

Influence of Dynamic Disturbance on Evolvement and Stability of Rock Mechanics System

Y. C. Shao^{1, a}, P. Shao^{1,2, b}

¹ School of Mechanics and Civil Engineering, China University of Mining and Technology, Xuzhou, China

² State Key Laboratory for Geomechanics and Deep Underground Engineering, China University of Mining and Technology, Xuzhou, China

^a674447226@ qq.com, ^bspsyc@163.com

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Abstract. According to the self-organizing characteristic of rock mechanics system during evolution, synergetics was applied to investigate the evolution process by taking dynamic disturbance into account, and a generalized dynamical equation was established. Based on analysis of phase transition theory and damage thermodynamics theory, the major parameters which affect evolution of rock mechanics system, including order parameter, control parameter and outer field were discussed and their expressions were given. The influence of dynamic disturbance on evolution and stability of rock system was analyzed using potential function method. The critical instability point and the relationship between control parameter, outer field and disturbance intensity were discussed. Results indicated that dynamic disturbance could not only decrease the stability but also induce the instability of a rock mechanics system under certain conditions. The critical instability point is together determined by disturbance intensity, differential stress and differential damage.

Introduction

It is generally believed that the rock mechanics system is a mechanical system, which is composed of the rock structure, the surrounding rock, the support, the engineering geology and the natural environment. The stability of this system refers to the ability of the rock mass to maintain the original balance state under external force. Recent studies showed that the evolution of rock mechanics system has obvious self-organizing characters: i) it is an open system; ii) during evolution, the system is always accompanied by the conversion of energy and information; iii) the deformation and failure of system are the result of the interaction of various nonlinear mechanism far from the equilibrium condition. For this kind of system which has the function of self-organization, it is suitable for using self-organization theory to research. For example, based on the assumption that there exists a proportional relation between the numbers of microseismic, catastrophe theory was applied to analyzed rock burst [1]. Tan et al. [2] established an order parameter equation to describe the evolution process of roof movement using synergetics method, and put out that the prediction of mechanical action of roof movement at critical state is unreliable. Tiampo et al. [3] described a new method based on phase dynamics techniques to analyze and forecast the space-time patterns of activity in earthquake fault systems. Liu et al. [4] established an energy model using catastrophe theory to obtain the instability criteria of system and studied the interaction between backfill and rock mass. Bao et al. [5] established a catastrophe model of plastic strain energy based on strength reduction factor. The model was applied to the sliding stability analysis of a concrete gravity dam. Moreover, many works have been done by other researchers [6-7].

Evolution process of rock mechanics system is often associated with various random dynamic loads. These dynamic disturbances usually result in catastrophic consequences, especially in mining engineering, water resources and hydropower engineering, protective engineering, etc. Therefore, it is of practical significance to reveal the effect of dynamic disturbances on the evolution mechanism and stability of the rock mechanics system under nonlinear condition.

Evolutionary Process of Rock Mechanics System

General Dynamic Equations. Assuming that $q_1(u,t), q_2(u,t), \dots, q_n(u,t)$ are space-time variables representing the states of rock mechanics system, then the system state with vector form is $\mathbf{q} = \{q_1, q_2, \dots, q_n\}$. The change rate of variables is

$$\begin{cases} \dot{\mathbf{q}}_1 = \mathbf{a}_1 q_1 + F_1(q_1, q_2, \dots, q_n) \\ \mathbf{L} \\ \dot{\mathbf{q}}_i = \mathbf{a}_i q_i + F_i(q_1, q_2, \dots, q_n) \\ \mathbf{L} \\ \dot{\mathbf{q}}_n = \mathbf{a}_n q_n + F_n(q_1, q_2, \dots, q_n) \end{cases} \quad (1)$$

In accordance with synergetics, the system state variables are separated into fast and slow variables. Fast variables are large in number but attenuating rapidly, so they cannot play a leading role in the evolution of the system. Conversely, a system usually contains only a single slow variable, which determines the evolution trend. The slow variable is also called order parameter. To obtain the order parameter, adiabatic elimination technique is the main method in synergetic theory. If q_1 is the order parameter in a rock mechanics system, we can let

$$\dot{\mathbf{q}}_2 = \mathbf{L} = \dot{\mathbf{q}}_i = \mathbf{L} = \dot{\mathbf{q}}_n = 0 \quad (2)$$

