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Reliability Prediction for CNC Machine of Spindle System

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Abstract. Grey prediction of CNC machine spindle system failure is modeled with GM(1,1) grey prediction as data accumulation weaken randomness and strengthen regularity. The model result provides preventative maintenance of CNC machine spindle system with theoretical foundation. The example analysis shows that grey prediction model can exactly predict time of failure quickly. It is effective to be used in reliability prediction of CNC machine spindle system. The grey prediction model also can predict failure time of CNC machine and the other subsystem.

Introduction

Spindle system is one of the key CNC machine subsystems. It is related to CNC machines' capability and the quality of machined workpieces. Its reliability is very important to CNC machine.

Spindle system of CNC machine includes spindle and spindle components (bearing, spindle encoder, spindle motor etc). In past surveys, its dynamic behavior can be got by FEA, the establishment of the dynamic model and transient response of its parts. The data structure of spindle parts provide the basis for design and assemble of spindle system. The reliability of spindle system can be analyzed by FTA [1-3]. There is less report of spindle failure time prediction. The prediction time of spindle system is of great value to theoretical foundation of spindle system preventative maintenance and CNC machine reliability improving.

The paper has a certain type of CNC machine spindle system fault information as the known information. Subsequent failure time of spindle system can be predicted by grey prediction theory.

Data Acquisition and Processing of Spindle System Failure

Data Acquisition of Spindle System Failure. The failure data is from a certain CNC machine that is tracked on the scene. It includes reception time, starting downtime, termination of the downtime and maintenance time [4]. The fault message of measured CNC machine can be got by failure data record. The paper researches the example that is spindle failure data of 15 CNC machines as Table 1.

Table 1. Failure time of spindle system.

No.	Failure time/h		No.	Failure time/h	
01	5121.20		09	3342.78	
02	2041.89	2102.27	10	3787.38	
03	823.34	2695.08	11	2552.37	
04	4923.60		12	2552.37	2865.24
05	2239.50		13	378.74	395.21
06	1267.95		14	131.74	1020.95
07	3589.78		15	1053.88	
08	675.14	1805.87			

Generating Process of Failure Data. Subsequent failure time can be got by generating process of the collected data. The method of generating process includes accumulation generation method, B-b generated method and mean value generating method [5-7].

Accumulation generation method. Original failure data column as follows:

$$X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots\ldots, x^{(0)}(n)\} \tag{1}$$

Formula: superscript (0) is original failure data column, n is the number of values.
Accumulation generation data column as follows:

$$X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \ldots\ldots, x^{(1)}(n)\} \tag{2}$$

Formula:

$$x^{(1)}(t) = \sum_{i=1}^{t} x^{(0)}(i) = x^{(1)}(t-1) + x^{(0)}(t), t = 1, 2, \ldots\ldots, n \tag{3}$$

Superscript (1) expresses once accumulation generation data.

B-b generated method. B-b generated method is that front and back data of original failure data subtract. B-b method is inverse operation of accumulation method. The accumulated data can be restored by B-b method. The B-b generated formula as follows:

$$X^{(1)} = x^{(0)}(t) - x^{(0)}(t-1), t = 1, 2, \ldots\ldots, n \tag{4}$$

In the formula: $x^{(0)}(0) = 0$

Mean value generating method. The new data can be generated by mean value of adjacent data. The generated formula as follows:

$$Z(t) = \frac{1}{2}[x^{(0)}(t) + x^{(0)}(t-1)], t = 2, 3, \ldots\ldots, n \tag{5}$$

Prediction of Spindle System Reliability

The prediction is to predict subsequent failure time of spindle system while building model by grey theory. The model can predict the subsequent failure.

The grey model is used to process the data which accord with smooth discrete function and the generated data possesses index-to-be law. Therefore, the data should be checked whether according with smooth discrete and index-to-be law before building grey prediction model. While the data accords with the law, the grey prediction model can be builded.

Checking Smooth Discrete and Index-to-be Law of the Data.

