Coordination of Dual-Channel Supply Chain Under Demand Sensitive to Price and Order-To-Delivery Cycle

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Abstract
The issue of channel coordination is studied with the demand sensitive to price and order-to-delivery cycle. Combining with the Stackelberg Game Theory, the paper analyses the manufacturer and the retailer's optimal pricing and profits in decentralized decision model and centralized decision model. Then the coordination contract of the wholesale price discount was designed to achieve the coordination of the dual-channel supply chain. Finally the paper analyses the impact of order-to-delivery cycle and sensitivity of demand to order-to-delivery cycle to the pricing and profits through a numerical example, and verifies the effectiveness of the coordination contract.

Key words: *dual-channel supply chain; order-to-delivery cycle; Stackelberg Game Theory; channel coordination; coordination contract*

1 Introduction

In recent years, the E-commerce and Internet information technology has been widely used and developed rapidly. On the basis of the traditional retail channel, many enterprises such as DELL, IBM, and Cisco have opened online direct channel, which constitute the dual channel supply chain network (*e.g. AA Tsay et al.*[1]). In the build-to-order (BTO) production environment, the time sensitivity of demand to the order-to-delivery cycle has become more intense. Manufacturers continue to invest resources to shorten the order-to-delivery cycle (*e.g. Heinonen et al.*[2]). Many scholars have made some achievements in the research of dual-channel supply chain, and the time sensitivity has become a hot research topic.

Early in 1988, *Stalk*[3] point out that the time has become a strategic support strength of the enterprise competitive power, the rapid response to customer demand brings more powerful competitive advantage for enterprises. *Bibo* and *Joseph*[4] study the decision problem of inventory and lead time in the demand sensitive to price and sales lead time, and provide the model and solving method to obtain the optimal pricing and lead time. *Hua et al.*[5] study the effect of lead time on pricing behaviour of dual-channel supply chain members, and point out that the lead time can influence the pricing strategy and the profits of manufacturers and retailers. *Zhu* and *Qi*[6] study the pricing strategy of dual-channel supply chain in two kinds of situations: certainty and uncertainty of consumers' sensitivity to time. *Shi et al.*[7] discuss the coordination of dual-channel supply chain with different coordination contracts, and discuss the influence of the change of lead time on the decision variables. *Cun* and *Xu*[8] study the relationship between pricing and delivery time in two cases: centralized decision-making and decentralized decision-making, and point out the existence of optimal distribution time strategy. *Shao*[9, 10], *Ma*[11] and *Gao*[12] also analyse the decision problem of the two-echelon BTO supply chain when demand is sensitive to price and time.

From the above literature, we can see that the research mainly focuses on the sensitivity of demand to time, the enterprise investment cost in improving the time factor is not fully considered yet. *Moori et al.*[13] and *Roberts*[14] point out that the promotion of logistics capability and business process reengineering (BPR) have a positive impact to shorten the order-to-delivery cycle. But the cost will also affect the channel profit. Therefore, how to coordinate the dual-channel supply chain and reasonably determine the product price of each channel are problems with practical and theoretical significance.

2 The basic model

Consider the two-echelon dual-channel supply chain consisting of a single BTO manufacturer and a single retailer, which manufacturer and retailer sell homogeneous products. The manufacturer sells the product to consumers through traditional retail channel and online direct channel, the retailer sells the product to consumers only through traditional retail channel. We denote the online direct channel and the tradition retail channel as channel 1 and 2, respectively, i.e. $i = 1, 2$.

For convenience, we use the following notation ($i = 1, 2$) throughout the paper:

c the marginal production cost of manufacture, $c > 0$;

t the order-to-delivery cycle of dual-channel supply chain, $t > 0$;

w the unit wholesale price announced by manufacturer, $w \geq c$;

m the sensitivity of demand to order-to-delivery cycle, termed as the time sensitivity, $m > 0$;

k_i the sensitivity of demand d_i to retail price p_i, $0 < k_i \leq 1$;

l the sensitivity of demand d_i to retail price p_{3-i}, $0 < l < k_i$.