Substituting Eq. 2 into Eq. 1, we have

$$\begin{cases} q_2 = H_2(q_1) \\ \mathbf{L} \\ q_i = H_i(q_1) \\ \mathbf{L} \\ q_n = H_n(q_1) \end{cases} \quad (3)$$

By substituting Eq. 3 in Eq. 1, we therefore obtain

$$\dot{\mathbf{q}}_1 = \mathbf{a} q_1 + g_1(q_1, q_2, \dots, q_n) = \mathbf{a} q_1 + G(q_1) \quad (4)$$

where \mathbf{a} is a control parameter and $G(q_1)$ is the nonlinear function of q_1 .

For rock mechanics system, $G(q_1)$ is defined by [8]

$$G(q_1) = -fy(q_1) \quad (5)$$

where f is a coefficient and the function of $y(q_1)$ can be written as

$$y = -ky + q_1^2 \quad (6)$$

herein k is a constant. For Eq. 5, the integrated form of rate is

$$y(t) = \int_{\infty}^{-y(t-t)q_1^2} dt \quad (7)$$

Using integration by parts, we have

$$y(t) = \frac{1}{k} q_1^2(t) - \frac{1}{k} \int_{\infty}^{-y(t-t)} 2(q_1 \dot{\mathbf{q}}_1) dt \quad (8)$$

Due to the variation of q_1 being slow, we get $y(t) = \frac{1}{k} q_1^2(t)$ by ignoring the integral part in Eq. 8.

Thus, Eq. 4 is rewritten as

$$\dot{\mathbf{q}}_1 = \mathbf{a} q_1 - b q_1^2 \quad (9)$$

where $b=f/k$. When dynamic disturbance outside rock mechanics system is considered, Eq. 9 can be expressed by using generalized Langevin equation as

$$\dot{\phi}_1 = aq_1 - bq_1^2 + q_1h(t) \quad (10)$$

where $h(t)$ denotes the fluctuating force of dynamic disturbance with the properties given by

$$\langle h(t) \rangle = 0, \langle h(t)h(t') \rangle = 2D\delta(t-t'), \quad (11)$$

herein δ is a correlation function and D is fluctuation intensity.

Variables analysis. In Eq.(11), q_1 , a and b represent the order parameter, outfield parameter and control parameter, respectively. To determine their meaning, Shao et al. [12] gave a primary discussion based on phase transition theory and thermodynamics. The obtained functional form of outfield parameter is $a=f(s_1-s_3)$ and of control parameter is $b=f(w)$. s_1-s_3 is the differential stress, s_1 and s_3 are the maximum and minimum principal stress in rock mass, is rock damage.

According to the fundamental principle of damage thermodynamics and referring to the model suggested by Rundle[9-11], the meaning of order parameter as well as the expressions about a and b were furtherly explored. We considere that the order parameter q_1 is actually the plastic strain x of rock mass and the specific expressions about a and b associated with Eq.9 and Eq. 10 could be expressed as

$$a=1-(s_1-s_3)/(s_1-s_3)_c, \quad (12)$$

$$b=w_c-w, \quad (13)$$

herein $(s_1-s_3)_c$ is critical differential stress and w_c is critical damage. Furthermore, we call b the differential damage, which represents the approaching degree of rock damage to its critical value.

Then, Eq.10 could be rewritten as

$$\dot{x} = ax - bx^2 + xh(t). \quad (14)$$

Impact Analysis of Dynamic Disturbance

Potential function. In self-organization theory, the potential function of a system under the influence of internal fluctuation is usually called deterministic potential function. On the contrary, if a system suffering the effect of external fluctuation, the potential function is named random potential function. The most probable state of a system can be described through potential function analysis. For the influence of dynamic disturbance on the stability of rock mechanics system, random potential function analysis is suitable. Let $V(x)$ expressing the deterministic potential function as well as $U(x)$ denoting random potential function, the $V(x)$ of rock mechanics system is

$$V(x) = -\frac{1}{2}ax^2 + \frac{1}{3}bx^3. \quad (15)$$

According to the relationship between deterministic potential function and random potential function, $U(x)$ can be represented as

$$U(x) = \int \frac{V'(x)}{f^2(x)} dx - D \ln|f(x)|, \quad (16)$$

where $f(x)=ax-bx^2$. Substituting Eq. 15 in Eq.16, we have

$$U(x) = bx - a \ln x + D \ln x. \quad (17)$$

Thus, the influence of dynamic disturbance on system evolution and stability can be analyzed by potential function equation shown as Eq.17.