Checking smooth discrete. The formula of checking smooth discrete as follows:

$$\rho(t) = \frac{x^{(0)}(t)}{x^{(1)}(t-1)} \tag{6}$$

Checking index-to-be law. The formula of checking index-to-be law as follows:

$$\sigma^{(1)}(t) = \frac{x^{(1)}(t)}{x^{(1)}(t-1)} \tag{7}$$

The failure time data of table 1 is checked whether according with smooth discrete. The result as follows:

ρ(t) =(2.87,0.77,0.75,0.52,0.42,0.31,0.28,0.31,0.27,0.22,0.19,0.18,0.15,0.14,0.13,0.14,0.13,0.12, 0.14,0.13), The $\rho(4) = 0.52$ $\rho(5) = 0.42 < 0.5$ ……….. $\rho(21) = 0.13 < 0.5$, While $t > 4$, the data accords with smooth discrete law.

The failure time data of Table 1 is checked whether according with index-to-be law. The result as follows:

$\sigma^{(1)}(t) =$ (3.87,1.77,1.75,1.52,1.42,1.31,1.28,1.31,1.27,1.22,1.19,1.18,1.15,1.14,1.13,1.14,1.13,1.12, 1.14,1.13), $\sigma^{(1)}(4) = 1.52$ $\sigma^{(1)}(5) = 1.42 \in [1,1.5]$,……, $\sigma^{(1)}(21) = 1.13 \in [1,1.5]$, While $t > 4$, the data accords with index-to-be law.

Building Grey Prediction Model. The GM (1,1) is usually applied to grey prediction model. Its construction is simple and convenient to build but its result is exact. The model (1,1) presents differential equation of first order and a variable.

Building the basic form of GM (1,1). The failure time column $X^{(0)}$ has n observed value, as $X^{(0)}=\{x^{(0)}(1),x^{(0)}(2),......,x^{(0)}(n)\}$.

The new accumulation generation data column as $X^{(1)}=\{x^{(1)}(1),x^{(1)}(2),......,x^{(1)}(n)\}$.

$X^{(1)}$ is continuous function of t. The building differential equation as follows:

$$\frac{dx^{(1)}}{dt}+ax^{(1)}=b \tag{8}$$

In the formula: a is progress grey number, b is endogeny command grey number.

The basic form of grey prediction model can be got by the formula (8)

$$x^{(0)}(t)+az^{(1)}(t)=b \tag{9}$$

The parameter estimation of model.

$$Y=\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ .... \\ .... \\ x^{(0)}(n) \end{bmatrix}, \quad B=\begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ ...... & ...... \\ ...... & ...... \\ -z^{(1)}(n) & 1 \end{bmatrix}$$

To predict the progress grey number by least square method as follows:

$$\hat{a}=(B^TB)^{-1}B^TY \tag{10}$$

Following

$$Y=B\hat{a} \tag{11}$$

Final result as follows:

$$\begin{cases} a=\dfrac{\frac{1}{n-1}\sum\limits_{t=2}^{n}x^{(0)}(t)\sum\limits_{t=2}^{n}z^{(1)}(t)-\sum\limits_{t=2}^{n}x^{(0)}(t)z^{(1)}(t)}{\sum\limits_{t=2}^{n}(z^{(1)}(t))^2-\frac{1}{n-1}(\sum\limits_{t=2}^{n}z^{(1)})^2} \\ b=\dfrac{1}{n-1}[\sum\limits_{t=2}^{n}x^{(0)}(t)+a\sum\limits_{t=2}^{n}z^{(1)}(t)] \end{cases} \tag{12}$$

Put the predicted a, b into prediction model, and the grey prediction model GM (1,1) as follows:

$$\hat{X}^{(1)}(t+1)=(X^{(0)}(1)-\frac{\hat{b}}{\hat{a}})e^{-\hat{a}t}+\frac{\hat{b}}{\hat{a}} \tag{13}$$

So prediction model of the original column as follows:

$$\hat{X}^{(0)}(t+1)=\hat{X}^{(1)}(t+1)-\hat{X}^{(1)}(t)=(X^{(0)}(1)-\frac{\hat{b}}{\hat{a}})\times(e^{-\hat{a}t}-e^{-\hat{a}(t-1)}) \tag{14}$$

The final prediction model of spindle system as follows:

$$\hat{x}^{(1)}(t)=(x^{(0)}(1)-\frac{b}{a})\times e^{-at}+\frac{b}{a}=(x^{(0)}+6805.37)\times e^{0.1034t}-6805.37 \tag{15}$$

$X^{(0)}$ can be restored by the formula (4) and (5).

