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Correction of axis tilt error for 40Cr ring gear measurement based on collinear measuring heads

Chengzhi Su[1, a *], Fangchao Shao[1, b], Biao Zhou[1, c], Chun Yan[1, d] and Lei Zhong[1]

[1]College of Mechanical and Electric Engineering, Changchun University of Science and Technology, Jilin Changchun 130022, China

[a]chengzhi_su@126.com, [b]905533634@qq.com, [c]350537623@qq.com, [d]760854231@qq.com

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Abstract. The ring gear and flywheel are interference fit. In order to verify whether magnitude of interference meets the processing requirements, the size change of the 40Cr ring gear before and after heating is measured. Measurement accuracy of ring gear measurement equipment should reach 0.01mm. Due to the machining errors of the components, assembly errors, elastic deformation and so on, the tilt of axis is almost universal. Therefore, the measurement accuracy of ring gear measurement equipment is reduced. The correction method of axis tilt error based on collinear measuring heads is proposed for the problem of measurement accuracy reduction due to the tilt of axis. Firstly, the physical model of axis tilt error is studied, and the corresponding mathematical model is established according to multi-body system theory. Secondly, the mathematical model of correction of axis tilt error based on collinear measuring heads is established and the tilt correction factor is solved. Finally, the system consisting of 2 eddy current sensors and 1 standard bar is used to identify the parameters of algorithm. Simulation results show that this algorithm can be very good for correction of axis tilt error when shaft inclination angle is in the range of 5^{o}. The ring gear measurement accuracy after correction of tilt error can meet the processing requirements of interference magnitude.

Introduction

The ring gear and flywheel are interference fit. In order to verify whether magnitude of interference meets the processing requirements, the size change of the 40Cr ring gear before and after heating is measured. Measurement accuracy of ring gear measurement equipment should reach 0.01mm. Due to the machining errors of the components, assembly errors, elastic deformation and so on, the tilt of axis is almost universal. Therefore, the measurement accuracy of ring gear measurement equipment is reduced. The main reasons for the tilt of axis are perpendicularity error and wobble error. Therefore, many scholars at home and abroad had done a lot of research for measurement and error separation of perpendicularity error and wobble error causing tilt of axis. The method of two self collimation electronic theodolite with reflector to measure and the optimal datum fitting processing is proposed for perpendicularity error and wobble error of revolving spindle of laser gyro inertial stabilization platform [1]. Document [2] used the reversal method based on leveling instrument to measure the perpendicularity error of revolving spindle. The model of wobble error based on geometry and the method of error separation based on curve-fitting are proposed for wobble error of revolving spindle [3]. But correction of axis tilt error caused by perpendicularity error and wobble error has not been researched deeply. The correction method of axis tilt error based on collinear measuring heads is proposed for the problem of measurement accuracy reduction due to the tilt of axis. Firstly, the physical model of axis tilt error is studied, and the corresponding mathematical model is established according to multi-body system theory. Secondly, the mathematical model of correction of axis tilt error based on collinear measuring heads is established and the tilt correction factor is solved. Finally, the method of document [4] is used to identify the parameters of algorithm.

Modeling

The mathematical model which contains six error parameters is established according to multi-body system theory [5, 6]. It prepared for the following correction of axis tilt error. Fig. 1 shows schematic diagram of tilt of axis. The black line is the ideal location of axis and the red line is the actual location of axis. Perpendicularity error and wobble error is the main cause of tilt of axis. Perpendicularity error belongs to system error and wobble error belongs to random error.

The coordinate system $o-xyz$ is established in benchmark plane. The shaft line of measured rigid body is as z-axis. x- axis and y-axis consist in benchmark plane which is perpendicular to z-axis. The intersection point of z-axis and benchmark plane is as the origin of coordinate system o. The coordinate system $o_1-x_1y_1z_1$ is established in the rotary table. The shaft line of the spindle of rotary table is as z_1-axis. The laser beam which is perpendicular to z_1-axis is as x_1-axis. The line which is perpendicular to z_1-axis and x_1-axis is as y_1-axis. The intersection point of z_1-axis and plane that contains x_1-axis and y_1-axis is as the origin of coordinate system o_1.

