Deduction and modification of experimental similarity criteria for rheological model of soft rock under temperature

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Abstract. With the increasing of mining depth, the temperature has become one of the main factors influencing the deep mining engineering. As the surrounding rock of deep soft rock has the characteristics of strong rheology and large deformation under the complex environment of high temperature and high pressure, the key to the support is to master the rheological law. Based on the three laws of similarity, we briefly analysed the influence mechanism of temperature on the soft rock mechanical characteristics, and deduced in detail the similarity criteria which must be met by the geometric quantities and physical quantities of the prototype and the model in the rheological model test of soft rock under temperature, and then obtained the similarity relation which must be met among various physical quantities of the prototype and the model in the rheological model test of soft rock and enriched the application field of similarity criterion, which has important practical engineering value for support of deep soft rock engineering.

1 Introduction

Geological mechanical model experiment is a kind of research methods for the rock mechanics properties with development early, widely used and image visual and is also the important means to solve and research complex engineering topic, and has been widely used especially in the deep undergroumd geotechnical engineering field of complicated conditions. Based on the similarity theory, we made a series of laboratory mechanics experiment by making similar specimens as the research object and deduced the prototype mechanical properties through the model mechanical properties, which saved the experiment cost and shorted the experiment cycle. Thereby the results show that geological mechanical model experiment is a a simple, efficient and energy saving method of rock mechanics research[1].

Problems often appear but unable to use mathematical equations to describe in engineering practice and has no equation solution phenomenon or geotechnical problems[2,3]. Model test has strong application value especially for complex engineering problems that cannot be established mathematical model. So far, most of the research has focused on the statics model experiment of brittleness similar materials and the mechanical behavior and failure law of a single rock in elastic state[4], and has less study for the rheological properties of the soft rock similarity theory. It mainly included that Li Yuan[5] deduced the similarity criterion that model and the prototype should be met in the soft rock model test using the similarity theory of dimension analysis and obtained the similar relationship between various physical quantities that the model and the prototype must be met in the process of the elastic stage, yield stage, the creep and stress relaxation in the soft rock model test, and Ma Zhenguo[6] derived the similarity criterion of soft rock creep stage using the similar principle and preliminary discussed the similarity criterion considering the influence of temperature on the properties of soft rock creep.

Traditional theories about the similar model of the rock have not considered the temperature factor into the rock similarity theory system category, or even studied, not analysed deeply in nature. Considering the complexity of deep soft rock engineering (such as material anisotropy, complex stress field and environment field and its complex boundary conditions, etc), it is necessary to

derive the similar simulation theory system including temperature,which lay the foundation of solid flow transformation theory research for the deep soft rock.

2 Similar principles

2.1 The basic content

(1) Similar to the first theorem: the phenomenon of similar to each other describes the phenomenon of the same characteristics (the similarity criterion)correspondent equal.

(2) Similar to the second theorem, also called E Buckingham law: if a physical system contains nphysical quantities and k basic dimensions, then n physical quantities can be expressed as (n-k) independent function relation between similarity criterion π_1 、 π_2 …… π_{n-k} , namely $f(\pi_1\pi_2\cdots\cdots\pi_{n-k})=0$.The similarity phenomenon each other should be equal to similarity criterion in the corresponding points and the corresponding time, so is their π relation. The relaiton can promote the test results to the same phenomenon function, and as it is a π dimensionless equation with less yuan converted from a multivariate physical function relatio.(3)Similar to the third theorem: If the two phenomenas can be described by the relation with the same text, and the single valued condition is similar and the similarity criterion consisted by it is equal, then the two phenomenas are similar.

2.2 Derivation method of similarity criterion

Three kinds of method of derivation similarity criterion are as follows: Law analysis method、equation analysis method(similar analysis method) and dimensional analysis method. (1) Law analysis method: It requires people to fully grasp all the the laws of physics for the phenomenon and distinguish between primary and secondary. (2)Equation analysis method: It is a derivative similar characteristics method according to the basic differential equations and all the single valued conditions using some known rules in physical phenomena.(3)Dimensional analysis method: It generated in the process of similar phenomenon through investigating the dimensions of each physical quantity, which is based on the theory of odd equation dimension[7,8].

3 The similarity criterion of soft rock model test without considering temperature

Rheology means that objects exist deformation properties related with time in the forced deformation. Soft rock rheological phenomenon contains creep, relaxation, strain rate effect and the long-term strength. Soft rock rheological phenomenon is very complicated. Theorys of studying the rheology in rocks include aging theory, flow theory, hardening theory and theory of hereditary creep, which all take a different form of equation and have some limitations for the rheology. And it doesn't achieve the level of expressing with unified mathematical equation for the understanding of the rheological, so it is fit for deducing the similarity criterion with dimension analysis method.