We consider the cost of the manufacturer to shorten the order-to-delivery cycle as $c(t)$, which includes the investment cost for logistics, business process, information system, order management, and so on (*e.g. S Cui et al.*[15]). The greater the cost of enterprise investment, the corresponding order-to-delivery cycle will be shorter, $c(t)$ is a decreasing function of order-to-delivery cycle, i.e. $c(t) = kt + b$, where b ($b > 0$) is the maximum investment cost of the manufacturer, k ($k < 0$) is the time elasticity of the cost $c(t)$. The investment cost of the manufacturer is always non-negative, i.e. $c(t) \geq 0$, so the order-to-delivery cycle *t* meet the condition $0 < t \leq \frac{b}{-k}$. In the whole supply chain, we assume that all information is public information, supply chain members are risk neutral and fully rational, and other variables in the model are non-negative. The product demand is given by the following.

$$d_1 = \lambda a - k_1 p_1 + lp_2 - mt \tag{1}$$

$$d_2 = (1-\lambda)a - k_2 p_2 + lp_1 - mt \tag{2}$$

Where a is the total market demand for the product when the product price $p_i\,(i = 1,2)$ and the order-to-delivery cycle t are zero. λ ($0 \leq \lambda \leq 1$) is the consumers' preference for the online direct channel, so the consumers' preference for the traditional retail channel is $(1-\lambda)$. We assume that the two kinds of distribution channels have the same order-to-delivery cycle, and the sensitivity coefficient of each channel product demand to the order-to-delivery cycle is the same.

3 Pricing and profit analysis of dual-channel supply chain

3.1 The decentralized decision (DD) model

The manufacturer and retailer can form two kinds of decision-making models, namely centralized decision (CD) and decentralized decision (DD).In the decentralized decision (DD) model, the manufacturer will choose Stackelberg game and act as the game leader, and the retailer will act as the game follower. The manufacturer alone bear all the investment cost which used to reduce order delivery cycle, so the retailer can free riding in the dual-channel supply chain. The profit function of the manufacturer and the retailer can be respectively written as

$$\pi_{1DD} = (p_1 - c)d_1 + (w - c)d_2 - c(t) \tag{3}$$

$$\pi_{2DD} = (p_2 - w)d_2 \tag{4}$$

And the total profit function of dual-channel supply chain in the DD model is

$$\pi_{DD} = \pi_{1DD} + \pi_{2DD} \tag{5}$$

Noticed that the second derivative of decision variables on the retailer's profit is $\partial^2 \pi_{2DD} / \partial p_2^{\,2} = -2k_2 < 0$, so the retailer's profit function is a concave function of the price. By solving the first-order condition $\partial\pi_{2DD}/\partial p_2=0$, the retailer's response function is

$$p_2(p_1,w)=\frac{(1-\lambda)a+lp_1+k_2w-mt}{2k_2} \tag{6}$$

The manufacturer can make decisions based on the retailer's response function. Substituting Eq. (6) into Eq. (3), we obtain the Hessian matrix of the manufacturer's profit function as follows:

$$H_{\pi_{1DD}}=\begin{bmatrix}\dfrac{l^2-2k_1k_2}{k_2} & l\\ l & -k_2\end{bmatrix} \tag{7}$$

As mentioned above, the parameters k_i and l meet the conditions $k_i>l>0$, the Hessian matrix $H_{\pi_{1DD}}$ is negative definite. So the manufacturer's profit function is a concave function of the product price and the wholesale price. By solving the first-order condition $\partial\pi_{1DD}/\partial p_1=\partial\pi_{1DD}/\partial w=0$, we can get the product price and the wholesale price of the manufacturer as follows:

$$p_{1DD}^*=\frac{\left((1-\lambda)l+\lambda k_2\right)a+\left(k_1k_2-l^2\right)(c+kt+b)-(k_2+l)mt}{2\left(k_1k_2-l^2\right)} \tag{8}$$

$$w_{DD}^*=\frac{\left((1-\lambda)k_1+\lambda l\right)a+\left(k_1k_2-l^2\right)(c+kt+b)-(k_1+l)mt}{2\left(k_1k_2-l^2\right)} \tag{9}$$

