Reliability analysis of aerostatic instability of suspension bridge under stochastic wind loading on Bayesian theory

Wang Huasheng[1, a *]

[1]Inner Mongolia Communications Planningand Design Institute Co., Ltd, Huhhot, Inner Mongolia, China

[a]tongjidfh@163.com

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Abstract. Based on the structural reliability theory, the extreme wind speed forecast method was used to establish the reliability model for analyzing aerostatic instability of long-span suspension bridge. The limit state equation of reliability analysis model is a function of conversion factor, critical wind speed of the aerostatic instability, gust factor and extreme wind speed at the bridge site. In this paper, JC method based on the first order second moment reliability theory was used to calculate reliability indices of the aerostatic instability of the suspension bridge. The results indicate that probability assessment and reliability analysis of aerostatic instability of suspension bridge under stochastic wind loading based on Bayesian theory are more precise and practical, which is more valuable in engineering application.

Introduction

With the ever-growing span-length of bridges, aerostatic instability is becoming a matter of significant design consideration for long-span suspension bridges again. Aerostatic instability takes place when bridges are exposed to wind speeds beyond a certain critical value that could be got by wind tunnel experiments and theoretical calculations with tests-obtained parameters. Nevertheless, because most of these parameters from the prediction are actually indefinite variables or/and empirically assumed values from researchers due to lack of complete theory, it is more rational to conduct an analysis of probabilistic reliability to determine the probability of the bridge failure resulting from aerostatic instability for a given return period rather than stating a single critical wind speed [1].

In this paper, a reliability analysis model was established by taking account of four random and mutually independent variables, and safety margin, also a random variable, which only depends on stochastic nature of these variables [1]. The forecast method of extreme wind speed is introduced in the present paper, which aims to obtain the true value of wind speed at the bridge site because the samples of wind speed are always not enough. Based on the wind speed of Bayesian estimates, the reliability indices of aerostatic instability for four long-span cable-stayed bridges were calculated and compared to the results of maximum likelihood estimates.

The Gumbel Model

The extreme value of wind speed is for Gumbel (extreme value of type-I) distribution [2, 3] in reliability analysis of aerostatic instability. For the Gumbel model, the probability distribution function (PDF) and the cumulative distribution function (CDF) of the wind speed T are given respectively, by

$$f(t)=\frac{1}{\sigma}e^{(-\frac{t-\mu}{\sigma}-e^{-\frac{t-\mu}{\sigma}})}, F(t)=\exp\left\{-\exp[-(\frac{t-\mu}{\sigma})]\right\}, -\infty<t, \mu<\infty, \sigma>0 \quad (1)$$

where, σ and μ are the scale parameters and location [4], respectively.

The wind speed forecast value which recurrence is T years (assurance rate is 1-$\frac{1}{T}$) can be obtained by the logarithmic form of function (2)

$$t_{\frac{1}{T}} = \mu - \sigma \left\{ \ln[-\ln(1-\frac{1}{T})] \right\}. \tag{2}$$

Bayesian Theory

In the Bayesian method, we regard μ and σ behaving as random variables with a joint PDF $\pi(\mu,\sigma)$ [4]. For extreme value of Gumbel distribution, the Jeffrey's non-informative prior [5, 6] is given by

$$\pi(\mu,\sigma) = \frac{1}{\sigma^2}. \tag{3}$$

To evaluate the expression above to acquire approximate Bayesian estimates of $t_{\frac{1}{T}}$, we would use Lindley's approximation method [6-8].

Lindley Approximation

Let

$$I = \frac{\int u(\theta)v(\theta)e^{L(\theta)}d\theta}{\int v(\theta)e^{L(\theta)}d\theta}. \tag{4}$$

where, $\theta = (\theta_1, \theta_2, ..., \theta_k)$, a vector of parameters. Also, let L= Log (likelihood function). Note that Iis the posterior expectation of $u(\theta)$ given the failure data $v(\theta)$. Denote by

$$u_1 = \frac{\partial u}{\partial \theta_1}, u_2 = \frac{\partial u}{\partial \theta_2}, u_{11} = \frac{\partial^2 u}{\partial \theta_1^2}, u_{22} = \frac{\partial^2 u}{\partial \theta_2^2}. \tag{5}$$

$$p = \pi(\theta_1, \theta_2), p_1 = \frac{\partial p}{\partial \theta_1}, p_2 = \frac{\partial p}{\partial \theta_2}. \tag{6}$$

$$L_{20} = \frac{\partial^2 L}{\partial \theta_1^2}, L_{02} = \frac{\partial^2 L}{\partial \theta_2^2}, L_{30} = \frac{\partial^3 L}{\partial \theta_1^3}, L_{03} = \frac{\partial^3 L}{\partial \theta_2^3}. \tag{7}$$

$$\sigma_{11} = (-L_{20})^{-1}, \sigma_{22} = (-L_{02})^{-1}. \tag{8}$$

and

$$E(u(\theta)|\vec{t}) = u(\overline{\theta_1}, \overline{\theta_2}) + \frac{1}{2}(u_{11}\sigma_{11} + u_{22}\sigma_{22}) + p_1 u_1 \sigma_{11} + p_2 u_2 \sigma_{22}$$
$$+ \frac{1}{2}(L_{30}u_1\sigma_{11}^2 + L_{03}u_2\sigma_{22}^2 + L_{21}u_2\sigma_{11}\sigma_{22} + L_{12}u_1\sigma_{22}\sigma_{11}). \tag{9}$$

where, $\overline{\theta_1}$ and $\overline{\theta_2}$ are the classical MLEs for θ_1 and θ_2, respectively [6].

