The bi-directional ICM method for topology optimization with the strain energy constraints

Jun Tie[1,a] and Yunkang Sui[2]

[1]*Department of Mathematics, Tianjin University of Finance and Economics, Tianjin, China*
[2]*College of Mechanical Engineering and Applied Electronics Technology, Beijing University of Technology, Beijing, China*

Abstract. To prevent the numerical instabilities about checkerboards, mesh-dependency and gray-scale elements in structural topology optimization with strain energy constraints, an improved ICM method based on the bi-directional filter function is presented. The sequential quadratic approximate algorithm is applied to solve the optimal model for continuum structure established in this paper. A classical numerical example is discussed which shows that the present method is effective and efficient.

Keywords: topology optimization, the strain energy constraints, independent continuous mapping (ICM) method, bi-directional ICM method.

1 Introduction

The essential of structural topology optimization lies in providing rational loads-pass roads for structures, so it is widely used in the period of conceptual design of loaded structures or components, in order to save material and reduce the cost of manufacture. There are mainly homogenization method [1], thickness-change method [2], density-change method [3], and ICM method [4] (Independent Continuous and Mapping) in continuum structural topological optimization, etc. ICM method abstracts the topology variables from size optimization scale in which topology variables adhere to area, density, etc. Firstly, the discrete 0-1 topology variables are expanded to [0,1] continuum domain by using step functions. Then step functions are approximated by use of polish function and filter function, changing the original discrete optimization model to continuous differentiable model.

However, Sui Yun-kang et al. [5] and Tie Jun et al. [6-7] study topology optimization of continuum structure with globalization of stress constraints using an efficient strain energy method.

In fact, most of papers referred to the above mentioned and the monographs [1-3] have been focused on global objectives, such as compliance, displacement, frequency and so on. It is fewer about the strain energy method in the structural topology optimization. Clearly the stress of the structural elements is a local quantity, it is necessary to introduce an alternative strain energy method to transform the local stress constraints into the strain energy constraints of the structural elements by means of the von Mises criteria in the theory of elastic failure.

In this paper, the bi-directional ICM method for topology optimization with the strain energy constraints is presented in which a bi-directional filter function is adopted to weaken the

[a] Corresponding author: tielaoshi@sina.com

low-topological-variable elements while enhancing the high-topological-variable elements by polarization.The sequential quadratic programming is adopted to solve it.

2 The bi-directional filter function

This paper study Sigmoid function to express the polish function and the filter function, and the expression of Sigmoid function is as follows

$$y = \frac{1}{1+e^{-\alpha x}} \ (\alpha > 0) \tag{1}$$

Sigmoid function (1) is shown in Fig.1 ($\alpha = 6, 7, 8$,respectively). When α is sufficiently large, $y = \frac{1}{1+e^{-\alpha x}}$ tends to step function. Since topological variable cannot be negative, we modify the half of the Sigmoid function so as to y = 1/2 correspond to t = 0 and it is a one-to-one correspondence relationship between the range of values of the Sigmoid function $[0.5,1]$ and the range of values of the polish function $[0,1]$ Let $t = 2(y-0.5)$, then we have

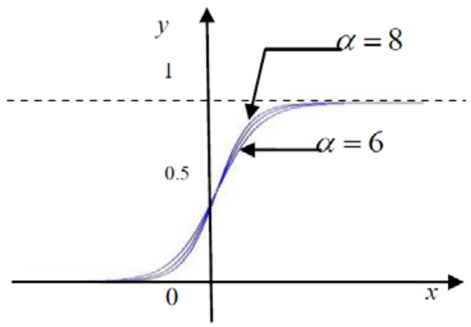

Figure 1. Sigmoid function.

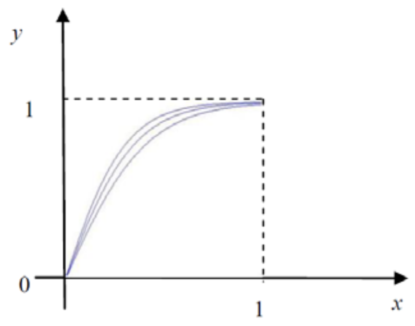

Figure 2. Modified Sigmoid function.