Substituting Eqs. (8) and (9) into Eq. (6), we obtain the product price of the retailer as follows

$$p_{2DD}^*=\frac{\left((1-\lambda)\left(3k_1k_2-l^2\right)+2\lambda lk_2\right)a+\left(k_1k_2-l^2\right)(k_2+l)(c+kt+b)-\left(3k_1k_2+2lk_2-l^2\right)mt}{4k_2\left(k_1k_2-l^2\right)} \tag{10}$$

From Eqs. (8) - (10), we can get the profit of the manufacturer and the retailer as follows

$$\pi_{1DD}=\left(p_{1DD}^*-c\right)\left(\lambda a-k_1p_{1DD}^*+lp_{2DD}^*-mt\right)+\left(w_{DD}^*-c\right)\left((1-\lambda)a-k_2p_{2DD}^*+lp_{1DD}^*-mt\right)-c(t) \tag{11}$$

$$\pi_{2DD}=\left(p_{2DD}^*-w_{DD}^*\right)\left((1-\lambda)a-k_2p_{2DD}^*+lp_{1DD}^*-mt\right) \tag{12}$$

3.2 The centralized decision (CD) model

In the centralized decision (CD) model, the manufacturer and the retailer become a unified decision-making body, they will make decision simultaneously to maximize the total profit of dual-channel supply chain. The total profit function of dual-channel supply chain is

$$\pi_{CD}=(p_1-c)d_1+(p_2-c)d_2-c(t) \tag{13}$$

The Hessian matrix of the total profit function is

$$H_{\pi_{CD}} = \begin{bmatrix} -2k_1 & 2l \\ 2l & -2k_2 \end{bmatrix} \tag{14}$$

As mentioned above, we obtain the Hessian matrix $H_{\pi_{CD}}$ is negative definite. So the total profit function is a concave function of the product price. By solving the first-order condition $\partial\pi_{CD}/\partial p_1 = \partial\pi_{CD}/\partial p_2 = 0$, we can obtain the product price as follows

$$p_{1CD}^* = \frac{\left((1-\lambda)l + \lambda k_2\right)a + \left(k_1k_2 - l^2\right)\left(c + kt + b\right) - \left(k_2 + l\right)mt}{2\left(k_1k_2 - l^2\right)} \tag{15}$$

$$p_{2CD}^* = \frac{\left((1-\lambda)k_1 + \lambda l\right)a + \left(k_1k_2 - l^2\right)\left(c + kt + b\right) - \left(k_1 + l\right)mt}{2\left(k_1k_2 - l^2\right)} \tag{16}$$

So the total profit of dual-channel supply chain in the CD model is

$$\pi_{CD} = \left(p_{1CD}^* - c\right)d_1 + \left(p_{2CD}^* - c\right)d_2 - c(t) \tag{17}$$

By comparing the total profit of dual-channel supply chain in the DD model and the CD model, we can obtain that

$$\pi_{CD} - \pi_{DD} = \frac{\left(\left(k_2 - l\right)c + mt - (1-\lambda)a\right)^2}{16k_2} \tag{18}$$

It is obvious that $\pi_{CD} - \pi_{DD} > 0$, this shows that under the decentralized decision-making model, the overall income level of the dual-channel supply chain is lower, and the overall optimization is not achieved. So the manufacturer and the retailer can increase their profits, improve the efficiency of the dual-channel supply chain, and achieve the Pareto improvement through channel coordination.

4 Coordination decision model with wholesale price discount contract

From the above analysis, we show that the manufacturer and the retailer in the dual-channel supply chain can achieve the maximum profit in the decentralized decision model, and the total profit of the supply chain is obviously lower than that under the centralized decision model. We design the wholesale price discount contract (α, β) to coordinate the dual-channel supply chain in decentralized decision model, where $\alpha\,(0 < \alpha < 1)$ is the discount percentage of wholesale price, $\beta\,(\beta > 0)$ is the franchise fee paid by the retailer to the manufacturer to obtain a lower wholesale price. The subscript "C" indicates the coordination decision model. In the coordination decision model, according to the coordination contract requirement, the manufacturer provide a lower wholesale price w_C to the retailer before the sales season, i.e. $w_C = \alpha \cdot w_{DD}^*$, and the retailer pay a certain amount of franchise fee to the manufacturer to obtain a lower wholesale price and also make up for the loss of the manufacturer at the end of the sales season. The profit function of the retailer in the coordination decision model is

$$\pi_{2C} = \left(p_{2C} - w_C\right)\left((1-\lambda)a - k_2 p_{2C} + lp_{1C} - mt\right) - \beta \tag{19}$$