Additional, the critical point of instability may be calculated using

$$x_c = (a - D)b \tag{18}$$

Influence of dynamic disturbance intensity. Fig.1 shows the variation of random potential function curves of a rock mechanics system when a and b were set as a fixed value ($a=b=0.1$). As $D=0.05, 0.07$ and 0.09 , the curves have obvious valley bottoms, but the depth of bottoms decrease along with the increment of D . This illustrates that the system is stable, but the stability tends to decrease with the increasing of disturbance intensity. As $D=0.11$ and 0.13 , the valley bottom disappeared. This indicates that disturbance intensity has already exceeded the tolerance of system and failure will happen. Thus it can be seen that, under the same differential stress and damage conditions, the impact of dynamic disturbance on rock mechanics relates to disturbance intensity.

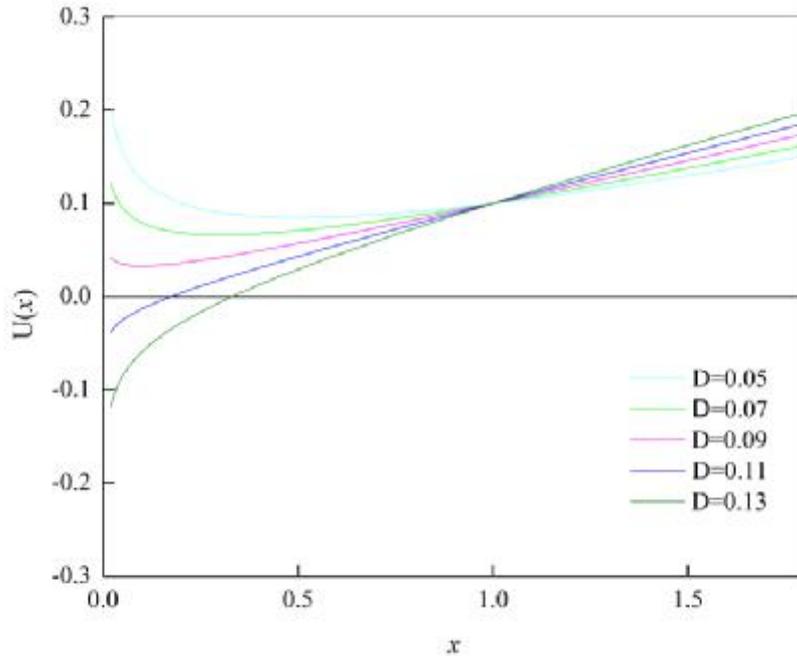


Fig.1 Influence of disturbance intensity ($a=b=0.1$)

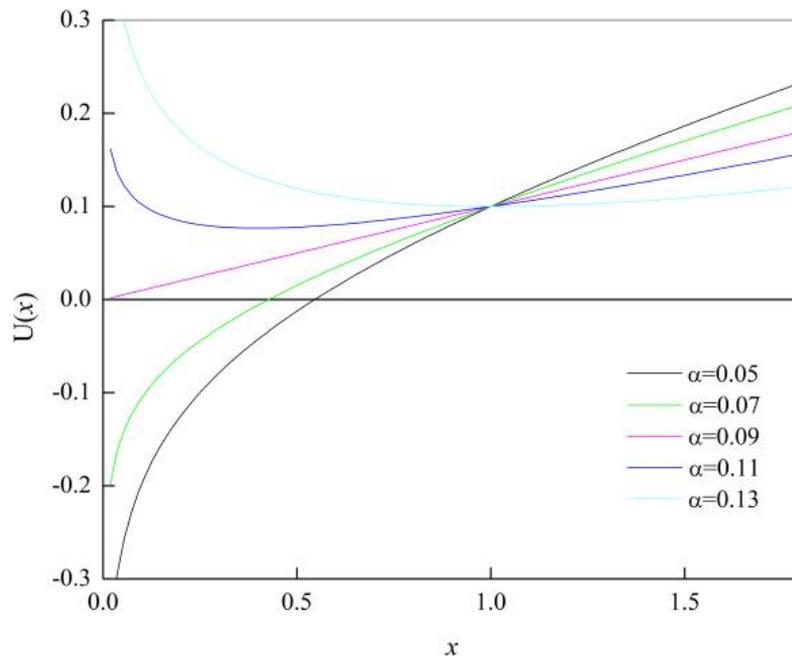


Fig.2 Influence of differential stress ($b=D=0.1$)

