A new method of fitting *P-S-N* curve for ultrahigh strength sucker rod

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Abstract. It is commonly believed that the fatigue life and cyclic stress is logarithm linear dependence when fitting *P-S-N* curve, but data of fatigue tests indicate that the linear correlation isn't evident for ultrahigh strength sucker rod. It is necessary to present a new fitting method. The scatter diagram of $\lg S_P$ and $\lg N_P$ is fitted by several nonlinear models in SPSS, and the model S has the highest goodness of fitting. Finally modified equations of *P-S-N* curve for ultrahigh strength sucker rod are obtained.

Keywords: *P-S-N* curve; ultrahigh strength sucker rod; correlation coefficient; nonlinear fitting.

1 Introduction

P-S-N curve is relation curve between fatigue life and cyclic stress processed by fatigue tests in the survival rate at a given value [1]. Under adequate alternating stress, the cross section of sucker rod gradually is weakened, and sucker rod will have fatigue failures when stress reaches a certain limit. Additionally the statistical data of oil fields suggests that about 70% failures of sucker rod are due to fatigue damage, so the *P-S-N* curve has practical significance.

Previous *P-S-N* curve of sucker rod is on the basis of assumptions:

1) At an equivalent stress level fatigue life follows the normal distribution.

2) At a given survival rate, the relation of cyclic stress S_P and fatigue life N_P follows this equation,

$$N_P S_P{}^{\mathrm{m}} = C \tag{1}$$

where *m* and *C* are constant, and $\lg S_P$ and $\lg N_P$ is linear dependent.

Based on these assumptions, *P-S-N* curve of sucker rod is fitted by methods such as Least Square, Weighted Least Square, Maximum Likelihood [1-3], etc. However according to a great deal of fatigue tests of metal materials, $\lg S_P$ and $\lg N_P$ is not linear dependent for *P-S-N* curve in medium or long life region [4].

Because of high fatigue strength, ultrahigh strength sucker rod doesn't meet assumptions and it's necessary to correct the method of fitting *P-S-N* curve.

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2 Analysis of data of fatigue tests

Using PLG-300 fatigue-testing machine, data of fatigue life under different stress level, shown in Table 1, are acquired for ultrahigh strength sucker rod.

Table 1. Data of fatigue tests.

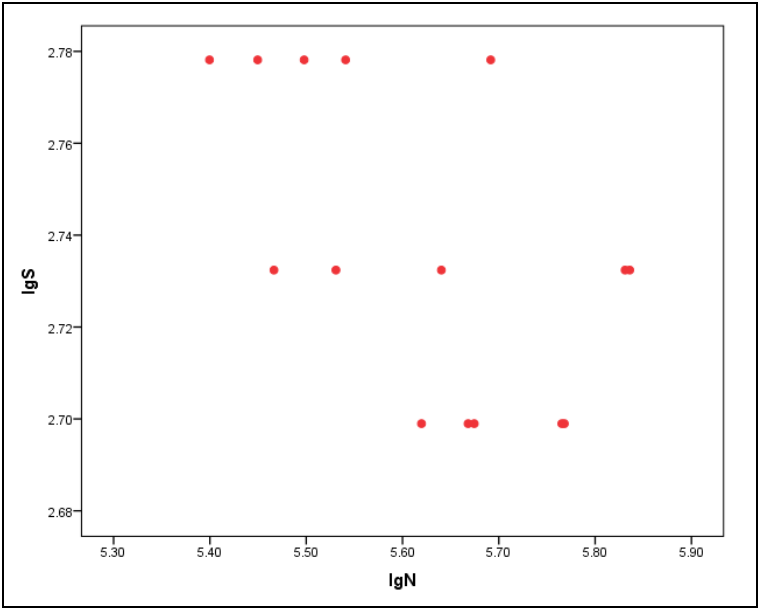| Grade	Stress level (MPa)	Fatigue life (time)
HY	500	465674, 472523, 582672, 416596, 586241
	540	339481, 436807, 677751, 685322, 292733
	600	250950, 314635, 347449, 281548, 491519
HL	500	342935, 249874, 433111, 413256, 408352
	540	274785, 192247, 168331, 243823, 240933
	600	197732, 285623, 294802, 320136, 302025