$$t = \begin{cases} 0, & y = 0.5 \\ 1, & y = 1 \end{cases} \tag{2}$$

Substituting Eq.(2) into Sigmoid function $y = \frac{1}{1+e^{-\alpha x}}$, then we obtain a polish function as follows (see Fig.2):

$$p(x) = \beta \frac{1-e^{-\alpha x}}{1+e^{-\alpha x}} \ (\alpha > 0, \quad 0 \le x \le 1) \tag{3}$$

where $\beta = \frac{1+e^{-\alpha}}{1-e^{-\alpha}} \ (\alpha > 0)$ is a correction coefficient to let the polish function (16) meet the properties $p(1) = 1$. Moreover, the inverse function of polish function (4) is taken as the filter function as follows (see Fig.2):

$$f(x)=\frac{1}{\alpha}\ln\frac{1+x/\beta}{1-x/\beta},\ (\alpha>0,0\le\beta\le 1) \tag{4}$$

Now we propose a new ICM method which is bi-directional approaching to the step function combining polish function (3) and its modified function. As it is shown in Fig.2, the new bi-directional filter function has several other advantages, such as stable and fast convergence, black and white solutions, etc. The new bi-directional filter function can be written as:

$$y=s(x)=\begin{cases}\delta[1-\beta\dfrac{1-e^{-\alpha(1-\frac{x}{\delta})}}{1+e^{-\alpha(1-\frac{x}{\delta})}}], & 0\le x<\delta\\ \delta, & x=\delta\\ \delta+(1-\delta)\beta\dfrac{1-e^{-\alpha(\frac{x-\delta}{1-\delta})}}{1+e^{-\alpha(\frac{x-\delta}{1-\delta})}}, & \delta<x\le 1\end{cases} \tag{5}$$

In Eq. (5), the new bi-directional filter function $s(x)\to p(x)$ when $\delta\to 0$, and $s(x)\to f(x)$ when $\delta\to 1$, thus the new bi-directional filter function (5) may obtain the optimization model (6) by the classical ICM method as:

Find topological variables $\mathbf{t}=(t_1,t_2,\cdots,t_n)$

Minimize: $W(t)=\sum_{i=1}^{n}s_w(t_i)w_i^0$

Subject to:

$$\begin{gathered} e_{ij}\le s_e(t_i)\overline{e}_i \\ 0<t_{\min}\le t_i\le 1 \quad (i=1,\cdots,n \\ 0<t_{\min}\le t_i\le 1\ ,\ j\in J_m=\{1,\cdots,m\} \end{gathered} \tag{6}$$

where w_i^0 is the original weight of the *i-th* element for structural material, t_i is the *i-th* independent continuous topological design variable, and $t=(t_1,\cdots,t_n)$ is the independent continuous topological design vector, the total number of design variables is n, the total number of external loads is m, e_{ij} is the strain energy density and $\overline{e}_i^0$ is defined as the original allowable strain energy of the i-th element which is also unique under multi-load case.

In Eq.(5), let $\alpha=4,\beta=\dfrac{1+e^{-4}}{1-e^{-4}}=1.0373\approx 1$,then the weight filter function is as follow:

$$s_w(t)=\begin{cases}\delta[1-\beta\dfrac{1-e^{-3(1-\frac{t}{\delta})}}{1+e^{-3(1-\frac{t}{\delta})}}], & 0\le t<\delta\\ \delta, & t=\delta\\ \delta+(1-\delta)\dfrac{1-e^{-3(\frac{t-\delta}{1-\delta})}}{1+e^{-3(\frac{t-\delta}{1-\delta})}}, & \delta<t\le 1\end{cases} \tag{7}$$

In Eq.(5), let $\alpha=10,\beta=\dfrac{1+e^{-10}}{1-e^{-10}}=1.0001\approx 1$,then the strain energy filter function is as follows:

$$s_e(t)=\begin{cases}\delta[1-\beta\dfrac{1-e^{-10(1-\frac{t}{\delta})}}{1+e^{-10(1-\frac{t}{\delta})}}], & 0\le t<\delta\\ \delta, & t=\delta\\ \delta+(1-\delta)\beta\dfrac{1-e^{-10(\frac{t-\delta}{1-\delta})}}{1+e^{-10(\frac{t-\delta}{1-\delta})}}, & \delta<t\le 1\end{cases} \tag{8}$$

3 Solutions for the optimization model

Take the reciprocal of stiffness filter function as design variables as follows

$$x_i=\frac{1}{s_e(t_i)} \tag{9}$$

We have

$$t_i=s_e^{-1}(x_i) \tag{10}$$

Therefore, the stiffness matrix filter function and weight filter function are given as

$$s_e(t_i)=\frac{1}{x_i}\ \ ;\ \ s_w(t_i)=s_w[s_e^{-1}(\frac{1}{x_i})]=F(x_i) \tag{11}$$

The weight W can be calculated by

$$W=\sum_{i=1}^{n}w_i=\sum_{i=1}^{n}s_w(t_i)w_i^0=\sum_{i=1}^{n}F(x_i)w_i^0 \tag{12}$$

However, we use second-order Taylor series expansion to approximate the objective function and ignore the constant item.