The check of prediction model. The building model should be checked to ensure it is right or wrong. The checking method includes residual examination, correlation test and the after test rule. The building prediction model should be checked by the above three methods to ensure it reasonable.

Residual examination. Residual examination is to calculate the absolute error and relative error between the original data column $X^{(0)}(t)$ and prediction, column $\hat{X}^{(0)}(t)$. The formulas are as follows:

The absolute error:

$$\Delta^{(0)}(t) = \left| X^{(0)}(t) - \hat{X}^{(0)}(t) \right|, t = 1, 2, ..., n \tag{16}$$

The relative error:

$$\Phi(t) = \frac{\Delta^{(0)}(t)}{X^{(0)}(t)} \times 100\%, t = 1, 2, ..., n \tag{17}$$

The model precision is higher while the absolute error and relative error is smaller according to experience.

Correlation test. The correlation test is to predict correlation between the prediction column $\hat{X}^{(0)}(t)$ and the original column $X^{(0)}(t)$ .The computational process as follows:

① Calculating correlation coefficient

Supposing the prediction failure time column as follows:

$\hat{X}^{(0)}(t) = \{\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), ...\hat{x}^{(0)}(t)...., \hat{x}^{(0)}(n)\}$ The original failure time column as follows:

$X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), ...x^{(0)}(t)..., x^{(0)}(n)\}$

The definition of correlation coefficient as follows:

$$\eta(j) = \frac{\min_j\{\Delta(j)\} + \rho \max_j\{\Delta(j)\}}{\Delta(j) + \rho \max_j\{\Delta(j)\}} \tag{18}$$

In the formula: (a) $\min_j\{\Delta(j)\}$ is minimum value of absolute error, $j = 1, 2, ..., n$, (b) $\max_j\{\Delta(j)\}$ is maximum value of absolute error, $j = 1, 2, ..., n$, (c) ρ is resolution ratio, $0 < \rho < 1$,usually $\rho = 0.5$.

② Calculating correlation degree

The formula of correlation degree as follows:

$$r = \frac{1}{n} \sum_{j=1}^{n} \eta(j) \tag{19}$$

The after test rule.

①The after test rule should firstly calculate the standard deviation of the original data column. The formula as follows:

$$S_1 = \sqrt{\frac{1}{n-1} \sum_{t=1}^{n} (X^{(0)}(t) - \bar{X}^{(0)})^2} \tag{20}$$

In the formula:

$$\bar{X}^{(0)} = \frac{1}{n} \sum_{t=1}^{n} X^{(0)}(t) \tag{21}$$

② Calculating the standard deviation of absolute error column. The formula as follows:

$$S_2 = \sqrt{\frac{1}{n-1} \sum_{t=1}^{n} (\Delta^{(0)}(t) - \bar{\Delta}^{(0)})^2} \tag{22}$$

In the formula:

$$\bar{\Delta}^{(0)} = \frac{1}{n}\sum_{t=1}^{n}\Delta^{(0)}(t) \quad (23)$$

③ Calculating the ratio of standard deviations. The formula as follows：

$$C = S_2 / S_1 \quad (24)$$

④ Estimating the little probability of error:

Estimating the little probability of error according to $\left|\Delta^{(0)}(t) - \bar{\Delta}^{(0)}\right| < 0.6745S_1$ as follows:

$$P = P\{\left|\Delta^{(0)}(t) - \bar{\Delta}^{(0)}\right| < 0.645S_1\} \quad (25)$$

Accuracy grade referring to the Table 2.