Forecast Model of Extreme Value Wind Speed

Thus, a Bayesian approximate estimate for $\overline{t_B}$ is given by

$$\overline{t_B} = \overline{\mu} + 4.6\overline{\sigma} + p_2 u_2 \sigma_{22} + \frac{1}{2}(L_{30}\sigma_{11}^2 + L_{03}u_2\sigma_{22}^2 + L_{21}u_2\sigma_{11}\sigma_{22} + L_{12}u_1\sigma_{22}\sigma_{11}). \tag{10}$$

where, $\bar{\mu}$ and $\bar{\sigma}$ are the classical MLEs for μ and σ, respectively.

Numerical Analysis

In this section, a numerical study is presented to compare the maximum likelihood and Bayes estimates for determining the wind speed model subject to specified reliability. The numerical simulation was carried out in the following manner [6]:

Because of the size of the simulation, some of all the numerical results are listed in table 1 under 99% reliability. In Table 1, the size of the prior sample used to calculate the Bayes estimate μ_B was presented. $t_{\frac{1}{T}}$ is the true wind speed, $\bar{t}_{\frac{1}{T}}$ is the ML estimates of wind speed, $\bar{t}_B$ is the Bayes estimation value of wind speed, $\left|t_{\frac{1}{T}} - \bar{t}_{\frac{1}{T}}\right| / t_{\frac{1}{T}}$ and $\left|t_{\frac{1}{T}} - \bar{t}_B\right| / t_{\frac{1}{T}}$ represent the absolute value of the difference between the true wind speed, and maximum likelihood and Bayes wind speed estimates, respectively.

Table 1. Numerical Study of the Gumbel wind speed.

μ_B	$\bar{\mu}$	σ_B	$\bar{\sigma}$	t_α	$\bar{t}_\alpha$	$\bar{t}_B$	$\left\|t_{\frac{1}{T}} - \bar{t}_{\frac{1}{T}}\right\| / t_{\frac{1}{T}}$	$\left\|t_{\frac{1}{T}} - \bar{t}_B\right\| / t_{\frac{1}{T}}$
14.9081	15.1877	2	1.9201	24.2	24.0202	24.3331	0.74%	0.55%

In Table 1, the Bayes estimate is closer to the true wind speed than the maximum likelihood.

Engineering Application

Based on aerostatic instability model from [9], wind speed is obtained by the Bayesian method, and the reliability indices of Xihoumen Bridgeis was calculated by the JC method.

Limit State Function. A limit state function can be represented by

$$G = C_w U_f - G_s U_b . \tag{11}$$

in which C_w is the wind conversion factor, and U_f is the basic aerostatic instability speed, and G_s is the gust speed factor, and U_b is the basic wind speed at the bridge deck location.

The statistics of C_w, U_f, G_s and U_b of Xihoumen Bridge are shown in Table 2 [1,10].

Table 2. Random variables and their statistical properties of Xihoumen Bridge - L=1650m.

Random variables	Mean	Coefficientsof variation	Distribution type
C_w	1	0.1	Normal
U_f	194 (empirical formula) 115 (finite element) 95 (wind tunnel test)	0.1	Lognormal
G_s	1.2	0.12	Normal
U_b	23.17 (Bayes estimates) 24.23 (ML estimates)	0.3	Extreme type I

Reliability Analysis of Four Cable-Stayed Bridges. On the basis of the JC approach, the reliability index β are numerically listed in Table 3 and compared to the traditional safety factors defined as $K = [U_{cr}] / \mu_G U_d$ [11], which listed in Table 4.

Table 3. Reliability indices of suspension bridge.

Bridge name	empirical formula		Finite element		Wind tunnel test	
	Bayes	ML	Bayes	ML	Bayes	ML
Xihoumen Bridge	7.38	7.51	6.22	6.43	5.11	5.36

Table 4. Safety factors of suspension bridge.

Bridge name	empirical formula		Finite element		Wind tunnel test	
	Bayes	ML	Bayes	ML	Bayes	ML
Xihoumen Bridge	2.93	3.06	1.74	1.82	1.44	1.51

It can be concluded from Table 3 and 4 that the reliability indices of Bayes estimates are smaller than those of ML estimates for Xihoumen Bridge, which are all same as the safety factors of aerostatic stability, which indicates that the results of ML estimation are unsafe while the Bayes estimation could give the rational calculation results.

Conclusion

In this paper, a forecast method based on Bayesian theory of wind speed was introduced, upon which the reliability analysis model was established. The reliability indices were calculated using the JC approach based on the empirical formula, finite element analysis and wind tunnel test, in which the results of Bayes estimates and ML estimates were compared and indicate that probability assessment and reliability analysis of aerostatic instability of suspension bridge under stochastic wind loading based on Bayesian theory are more precise and scientific, which is significantly valuable in the application of practical engineering projects.

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