Noticed that the second derivative of decision variables on the retailer's profit is $\partial^2 \pi_{2C} / \partial p_{2C}{}^2 = -2k_2 < 0$, so the retailer's profit function is a concave function of the price. By solving the first-order condition $\partial \pi_{2C} / \partial p_{2C} = 0$, the retailer's product price is

$$p_{2C}^* = \frac{(1-\lambda)a + lp_{1C}^* + k_2 w_C - mt}{2k_2} \tag{20}$$

The coordination of the dual-channel supply chain requires that the optimal pricing decisions under coordination decision model with coordination contracts are consistent with the optimal pricing decisions in the centralized decision model, which means

$$p_{1C}^* = p_{1CD}^*, p_{2C}^* = p_{2CD}^* \tag{21}$$

In order to ensure the effective implementation of the coordination contract, the optimal product price of the manufacturer in the online direct channel is not determined by $\partial \pi_{1C} / \partial p_{1C} = 0$ A, but must be adjusted to p_{1CD}^*. Solving Eqs. (20) and (21), we obtain

$$\alpha = \frac{(k_2 + l)lmt - al\left(\lambda k_2 + (1-\lambda)l\right) - c\left(2k_2 - l\right)\left(k_1 k_2 - l^2\right)}{(k_1 + l)k_2 mt - ak_2\left(\lambda l + (1-\lambda)k_1\right) - ck_2\left(k_1 k_2 - l^2\right)} \tag{22}$$

So the profits of the manufacturer and the retailer are

$$\pi_{1C} = \left(p_{1CD}^* - c\right)\left(\lambda a - k_1 p_{1CD}^* + lp_{2CD}^* - mt\right) + \left(\alpha w_{DD}^* - c\right)\left((1-\lambda)a - k_2 p_{2CD}^* + lp_{1CD}^* - mt\right) - c(t) + \beta \tag{23}$$

$$\pi_{2C} = \left(p_{2CD}^* - \alpha w_{DD}^*\right)\left((1-\lambda)a - k_2 p_{2CD}^* + lp_{1CD}^* - mt\right) - \beta \tag{24}$$

To make the dual-channel supply chain members accept the coordination contract, in addition to meet the above condition Eq. (21), it must meet that the benefits of the members after the coordination is not less than the benefits before the coordination, which means

$$\begin{cases} \pi_{1H} - \pi_{1S} \ge 0 \\ \pi_{2H} - \pi_{2S} \ge 0 \end{cases} \tag{25}$$

Solving Eq. (25), we obtain

$$\frac{1}{8}\beta_0 \le \beta \le \frac{3}{16}\beta_0 \tag{26}$$

Where $\beta_0 = \dfrac{\left(ck_2 - a - cl + \lambda a + mt\right)^2}{k_2}$.

To sum up, in a given order-to-delivery cycle t, the decision variables of the coordination contract meet

$$\begin{cases} \alpha = \dfrac{\left(k_2 + l\right)lmt - al\left(\lambda k_2 + \left(1-\lambda\right)l\right) - c\left(2k_2 - l\right)\left(k_1k_2 - l^2\right)}{\left(k_1 + l\right)k_2mt - ak_2\left(\lambda l + \left(1-\lambda\right)k_1\right) - ck_2\left(k_1k_2 - l^2\right)} \\ \dfrac{1}{8}\beta_0 \le \beta \le \dfrac{3}{16}\beta_0 \end{cases} \tag{27}$$

Under the conditions (21) and (27), the wholesale price discount contract can achieve the coordination of the dual-channel supply chain, improve the profits of members, and achieve the Pareto improvement.

5 An illustrative numerical example

To further explain the effect of the sensitivity coefficient m and the order-to-delivery cycle t on the coordination pricing of the dual-channel supply chain, and the coordination effect of the proposed wholesale price discount coordination contract to the dual-channel supply chain, the following will be illustrated by a numerical example. For our simulation, we use the following parameter values: $a = 200, k_1 = k_2 = 1, l = 0.5, c = 10, \lambda = 0.2, k = -0.6, b = 3500$ .we vary t and m using the following data ranges:

$t \in \{10,15,20,25,30,35,40\}$, $m \in \{0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$.