Let $F(x_i)=[s_e^{-1}(\frac{1}{x_i})]$, $a_i=\frac{1}{2}w_i^0F''(x_i^{(v)})$, $b_i=w_i^0[F'(x_i^{(v)})-x_i^{(v)}F''(x_i^{(v)})]$,then the model (6) is transformed into the following quadratic programming model:

$$\begin{cases} find \quad \mathbf{t}=(t_1,t_2,\cdots,t_n) \\ \min \quad W=\sum_{i=1}^{n}(a_ix_i^2+b_ix_i) \\ s.t. \quad e_{ij}x_i \le \overline{e_j} \\ \qquad 1\le x_i \le \overline{x_i}=\dfrac{1}{f_e^{-1}(t_{\min})} \quad (i)=1,\cdots,n \end{cases} \tag{13}$$

where $x_i^{(v)}$ is the *i-th* design variable corresponding to the *i-th* element in the $v-th$ iteration.The sequential quadratic approximate algorithm is applied based on exact dual mapping to solve the optimal model for continuum structure established in this paper.

4 A classical numerical example

The ground structure is a plane structure with 1.0 m2.4 m0.09 m and is divided into cell-grid of 2048. Fig. 15 and Fig. 16 illustrate the geometric and finite element models of the structure respectively. The elastic modulus of the material is 6.88910^{10} Pa with poisson ratio of 0.3 and density of 110^{3} kg/m^{3}. The left edge of the structure is fixed, the mid point of the right edge P is taken on concentrated load of 15600 N, and with displacement constrained less than 1.510^{-4} m downwards from P . Convergence precision is 0.001.

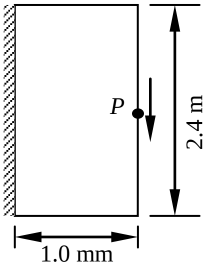

Figure 3. Ground structure.

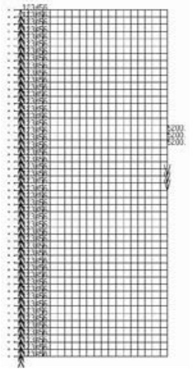

Figure 4. Finite element model.

The optimal topology configuration showed in Fig.5. is received after 15 times iterations. The configuration is similar to the configuration in Fig.6, which displays the result gotten by using power function as polish function in reference [5].

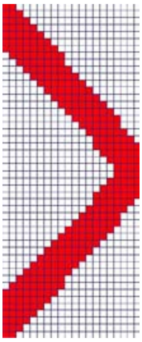

Figure 5. Optimal topology configuration in this paper.

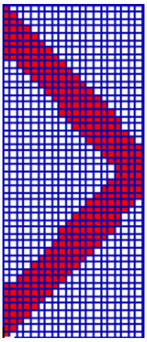

Figure 6. Optimal topology configuration [5] in reference [12].

5 Conclusions

The bi-directional ICM method for topology optimization with the strain energy constraints is effective and efficient to prevent the numerical instabilities about checkerboards, mesh-dependency and gray-scale elements in structural topology optimization. Moreover, a bi-directional filter function is adopted to weaken the low-topological-variable elements while enhancing the high-topological-variable elements by polarization. The classical numerical example is discussed which shows that the presented method is successful.

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References

1. BENDSOE M P, KIKUCHI N. Generating optimal topologies in structural design using a homogenization method [J].Comp. Meth. Appl. Mech. Engry, 1988, 71(1):197-224.
2. WANG Jian, Geng-dong. Optimal topology design of truck frame with stress constraints [J].Automotive Engineering, 1997, 19(1):15-19.(in Chinese)
3. ROZVANY GIN, ZHOU M.Generalized shape optimization without homogenization. Structural and multidisciplinary Optimization [J].1992, 1(4):250-253.
4. SUI Yun-kang.Modelling transformation and optimization new developments of structural synthesis method [M].Dalian: Dalian University of Technology Press,1996.(in Chinese)

5. Peng Xi-rong,Sui Yun-kang(2006) Topological optimization of continuum structure with static displacement and frequency constraints by ICM method, Chinese Journal of computational Machanics 23(4): 391-396.
6. Jun Tie, Sui Yun- kang. Topology Optimization Using Parabolic Aggregation Function with Independent-Continuous-Mapping Method [J]. Mathematical Problems in Engineering, 2013, 2013(3):1-18.
7. Tie Jun, Sui Yun-kang .An improved integrated optimization model for the structural topology problems.2010 International Conference on Machine Vision and Human-Machine Interface, MVHI 2010: 666-668.