Figure 1. Scatter diagram of $\lg S_P$ and $\lg N_P$ for grade HY sucker rod

In these scatter diagrams $\lg S_P$ is dependent variables and $\lg N_P$ is independent variables as Figure 1 and Figure 2. Correlation can't be directly got, so correlation coefficient is introduced.
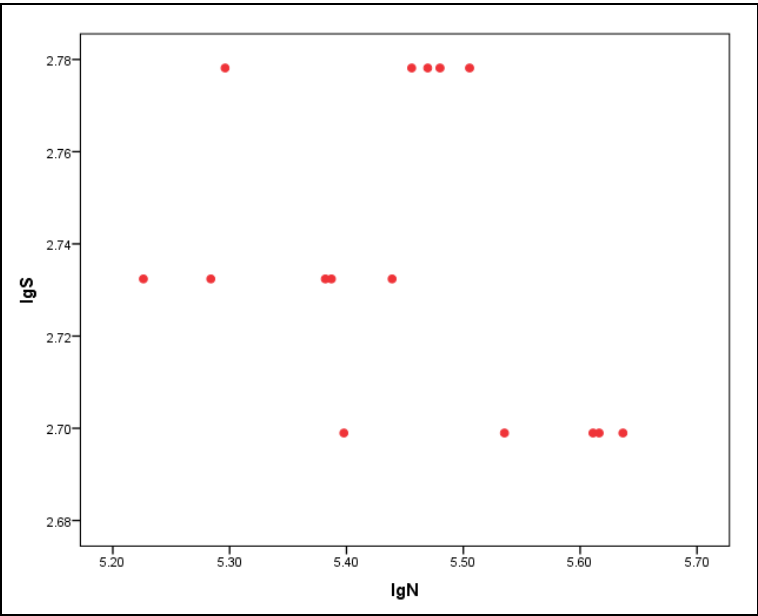
Figure 2. Scatter diagram of lgSP and lgNP for grade HL sucker rod

3 Analysis of correlation

The calculation results of K-S (Kolmogorov-Smirnov) show that at a equivalent stress level $\lg N_P$ follows the normal distribution when the significance level is 0.05. Taking the case of grade HY suck rod, Figure 3 is the normal P-P plot of $\lg N_P$ under 500MPa stress level. In Figure 3 splashes are near 45° curve, that is to say $\lg N_P$ follows normal distribution.
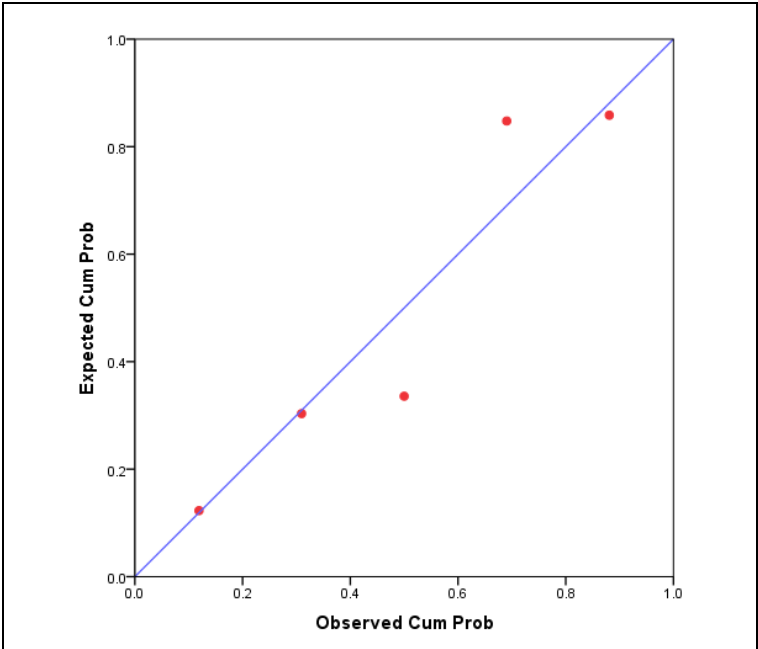

Figure 3. Normal P-P plot of $\lg N_P$

In order to evaluate the correlation of variables, correlation coefficient is introduced. Pearson simple correlation coefficient r is defined as

$$r = \frac{\sum_{i=1}^{n}\left(x_i - \overline{x}\right)\left(y_i - \overline{y}\right)}{\sqrt{\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2 \sum_{i=1}^{n}\left(y_i - \overline{y}\right)^2}} \tag{2}$$

Where n is sample capacity, x_i is dependent variables, y_i is independent variables. If $r=0$, there is no linear correlation. And the nearer an absolute value of r to 1, there is more obvious linear correlation.

We propose an assumption H_0: there is no obvious linear correlation, or there is zero correlation.

Test statistics t is defined as

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} \tag{3}$$

t follows the t-distribution with n-2 degrees of freedom.

SPSS (Statistical Product and Service Solutions) software calculate Pearson simple correlation coefficient r, test statistics t and corresponding probability P- shown in Table 3.

Table 2. Results of calculations by SPSS.

Grade	*r*	*t*	P-
HY	-0.567	0.027	0.05
HL	-0.347	0.205	

For HY sucker rod, r=-0.567 and P- less than 0.05, that is to say the assumption is valid, that is to say the assumption is invalid, so there is no obvious linear relationship. For HL sucker rod, r=-0.347 and P- more than 0.05, so there is no linear relationship.

So the method of linear fitting is not suitable for fitting *P-S-N* curve of ultrahigh strength sucker rod, and a new method of nonlinear fitting should be used.