In practice, the number of franchise fee paid by the retailer to the manufacturer is determined by the members of the dual-channel supply chain according to their bargaining power, this paper chooses the minimum threshold value to explain the problem, and we can obtain the equilibrium outcome in factors, summarized by Table 1 and 2.

Table 1 – The equilibrium outcome when t changes

t	p_{1C}^*	p_{2C}^*	α	β	π_{1DD}	π_{1C}	π_{2DD}	π_{2C}	π_{DD}	π_C
10	79	119	0.374	2775	2852	2852	1388	2775	4239	5627
15	76	116	0.371	2665	2441	2441	1332	2665	3773	5105
20	73	113	0.367	2556	2045	2045	1278	2556	3323	4601
25	70	110	0.364	2450	1665	1665	1225	2450	2890	4115
30	67	107	0.360	2346	1301	1301	1173	2346	2474	3647
35	64	104	0.356	2245	953	953	1122	2245	2075	3197
40	61	101	0.351	2145	620	620	1073	2145	1692	2765

From Table 1, we know that, the optimal pricing of the manufacturer and the retailer is reduced, the discount percentage of wholesale price decreases, the franchise fee paid by the retailer decreases, the manufacturer's profit and the retailer's profits are reduced, and the total profit of the supply chain is reduced when the order-to-delivery cycle is extended. The extension of the order-to-delivery cycle leads to a decrease in the product demand. The manufacturer and the retailer will attract consumers by lowering their prices, at the same time, the discount percentage of wholesale price offered by manufacturers will decrease to maintain their own profitability. The extension of the order-to-delivery cycle also makes the investment cost of the manufacturer also reduced, and the retailers' free riding behavior will also weaken. So the franchise fee paid by the retailer also reduces.

Table 2 – The equilibrium outcome when m changes

m	p_{1C}^*	p_{2C}^*	α	β	π_{1DD}	π_{1C}	π_{2DD}	π_{2C}	π_{DD}	π_C
0.2	80	120	0.375	2813	3003	3003	1406	2813	4409	5815
0.3	78	118	0.372	2720	2652	2652	1360	2720	4011	5371
0.4	75	115	0.370	2628	2312	2312	1314	2628	3626	4940
0.5	73	113	0.367	2538	1983	1983	1269	2538	3252	4521
0.6	70	110	0.364	2450	1665	1665	1225	2450	2890	4115
0.7	68	108	0.360	2363	1358	1358	1182	2363	2540	3721
0.8	65	105	0.357	2278	1062	1062	1139	2278	2201	3340
0.9	63	103	0.354	2195	777	777	1097	2195	1874	2971

From Table 2, we know that, as the time sensitivity m increases, the optimal pricing of the manufacturer and the retailer is reduced, the discount percentage of wholesale price decreases, the franchise fee paid by the retailer decreases, the manufacturer's profit and the retailer's profits are reduced. In the numerical analysis, the franchise fee paid by the retailer to the manufacturer is the minimum value of the threshold range, so the profit of the manufacturer is the same before and after the channel coordination.

By comparing the profits before and after the coordination, it is found that the profits after the coordination is not less than that before the coordination. This shows that the wholesale price discount contract can make the dual-channel supply chain to achieve coordination, improve the interests of the members and the general, and achieve the Pareto improvement.

6 Conclusions

This paper is based on the dual-channel supply chain consisting of a single manufacturer and a single retailer with the product demand sensitive to price and order-to-delivery cycle, we have considered the investment cost of the manufacturer to shorten the order-to-delivery cycle. The optimal pricing strategy and profits of dual-channel supply chain in the decentralized decision model and centralized decision model have been analyzed. And we have also designed the wholesale price discount contract to realize the channel coordination of the dual-channel supply chain. Finally, we have found that the pricing and profits of the dual-channel supply chain will reduce when the order-to-delivery cycle increases through a numerical example analysis, and the time sensitivity of demand has the same effect. The results have verified the effectiveness of the designed coordination contract.

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