Influence of differential stress. Fig.2 shows the variation of random potential function curves of a

rock mechanics system when b and D were set as a fixed value ($b=D=0.1$). As $a=0.05$ and 0.07 , the curves have obvious valley bottoms, but the depth of bottoms decrease along with the increment of a . This illustrates that the system is stable, but the stability tend to decrease with the increasing of differential stress. As $a=0.09$, 0.11 and 0.13 , the valley bottom disappeared. This indicates that differential stress is too large to ensure the system to resist the effect of dynamic disturbance.

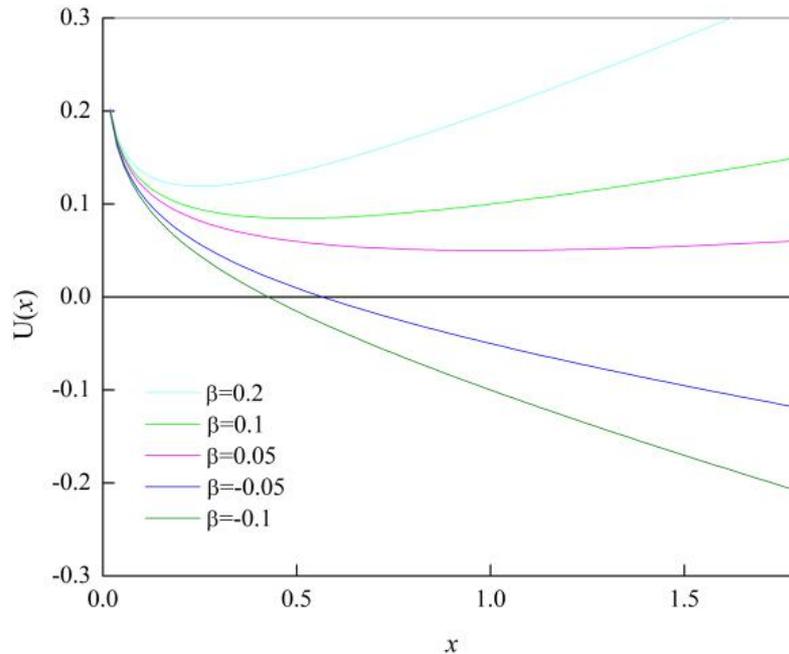


Fig.3 Influence of differential damage

Influence of differential damage. Fig.4 shows the variation of random potential function curves of a rock mechanics system when a and D were set as a fixed value ($a=0.1$, $D=0.05$). As $b=0.2$, 0.1 and 0.05 , the curves have valley bottoms, but the depth of bottoms decrease along with the decrease of b . This illustrates that the system can go back to the stable state after being disturbed, but the stability tend to decrease with the decreasing of differential damage. As $b=0.05$ and 0.03 , the curves reverse direction and the valley bottom disappear. This indicates that, under the same disturbance intensity and disturbance intensity, the stability of system decrease along with the decrease of differential damage.

In the above analysis, influence of dynamic disturbance intensity, differential stress and differential damage on the evolution and stability of rock mechanics system is discussed respectively. It should be noted that the state of a system is not determined by a single factor, but rather the result of interaction of multiple factors. The critical instability point should be determined according to Eq. 18, which reflects the interaction of disturbance intensity, differential stress and differential damage.

Conclusions

1. In accordance with the self-organizing characteristics of rock mechanics system, a generalized dynamic equation was set up to describe the state of system during evolution by combining dynamic with stochastic theory.
2. Phase change theory as well as thermodynamics was used to research the parameters in rock mechanics system. The meaning of order parameter was determined. The specific expressions of control parameter and external field were presented.
3. The concept of differential damage was put forward. It represents the approaching degree of rock damage to its critical value. The differential stress was used to describe state of system under static loads.
4. The evolution and stability of rock mechanics system was discussed in three different situations. The variation of disturbance intensity, differential stress and differential damage could affect the

trend of evolution. Under certain conditions this variation could result in the instability of system. The critical instability point relates to the interaction of disturbance intensity, differential stress and differential damage.

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