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Research on Analytic Hierarchy Process Based on New Utility Function- A Case Study of the National College Entrance Examination

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Abstract
For the shortcomings of classic Analytic Hierarchy Process (AHP), we propose a new type of utility function based on utility theory to research the situation where the huge change of utility value could seriously influence and even invert the final evaluation result when one or more indexes value gradually approach or reach its ideal optimum. So this article proposes an idea of dealing with this kind of questions by combining AHP with the new utility function. The experiment shows the AHP with new utility function could comprehensively make evaluation, make up for the lack of AHP when evaluating alternatives with particularly prominent index value, and meet the diversified demands.

Key words： *multi-criteria decision making; analytic hierarchy process; utility function; comprehensive evaluation method; national college entrance examination*

1 Introduction

Many decision making situations often need to consider multiple attributes or criteria in order to select suitable alternative. But how to fully analyze the alternatives under different criteria and integrate these properties is an important part of making assessment. With the wide applications in practice or theory, Multi-Criteria Decision Making (MCDM) has been an interesting region from practitioners to academics. Some MCDM tools have been developed, like TOPSIS,[1] AHP and DEA.[2] With further research, comprehensive methods have also been developed, like AHP/DEA,[3] integrating the good and avoiding the weaknesses of them. Among these common methods, the AHP approach is popular with the following features: consistence test of judgment, simplifying complex system problems, pairwise judgment matric for determining criteria weights, and the compatibility and expansibility with other comprehensive methods.[2] Therefore AHP is chosen to carry out relative importance judgment of indexes and make preparation for further work.

The use of constant index weights determined by AHP often ignores the effectiveness of the utility of index value, and even results in unreasonable evaluation results. For example, within the National College Entrance Examination in China, the weight of each subject is assumed to be equivalent and constant. However, the extremely special expertise in some field of a candidate is likely to be diluted by the general or bad performance in other subjects and miss those who have special talents and outstanding contributions in some aspect. Therefore, with regard to the weakness of conventionally comprehensive evaluation methods in coping with such problems, this article proposes a resolution way that treats these kinds of problem where significant transformation occurs when some index value approaches or achieve its ideal optimal by combining AHP with utility function. Not only could the new evaluation method make overall evaluation for each candidate, but it could fully consider alternatives with special talent to ensure that the special utility of expertise or specialty is not be "diluted", which guarantees these alternatives with particularly prominent performance in certain aspect is not be blundered away, meeting the diverse needs of the society for specialists or talents.

2 Brief of AHP based on Utility Theory

2.1 A Brief Review of AHP

As a great popular MCDM method, AHP has been widely used in many practical situations for many years since it was proposed by Saaty. AHP is designed to tack with complicated decision making problems involving multiple goals and alternatives. As a comprehensive evaluation method, AHP is used in a form of compensatory optimization method to make assessment by combining qualitative and quantitative criteria based on hierarchical structure. The main components of AHP include hierarchical and multilevel structure of goals or criteria, alternatives, reciprocal judgment matric being compiling, criteria weights and the way in which aggregation is performed to rank all the alternatives.[4] The advantages of AHP is also reviewed in the literature.[5] More details about the theory of AHP can refer to the literature.[6] Given the above, we choose AHP to further analyze the connection with utility theory.

2.2 Utility Theory

Since Bernoulli resolved the Paradox by using a logarithmic utility function based on the existing wealth, the concept of utility was first proposed in the word of preference not utility.[7] The utility or preference contains subjective and objective attributes, and is a kind of mental feeling influenced impressively by individual characteristic and experience. However, the utility or preference is not easy to quantitate and depends closely on the decision makers. So a variety of utility functions are constructed to calculate the utility of index value to stakeholders.[8] In decision making process, the introduction of utility conception provides a new solution method to dealing with these problems, which facilitates the decision making process.

2.3 AHP based on Utility Theory

The combination of AHP and utility theory in solving decision making problems has been researched in many references. And the utility foundation of AHP was also validated by constructing a framework which synthesizing AHP.[9] Based a new scaling method, AHP and multiple attribute utility theory was connected.[10] In the implementation of AHP method, many types of utility functions have been used to reflect the actual preference of decision makers. With the use of AHP, the data must be converted uniformly into the value between zero and one to get rid of the difference from dimensions of qualitative and quantitative indexes before they are put into utility function. The ultimate utility value based on conventional utility function using AHP generally is confined to be no bigger than one.

Whether the variable in utility function is continuous or not, the utility function itself is often considered to be continuous, which is unable to express the tremendous fluctuation in utility value, i.e., the utility curve becomes steeper with a same increase when some variable is close to its ideal optimum. While the value of conventional utility function may be discrete, this type of fluctuation is often limited to a certain range, like 0,1, and may still be diluted by weighting sum, causing the failure of selection an alternative with specialty in some aspect. Here, the utility is assumed to be infinite or enough large when the variable reaches its optimal.