The topology model of tilted axis is established according to multi-body system theory. The measurement equation of weekly scanning and measuring system can be obtained as follows:

$$\begin{bmatrix} \{^p r_o\} \\ 1 \end{bmatrix} = [A_{oo_1}] \begin{bmatrix} \{^p r_{o_1}\} \\ 1 \end{bmatrix} \tag{1}$$

$[A_{oo_1}]$ −transformation matrix from coordinate system o to coordinate system o_1;

$\{^p r_o\}$,$\{^p r_{o_1}\}$ −position vector of measured point p in coordinate system o and coordinate system o_1

Position and orientation transformation matrix from coordinate system o to coordinate system o_1, as follows:

$$[A_{oo_1}] = {}^{o_1}_{o}T_{xyz}\,{}^{o_1}_{o}R_{xyz} \tag{2}$$

${}^{o_1}_{o}T_{xyz}$ −the translational transformation matrix from coordinate system o to coordinate system o_1;

${}^{o_1}_{o}R_{xyz}$ −the rotational transformation matrix from coordinate system o to coordinate system o_1

$$\begin{aligned} {}^{o_1}_{o}T_{xyz} &= Trans(x, p_x)Trans(y, p_y)Trans(z, p_z) \\ &= \begin{bmatrix} 1 & 0 & 0 & p_x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & p_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \tag{3}$$

$\{p_x, p_y, p_z\}^T$ −position vector of origin of coordinate system o_1 in coordinate system o

$$\begin{aligned} {}^{o_1}_{o}R_{xyz} &= Rot(z,\gamma)Rot(y,\beta)Rot(x,\alpha) \\ &= \begin{bmatrix} c(\gamma) & -s(\gamma) & 0 & 0 \\ s(\gamma) & c(\gamma) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} c(\beta) & 0 & s(\beta) & 0 \\ 0 & 1 & 0 & 0 \\ -s(\beta) & 0 & c(\beta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c(\alpha) & -s(\alpha) & 0 \\ 0 & s(\alpha) & c(\alpha) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \tag{4}$$

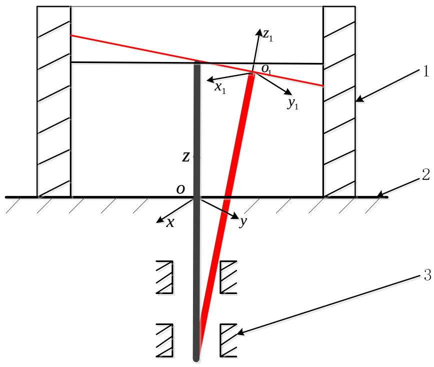
In the equation, s,c −the sine and cosine functions; α −the rotation angle of coordinate system o_1 rotate about the x-axis of coordinate system o; β −the rotation angle of coordinate system o_1 rotate about the y-axis of coordinate system o; γ −the rotation angle of coordinate system o_1 rotate about the z-axis of coordinate system o.

Position vector of measured point p in coordinate system o and in coordinate system o_1 is $\{^p r_o\}$ and $\{^p r_{o_1}\}$ respectively. Specifically expressed as follows:

$$\left\{{}^{p}r_{o}\right\}=\begin{bmatrix}x_R\\y_R\\z_R\end{bmatrix},\left\{{}^{p}r_{o_1}\right\}=\begin{bmatrix}t\\0\\0\end{bmatrix} \tag{5}$$

$\{x_R, y_R, z_R\}^T$ −position vector of measured point p in coordinate system o;
t −measurements of the displacement sensor
Measurement equations which contain six error parameters, as follows:

$$\begin{bmatrix}x_R\\y_R\\z_R\\1\end{bmatrix}=\begin{bmatrix}1&0&0&p_x\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\times\begin{bmatrix}1&0&0&0\\0&1&0&p_y\\0&0&1&0\\0&0&0&1\end{bmatrix}\times\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&p_z\\0&0&0&1\end{bmatrix}\times\begin{bmatrix}c(\gamma)&-s(\gamma)&0&0\\s(\gamma)&c(\gamma)&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\times$$
$$\begin{bmatrix}c(\beta)&0&s(\beta)&0\\0&1&0&0\\-s(\beta)&0&c(\beta)&0\\0&0&0&1\end{bmatrix}\times\begin{bmatrix}1&0&0&0\\0&c(\alpha)&-s(\alpha)&0\\0&s(\alpha)&c(\alpha)&0\\0&0&0&1\end{bmatrix}\times\begin{bmatrix}t\\0\\0\\1\end{bmatrix} \tag{6}$$



1-measured rigid body 2-benchmark plane 3-axis
Fig. 1 schematic diagram of tilt of axis

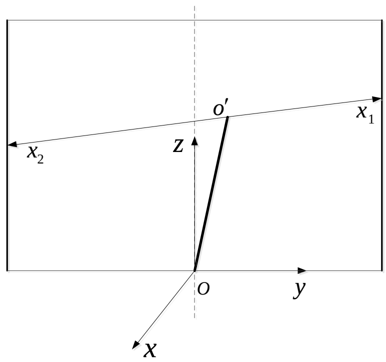


Fig. 2 schematic diagram of weekly scanning and measuring system based on collinear measuring heads