Physical quantities related to the soft rock rheological phenomenon without considering temperature have rock external forces F , physical quantity of geometric characteristics l , the rock internal stress σ , strain ε , stress effciency $\dot{\sigma}$, strain effciency. Physical quantities of rock viscous-elastic have elasticity modulus E , poisson's ratio μ , viscous coefficient C , frictional angle φ , yield strengh σ_s and time t . The relationship between each physical quantity can be expressed as $f(\varepsilon,\mu,\varphi,\dot{\varepsilon},\dot{\sigma},\sigma_s,\eta,\sigma,E,c,F,l,t)=0$, there are 10 similarity criterions[5]:

$$\pi_1=\varepsilon,\ \pi_2=\mu,\ \pi_3=\varphi,\ \pi_4=\dot{\varepsilon}t,\ \pi_5=\frac{\dot{\sigma}l^2t}{F},\ \pi_6=\frac{\sigma_s l^2}{F},\ \pi_7=\frac{\eta\, l^2}{Ft},\ \pi_8=\frac{\sigma\, l^2}{F},\ \pi_9=\frac{E\, l^2}{F},\ \pi_{10}=\frac{c\, l^2}{F}$$

4 Similarity criterion of soft rock model test under temperature

4.1 Temperature effect on the mechanism analysis of soft rock rheological

Rock mineral grains are complex and random and belong to anisotropy of mineral aggregate. All kinds of minerals in rock have different thermal expansion coefficient under high temperature, so the deformation of mineral grains is not the same after heated rock. Rock as a continuum, however, whose internal mineral grains can't change and deform according to their inherent thermal expansion coefficient with temperature in order to maintain the deformation continuity. Therefore, mineral grains produce constrains each other and form the thermal stress with large deformation in compression and small deformation in tension. When the thermal stress genetated by the rock exceeds the thensile stress yield strength between the rock particles, the internal structure of rock will be destroyed and generate new damage.[9-13].

4.2 The derivation of similarity criterion under temperature

Join the temperature and thermal physical parameters related to soft rock including density ρ ,thermal expansion coefficient α ,thermal conductivity λ ,specific heat capacity C_P, temperature Θ considering the influence of the 13 physical quantities on the rheological properties.The relationship between the physical quantities can be expressed as: $f(\varepsilon,\mu,\varphi,\dot{\varepsilon},\dot{\sigma},\sigma_s,\eta,\sigma,E,c,F,l,t,\rho,\alpha,\lambda,C_P,\Theta)=0$,the related physical quantities affected the rheological properties of the soft rock increased from 13 to 18, and 18 physical dimensions are composed of four basic dimensions—the time dimension T、the length of the dimension L、the quality of the dimension M and the temperature dimension Θ：n=18、r=4, which can form 14 dimensionless quantities. And we select four dimensional physical quantities F 、l 、t 、θ including four basic dimensions as the basic physical quantities and the dimensional matrix of the soft rock rheological model experiment is as follows:

	ε	μ	φ	$\dot{\varepsilon}$	$\dot{\sigma}$	σ_s	η	σ	E	c	F	l	λ	C_p	α	ρ	T	θ
M	0	0	0	0	1	1	1	1	1	1	1	0	1	0	0	1	0	0
L	0	0	0	0	−1	−1	−1	−1	−1	−1	1	1	1	2	0	−3	0	0
T	0	0	0	−1	−3	−2	−1	−2	−2	−2	−2	0	−3	−2	0	0	1	0
Θ	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	−1	0	0	1

Based on the similar to the second theorem, the 14 dimensionless quantity was the similarity criterion of soft rock rheological phenomenon and the corresponding matrix π is as follows:

	ε	μ	φ	$\dot{\varepsilon}$	$\dot{\sigma}$	σ_s	η	σ	E	c	λ	C_p	α	ρ	F	l	T	θ
π_1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
π_2	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
π_3	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
π_4	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0
π_5	0	0	0	0	1	0	0	0	0	0	0	0	0	0	-1	2	1	0
π_6	0	0	0	0	0	1	0	0	0	0	0	0	0	0	-1	2	0	0
π_7	0	0	0	0	0	0	1	0	0	0	0	0	0	0	-1	2	-1	0
π_8	0	0	0	0	0	0	0	1	0	0	0	0	0	0	-1	2	0	0
π_9	0	0	0	0	0	0	0	0	1	0	0	0	0	0	-1	2	0	0
π_{10}	0	0	0	0	0	0	0	0	0	1	0	0	0	0	-1	2	0	0
π_{11}	0	0	0	0	0	0	0	0	0	0	1	0	0	0	-1	0	1	1
π_{12}	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	-2	2	1
π_{13}	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1
π_{14}	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	4	-2	0

Then we can get 14 similarity criterion through sorting:

$$\pi_1=\varepsilon,\ \pi_2=\mu,\ \pi_3=\varphi,\ \pi_4=\dot{\varepsilon}t,\ \pi_5=\frac{\dot{\sigma}l^2t}{F},\ \pi_6=\frac{\sigma_s l^2}{F},\ \pi_7=\frac{\eta\, l^2}{Ft},\ \pi_8=\frac{\sigma\, l^2}{F},\ \pi_9=\frac{E\, l^2}{F},\ \pi_{10}=\frac{c\, l^2}{F},$$

$$\pi_{11}=\frac{\lambda t\theta}{F},\ \pi_{12}=\frac{C_p t^2\theta}{l^2},\ \pi_{13}=\alpha\theta,\ \pi_{14}=\frac{\rho l^4}{Ft^2}$$