4 Nonlinear fitting of *P-S-N* curve

Proper fitting of these experimental data is completed with some nonlinear models of SPSS software, shown in Table 3, and the reliability is 95%.

R^2 processed by SPSS is used to estimate the goodness of fitting with different nonlinear models. The physical significance of R^2 is correlation coefficient squared between independent variables and dependent variables, and the higher value of R^2 indicates a higher adoption of fitting models. It is assumed that the fitting model of grade HY and HL sucker rod is the same, and the results are shown in Table 4.

Finally equations of *P-S-N* curve for ultrahigh strength sucker rod are obtained when survival rate is 95%.

Table 3. Equations of nonlinear models in SPSS.

Model	Equation
Compound	$y = \beta_0 \beta_1^{\,x}$
Growth	$y = e^{\beta_0 + \beta_1 x}$
Logarithmic	$y = \beta_0 + \beta_1 \ln(x)$
S	$y = e^{\beta_0 + \beta_1 / x}$
Exponential	$y = \beta_0 e^{\beta_1 x}$
Power	$y = \beta_0 \left(x^{\beta_1} \right)$
Logistic	$y = \dfrac{1}{1/\mu + \beta_0 \beta_1 x}$

Table 4. Results of nonlinear fitting.

Grade	Model	β_0	β_1	Equations of fitting curve
HY	S	0.725	1.584	$\lg S = e^{0.725 + 1.584 / \lg N}$
HL		0.823	0.999	$\lg S = e^{0.823 + 0.999 / \lg N}$

For grade HY sucker rod,

$$\lg S = e^{0.725 + 1.584 / \lg N} \tag{4}$$

For grade HL sucker rod,

$$\lg S = e^{0.823 + 0.999 / \lg N} \tag{5}$$

And figures of *P-S-N* curve for ultrahigh strength sucker rod are shown in Figure 4 and Figure 5.
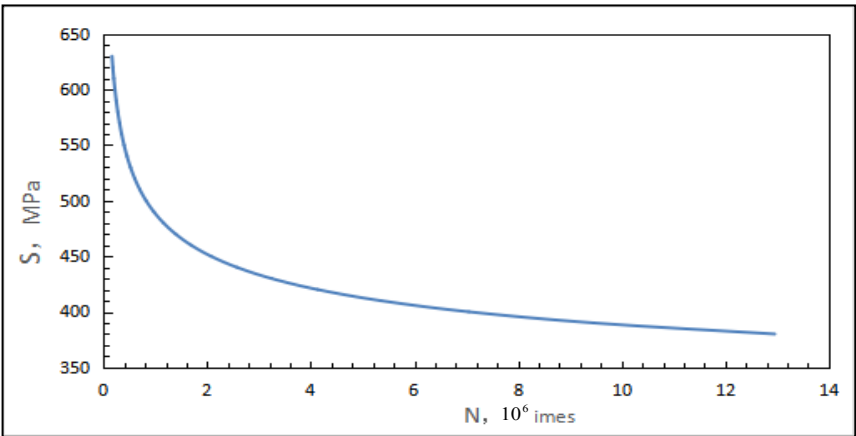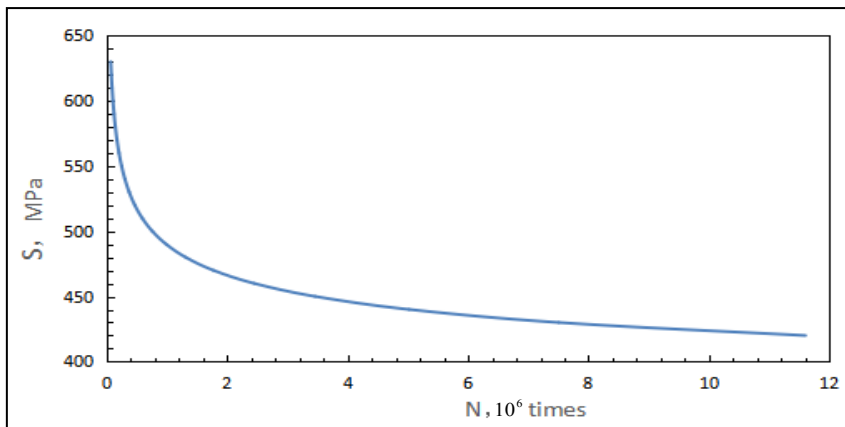

Figure 4. *P-S-N* curve of grade HY sucker rod.

Figure 5. P-S-N curve of grade HL sucker rod.

5 Summary

According to analysis of correlation coefficient, the relation of $\lg S_P$ and $\lg N_P$ is not simple linear correlation. A nonlinear method is used to fit data of fatigue tests for ultrahigh strength sucker rod, and equations of *S-P-N* curve are got. Equations can provide theoretical references for scientific use of ultrahigh strength sucker rod to avoid fatigue damage.

Acknowledgments

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