Correction Algorithm

Fig. 2 shows schematic diagram of weekly scanning and measuring system based on collinear measuring heads. The coordinate system $o'-x_1x_2y'$ is established in rigid body consisting of 2 collinear measuring heads. The shaft line of first measuring head is as x_1-axis. The shaft line of second measuring head is as x_2-axis. The intersection point of x_1-axis and x_2-axis is as the origin of coordinate system o'. The coordinate system $o-xyz$ is established in benchmark plane. The shaft line of measured rigid body is as z-axis. x-axis and y-axis consist in benchmark plane which is perpendicular to z-axis. The intersection point of z-axis and benchmark plane is as the origin of coordinate system o. When measuring i time, the pose matrix of coordinate system o' in coordinate system o, as follows:

$${}^{o'}_{o}T=\begin{bmatrix}n_x&o_x&-n_x&p_x\\n_y&o_y&-n_y&p_y\\n_z&o_z&-n_z&p_z\\0&0&0&1\end{bmatrix} \tag{7}$$

In the equation, $(n_x, n_y, n_z)^T$ -directional vector of x_1-axis in coordinate system o; $(o_x, o_y, o_z)^T$ -directional vector of y'-axis in coordinate system o; $(-n_x, -n_y, -n_z)^T$ -directional vector of x_2-axis in coordinate system o ; $(p_x, p_y, p_z)^T$ -position vector of origin of coordinate system o_1 in coordinate system o .

Parametric equations of x_1 -axis in coordinate system o , as follows:

$$\begin{cases} x = n_x t + p_x \\ y = n_y t + p_y \\ z = n_z t + p_z \end{cases} \quad (8)$$

We supposed equations of measured rigid body:

$$\begin{cases} x^2 + y^2 = R^2 \\ z = z_0 \end{cases} \quad (9)$$

R−radius of measured rigid body when measuring i time

Parameters when x_1 -axis and measured rigid body intersect can be obtained by combination of Eq. (8) and Eq. (9).

$$t_1 = \frac{-\left(n_x p_x + n_y p_y\right) + \sqrt{R^2\left(n_x^2 + n_y^2\right) - \left(n_x p_y - n_y p_x\right)^2}}{n_x^2 + n_y^2} \quad (10)$$

$t_1 - x_1$ -axis measurements of the displacement sensor

When p_x , p_y , t_1 and R in Eq. (10) is known, Eq. (10) can be seen as a function of n_x and n_y .

$$f_1\left(n_x, n_y; t_1, p_x, p_y, R\right) = 0 \quad (11)$$

Similarly, x_2 -axis measurements of the displacement sensor can be solved.

$$t_2 = \frac{\left(n_x p_x + n_y p_y\right) + \sqrt{R^2\left(n_x^2 + n_y^2\right) - \left(n_x p_y - n_y p_x\right)^2}}{n_x^2 + n_y^2} \quad (12)$$

$t_2 - x_2$ -axis measurements of the displacement sensor

Eq. (12) can be seen as a function of n_x and n_y .

$$f_2\left(n_x, n_y; t_2, p_x, p_y, R\right) = 0 \quad (13)$$

Eq. (10) and Eq. (12) are measurement equations. Eq. (11) and Eq. (13) are parameter identification equations. Parameters when measuring i time can be solved.

$$\begin{cases} n_x = n_{xi} \\ n_y = n_{yi} \end{cases} \quad (i=1,2,\ldots,N) \quad (14)$$

The tilt correction factor k_i can be solved according to Eq. (8) and Eq. (14).

$$k_i = \frac{\left(n_{xi}, n_{yi}, n_{zi}\right) \cdot \left(n_{xi}, n_{yi}, 0\right)}{\left|\left(n_{xi}, n_{yi}, n_{zi}\right)\right| \cdot \left|\left(n_{xi}, n_{yi}, 0\right)\right|} = \sqrt{n_{xi}^2 + n_{yi}^2} \quad (i=1,2,\ldots,N) \quad (15)$$

The measurements of the displacement sensor after correction, as follows:

$$t_i = k_i \frac{-\left(n_{xi} p_{xi} + n_{yi} p_{yi}\right) + \sqrt{R^2\left(n_{xi}^2 + n_{yi}^2\right) - \left(n_{xi} p_{yi} - n_{yi} p_{xi}\right)^2}}{n_{xi}^2 + n_{yi}^2} \quad (i=1,2,\ldots,N) \quad (16)$$

The axis tilt error can be corrected by Eq. (16).

Fig. 3 shows the basic flowchart of correction of axis tilt error. The basic flowchart includes the steps as follows:

(1) p_{xi} and p_{yi} (i=1,2,…,N) is solved by the method of document [4].