3 Construction of AHP Based on New Utility Function

With the mentioned problems, this article constructs a continuous utility function based on the concept of infinity or enough large number to cope with dramatic change of the utility with index value achieving the best optimal. Given a MCDM problem, the hierarchical model is first built according to its internal logic. The calculation process is as follows, in which the number of hierarchy levels is assumed to be three. Related symbols are defined as following:

M: total number of criteria;

N: total number of alternatives;

i: alternative, with i=1, 2, …, N;

j: criteria, with j=1, 2, …, M;

x_{ij}: value of alternative i on the criteria j, with i=1, 2, …,N and j=1, 2, …,M;

x_j^*: the ideal optimal value or level of criteria j, with j=1, 2, …,M;

y_{ij}: normalization of x_{ij} on criteria j, with i=1, 2, …,N and j=1, 2, …,M;

w_j: weight of the criteria j on the goal, with j=1, 2, …, M and $\sum_{j=1}^{M} w_j = 1$;

U_j: utility function on criteria j, with j=1, 2, …,M;

u_{ij}: utility value of alternative i on criteria j, where $u_{ij} = U_j(y_{ij})$;

H: infinite or enough large utility value of x_{ij} when it reaches its optimal under criteria j;

Z_i: ultimate utility value of alternative i on the highest goal of the hierarchy;

With the MCDM, a classic three-level hierarchy is used to represent the complex multiple attributes decision making process. The structure starts with the general object at the highest level, followed by a set of criteria, and ends with alternatives at the bottom level. The determination of criteria weights relates closely with the preferences of experts and stakeholders over these criteria. The weight w_j is determined for criteria *j*. And then, we can acquire original data x_{ij} and x_j* by investigating stakeholders or experts. Afterwards, the normalized data y_{ij} is calculated by using the formula $y_{ij}=x_{ij}/x_j*$. The utility u_{ij} of alternative *i* on criteria *j* is acquired by putting y_{ij} in the constructed utility function U_j. When x_{ij} reaches x_j*, the responding y_{ij}=1, and we assume related utility of x_{ij} is infinite or enough large, i.e., $u_{ij}=U_j(y_{ij,})= U_j(1)=H$. Completing the utility, we weight the utility u_{ij} of alternative *i* over all the criteria, $Z_i=\sum_{j=1}^{M} w_j * u_{ij}$, to score and rank alternatives in order to obtain the best one.

4 Application of AHP Based on New Utility Function

Many decision making problems in our everyday life could be explained by AHP based on the new utility function. A good example is the National College Entrance Examination in China. As one way of diversified admissions, the special admission does work. Besides, varieties of diversified policy of admission in American university are presented in the literature.[11] The practice of exceptional admission can be traced back to the Qing Dynasty, there is still no research analyzed from the utility perspective. In a broad way, exceptional admission is the behavior of adjusting admission standard to recruit people with special talent or expertise. As a complementary method, the occurrence of special admission breaks down the original standard and policy by establishing special procedures for admitting candidates. The outcome of special admission proved the new method reasonable. The recruitment of candidates with special talent or geek helps our country cultivate a group of Great Masters of Chinese Culture, like Qian Zhongshu, the writer of Fortress Besieged. There also are many similar instances which can be explained by the proposed theory and model of this article.

To simplify, five candidates of A, B, C, D and E is assumed in an examination for admission where the essential score is 620 points. The original information is shown in Table 1, where same priority is assigned to four subjects, i.e., $w_1=w_2=w_3=w_4$=0.25. The formula $y_{ij}=x_{ij}/x_j^*$ is used to normalize original data. Besides, the full mark of Chinese, Mathematics and English are separately 150 points, 300 points for General Science (Biology, Physics and Chemistry). The normalized data are shown in Table 2. Then calculating the utility function, $U_{ij}=U_j(y_{ij})=\frac{1}{1-y_{ij}}$, the utility u_{ij} of alternative *i* on criteria *j* is obtain. Weighting the utility u_{ij} of alternative *i* on all criteria by using expression $Z_i=\sum_{j=1}^{4} w_j * u_{ij}$, the ultimate utility of alternative *i* is acquired, with *i =A, B, C, D, E*. The result of utility calculation over the general object is structured and then sorted in Table 3, where the sign, H, represents an infinite or enough large number.

Table 1 - Score of each subject and score-based ranking

Candidate	Chinese	Mathematics	English	General Science	Total Score	Score-Based Ranking
A	126	128	137	234	625	3
B	122	138	133	240	633	1
C	122	138	124	246	630	2
D	127	133	122	236	618	5
E	112	150	120	240	622	4

Table 2 - Result of normalizing data

Candidate	Chinese	Mathematics	English	General Science
A	0.84	0.85	0.91	0.78
B	0.81	0.92	0.89	0.80
C	0.81	0.92	0.83	0.82
D	0.85	0.89	0.81	0.79
E	0.75	1.00	0.80	0.80

Table 3 - Ultimate utility score and utility-based ranking

Candidate	Chinese	Mathematics	English	General Science	Utility	Ranking
A	6	7	12	5	7.29	4
B	5	13	9	5	7.92	2
C	5	13	6	6	7.30	3
D	7	9	5	5	6.35	5
E	4	M	5	5	M	1

As shown in Table 3, the introduction of utility in AHP results in the priority order is E, B, C and A instead of B, C, A and E based on AHP. The rank change happens to A, B, C and E. For A, B and C, the decrease of rank results from being average of each item, and without outstanding performance in some aspect. Similarly, the increase of ranking of E is due to the fact he/she is outstanding in mathematics. The outcome could be interpreted from the point of utility: From the perspective of Mathematics potential, E, with full score in mathematics, is likely to be a potential genius in math and may be cultivated to be an outstanding person in mathematics. For B with higher total score, the possibility of making outstanding achievements in the future may be relatively lower than E in mathematics. In particular, from the perspective of the Mathematics, the A may be less preferred than other four ones by the decision makers or other people in charge.

5 Conclusion

Through the introduction of utility theory into AHP for multi-attribute decision making problems, this article describes in detail the theory and application of AHP based utility. In addition, the wide application of the method in practical problems indicates that the AHP based utility has extensive background and space in multi-attribute decision making field. Finally, a specific experiment illustrates and proves the feasibility and rationality of the proposed method, making up for the deficiency of AHP in dealing with alternatives with ideal optimal in certain aspect, providing another selection for decision makers to cope with the proposed situations.

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