4.3 The adjustment and application analysis of the similarity criterion under temperature

Assuming that we can take C to show the ratio of similitude of each physical quantities between the prototype (P) and the model (M) of soft rock, then:

$$C_\varepsilon=\frac{\varepsilon_p}{\varepsilon_M},C_\mu=\frac{\mu_p}{\mu_M},C_\varphi=\frac{\varphi_p}{\varphi_M},C_{\dot{\varepsilon}}=\frac{\dot{\varepsilon}_p}{\dot{\varepsilon}_M},C_{\dot{\sigma}}=\frac{\dot{\sigma}_p}{\dot{\sigma}_M},C_{\sigma_s}=\frac{\sigma_{s_P}}{\sigma_{s_M}},C_\eta=\frac{\eta_p}{\eta_M},C_\sigma=\frac{\sigma_p}{\sigma_M},C_E=\frac{E_p}{E_M},C_c=\frac{c_p}{c_M},C_\lambda=\frac{\lambda_p}{\lambda_M},$$

$$C_{C_P}=\frac{C_{P_P}}{C_{P_M}},C_\alpha=\frac{\alpha_p}{\alpha_M},C_\rho=\frac{\rho_p}{\rho_M},C_F=\frac{F_p}{F_M},\ C_l=\frac{l_p}{l_M},\ C_T=\frac{T_p}{T_M},\ C_\theta=\frac{\theta_p}{\theta_M}$$

The similarity criterion of the similar phenomena is equal according to the similar to the first theorem(similarity index is 1), which can get the similar condition met by the model and the prototype of soft rock rheological under high temperature in combination with 14 similarity criterion numbers:

$$C_\varepsilon=C_\mu=C_\varphi=1 \tag{1}$$

$$C_{\dot{\varepsilon}}C_t=1 \tag{2}$$

$$C_{\sigma_s}=C_\sigma=C_E=C_c=\frac{C_F}{C_l^{\,2}} \tag{3}$$

$$C_\sigma=C_{\dot{\sigma}}C_t \tag{4}$$

$$C_\eta=C_\sigma C_t \tag{5}$$

$$C_F=C_\lambda C_t C_\theta \tag{6}$$

$$C_l^{\,2}=C_{C_P}C_t^{\,2}C_\theta \tag{7}$$

$$C_\alpha C_\theta=1 \tag{8}$$

$$C_F=\frac{C_\rho C_l^{\,4}}{C_t^{\,2}} \tag{9}$$

The formula（6）divided by the formula （7）:

$$\frac{C_F}{C_l^{\,2}}==\frac{C_\lambda}{C_{C_P}C_t} \tag{10}$$

In general case, C_l is the constant value after the geometry size of prototype, model is sure, $C_\varepsilon,C_\mu,C_\varphi,C_\eta,C_c$ can be sure after the similar material is sure. Similar time constant of soft rock rheological is $C_t=\frac{1}{C_{\dot{\varepsilon}}}=\frac{C_\eta}{C_\sigma}$ by formula (2),(5), which illustrate the material of high creep rate can simulate the flow variables of lager soft rock in a short time, and the material of large viscous force and small internal stress can meet the requirement of large rheological in a short time; The force ratio of similar model experiment is determined after similar material and geometry size is determined by formula (3); the temperature similar multiple is determined accordingly after the force similar multiple, similar multiple of coefficient thermal conductivity and time are sure by formula (6) under high temperature; Formulas (8) mainly illutrate that the thermal expansion coefficient ratio of the prototype and the model directly affect the test temperature of prototype and the model, and it is best to choose the material of larger expansion coeffcient on more sensitive temperature to make the model experiment, which can provide theoretical support for the model making on the right material. the geometry of the model has very important effect on the model experiment by formula(3), (9). If the geometric size ratio of prototype and the model is 2:1, then the applied load ratio of the prototype and the model is 16:1, which can meet the experimental requirements, only to apply the applied load 1/16 of the prototype to the model, so we should take a full consideration to the precision of the experimental equipment and the bearable load scope when determining model geometry size to select the suitable scale ratio as much as possible. In the meantime, it can shorten the cycle model experiment and reduce the experiment cost by controling the model density, geometry size and thermal physical parameters when carrying out the creep

model experiment.

5 Summary

Based on the three similar laws, we deduced the similarity criterion suitable for the similarity test of soft rock rheological by using dimensional analysis without considering the similarity criterion of soft rock rheology on temperature through introducing the temperature parameters and material parameters of thermal and physical properties and got 14 similar criterion numbers, then obtained 10 similar conditions after adjusting and simplifying. Similar conditions has a guiding significance to determine the geometry size and material of the specimen in the model experiment. it can greatly short the experiment cycle and reduce the experiment cost by similarity simulation experiment, and the dimensional analysis has a great significance on the creep characteristics research of soft rock rheology.

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