(2) The measured data t_i (i=1,2,…,N) is obtained by scanning a circle.

(3) The semi-minor axis b of the ellipse is obtained by fitting t_i according to least square ellipse fitting method and it is used as initial value of R.

(4) n_{xi} and n_{yi} (i=1,2,…,N) is solved by Eq.(10) and Eq.(12).

(5) The tilt correction factor k_i (i=1,2,…,N) is solved by Eq.(15).

(6) Modified t_i (i=1,2,…,N) is solved by Eq.(16).

(7) The semi-minor axis b and semi-major axis a of the ellipse is obtained by fitting modified t_i according to least square ellipse fitting method.

(8) If the cutoff condition $\Delta R=(a-b)/3<\varepsilon$ (ε is correction accuracy) is met, the correction process is over. Otherwise, the correction process continues.

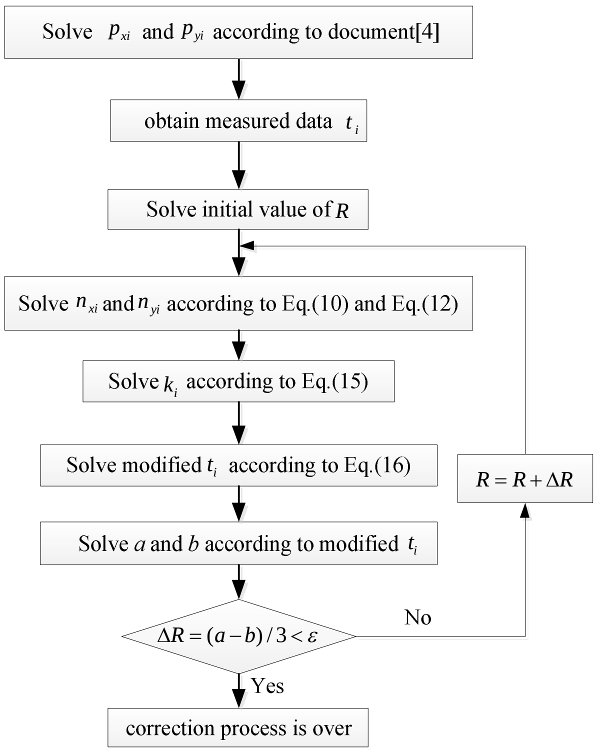


Fig. 3 Basic flowchart of correction of axis tilt error.

Simulation and Results Analysis

Materials. 40Cr steel is medium carbon steel. Its tensile strength is more than 980Mpa and hardness of HRC is 32~ 36. 40Cr steel has good quenching performance. After quenching and tempering treatment, it has good comprehensive mechanical properties. The coefficient of thermal expansion of 40Cr steel is $11.9-12.0\times10^{-6}$ at 20-200 Celsius degrees.

Methods. In order to verify that this algorithm can improve the measurement accuracy of ring gear measurement equipment, the simulation is done with MATLAB. We set radius of measured rigid body R =100mm, radial error p_x =0.03+0.1*rand(1,1)mm, and p_y =0.05+0.1*rand(1,1)mm. For reasons of space we only listed partial data. Table.1 show Partial data of before and after correction. Among them, R_1 stands for the measured value. Simulation results show that this algorithm can be very good for correction of axis tilt error when shaft inclination angle is in the range of 5^o.

Table 1. Partial data of before and after correction.

angle of Shaft inclination [^{o}]	R [mm]	Before correction		After correction	
		$R_1\,[mm]$	$(R_1 - R)/R\,[\%]$	$R_1\,[mm]$	$(R_1 - R)/R\,[\%]$
1.5	100	100.0175	0. 0175	100.0012	0.0012
2.0	100	100.0305	0. 0305	100.0024	0.0024
2.5	100	100.0476	0. 0476	100.0032	0.0032
3.0	100	100.0686	0. 0686	100.0048	0.0048
3.5	100	100.0934	0. 0934	100.0057	0.0057
4.0	100	100.1220	0. 1220	100.0064	0.0064
4.5	100	100.1545	0. 1545	100.0069	0.0069
5.0	100	100.1908	0. 1908	100.0073	0.0073

Summary

In this paper, the correction algorithm of axis tilt error based on collinear measuring heads is deduced in detail and the method of document [4] is used to identify the parameters of algorithm. Then, the tilt correction factor was solved for each location and the correction was completed. The simulation analysis is carried out before and after correction. Simulation results show that this algorithm can be very good for correction of axis tilt error when shaft inclination angle is in the range of 5^o.

The above analysis results are based on theoretical analysis, formula derivation and simulation, and the results will be verified by experiments. The ring gear measurement accuracy after correction of tilt error can meet the processing requirements of interference magnitude.

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