlation dimension could represent nonlinearity of material. With the loss of the SNR curves shifted upward along the y-axis illustrated the value gradually increased. Since correlation dimension measured the complexity of attractor, with the loss of the SNR the complexity degree of signal increased, so correlation dimension fortified.

While variation curve of K entropy seemed flat with the x-axis. Due to K entropy added about 8% with A_2 and the increasing range was relatively small comparing with the ordinate. Due to the K entropy measured the randomness of system. When the SNR was higher than 25, the randomness of system had little change, while the SNR was less than 20, the noise increased in exponential way and the randomness rapidly aggrandized, so K entropy enlarged quickly.

4.2 Chaotic characteristics analysis of signal x_1

Based on the simulation signal with ultrasonic attenuation, variation curves of four characteristics values with different SNR were showed in Figure 2.
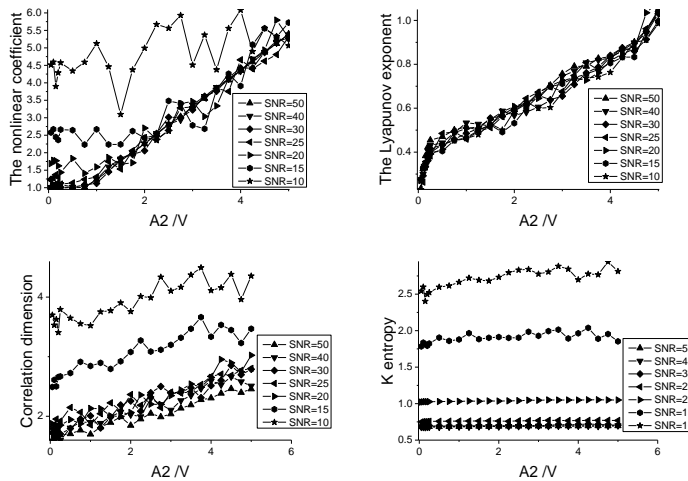
Figure 2. Characteristic values of simulation signals with attenuation

Contrasting analysis Figure 1 and Figure 2, variation trends of four characteristic values were the same, this indicated ultrasonic attenuation had little influence on them, but the value of them generally reduced for the reason of energy attenuation, for example, the ordinate of Lyapunov exponent reduced from 1.35 to 1.05.

5 Conclusion

Two kinds of simulation signals of SHG effect based on sine signals in nonlinear ultrasonic testing were constructed.

The biggest Lyapunov exponent were greater than zero indicated simulation signals had chaotic character.

The relative nonlinear coefficient wasn't sensitive to weaker nonlinearity and susceptible to noise of time series, while chaotic characteristic values could make up for inadequacies of the relative nonlinear coefficient and better evaluate the nonlinearity of specimens. Therefore the effectiveness of chaos and fractal theory to analyze the SHG signal was verified and a new method for nonlinear ultrasonic nondestructive testing was provided which chaos and fractal theory were used to analyze ultrasonic nonlinear time series.

Acknowledgments

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