The result can be got as Table 3 while checking the prediction model of spindle system by the above tests.

The relative error is less than 0.6 according to Table 3. The degree of association: $r = 0.5951 > 0.55$. The ratio of standard deviations: 0.096. The model is verified by the after test rule.

Table 2. Accuracy grade.

P	C	accuracy grade
>0.95	<0.35	First grade: good
>0.80	<0.50	Second grade: qualified
>0.70	<0.65	Third grade: Barely qualified
≤ 0.70	≥ 0.65	Fourth grade: unqualified

The prediction model of original data column is got preliminarily as follows:

$$x^{(0)}(t) = (x^{(0)}(1) - \frac{b}{a}) \times (e^{-at} - e^{-a(t-1)}), t = 1, 2, ..., n \quad (26)$$

So $x^{(0)}(t) = 681.46 \times e^{0.1034t}, t = 1, 2, ..., n$.

Table 3. The check result of prediction model.

Original data	Prediction data	Relative error	Correlation coefficient	Original data	Prediction data	Relative error	Correlation coefficient
131.74	131.74	0.0000	1.0000	2239.5	2123.902	0.0516	0.7190
378.74	604.5	0.5961	0.5672	2552.37	2355.196	0.0773	0.6001
395.21	615.68	0.5579	0.5730	2552.37	2611.678	0.0232	0.8330
675.14	1078.22	0.5970	0.4233	2695.08	2896.091	0.0746	0.5954
823.34	1253.849	0.5229	0.4073	2865.24	3211.477	0.1208	0.4608
1020.95	1142.297	0.1189	0.7091	3342.78	3561.209	0.0653	0.5753
1053.88	1266.694	0.2019	0.5816	3589.78	3949.026	0.1001	0.4516
1267.95	1404.637	0.1078	0.6840	3787.38	4379.077	0.1562	0.3333
1805.87	1557.603	0.1375	0.5437	4923.6	4855.961	0.0137	0.8139
2041.89	1727.227	0.1541	0.4846	5121.2	5384.778	0.0515	0.5288
2102.27	1915.323	0.0889	0.6128				

Building the final prediction model. In order to making the prediction model more reasonable and more correct, the final model can be got by weighting factor, method based on the original model.

The method steps as follows:

1) The data is divided into three groups by the whole data and deleting the last one and two data.

2) The models of other two group data can be got according to the method of building prediction model of whole data.

3) To predict the last one data by the three prediction models.

4) To give the parameters of three prediction models the empowerment according to the difference between the prediction and actual value. The principle is that the weight is big while the difference is small and the weight is small while the difference is big. Weight additive is equal to 1.

5) The addition of product that parameter multiply its weight is the parameter of final model. The final models as Table 4.

Table 4. Final prediction model.

Data column	Prediction model
Complete data	681.46*exp(0.103*t)
First group data (deleting the last one)	636*exp(0.1068*t)
Second group data (deleting the last two)	625*exp(0.1067*t)

The 21st failure time of spindle system is 5343h, 5384h, 5280h according to the three models in Table 4. Actually the 21st failure time of spindle system is 5121.2h. So the difference between the second prediction failure time and the real failure time is the minimum. The difference between prediction of complete data and the real failure time is the second minimum. The difference between prediction of first group data and the real failure time is the maximum.
The weights of three model parameters are 0.3, 0.2 and 0.5 according to the above analysis and weight principle. The final model as follows:

$$y = 644 \times \exp(0.1057t) \tag{27}$$

The 22st failure time of spindle system is about 5927.7h according to the prediction model as there is 21 failure time data in the assessment period.

Conclusions

(1) The grey prediction model of spindle system can predict subsequent failure time quickly and exactly. The model result provides preventative maintenance of CNC machine spindle system with theoretical foundation.
(2) The process proves that the feasibility method can be applied into the complete CNC machine and subsystem. The method is universal.
(3) The complete process does not need any assumption. The weight makes the result of prediction model more correct.

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