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The Analysis of Ricardian Model under the Production Possibility Frontier

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***Abstract*—Ricardian model as the cornerstone of the theory of comparative advantage considers a linear production possibility frontier and shows that the marginal rate of substitution is constant. However, production possibility frontier is often nonlinear in real life. In this paper we give a full mathematical analysis of Ricardian model especially under nonlinear production possibility frontier, find that comparative advantage makes total profits increasing and both win, and point out that both countries will win within the range of mutual beneficial trading prices. But, if both of the comparative advantages are not enough, the both countries will lose. For the line of formulating allocation is not parallel to the line of best optimal allocation, and the range of mutual beneficial trading makes the tangent of allocation lines within the tangents at the product equilibrium in closed system. Finally, several problems should be paid attention to the comparative advantage.**

Keywords—Ricardo Model; production possibility frontier; comparative advantage; mathematical analysis

I. Introduction

As a milestone of the theory of comparative advantage, Ricardian model[1] is the basic conjunct win theory in international trades, original economic trades and other social fields, in which the product possibility frontier is linear. Then, many others author followed the idea to study economic problems, see[2-4].

However, product possibility frontier is often nonlinear [5,6]. In this paper we give a full mathematical analysis of Ricardian model especially under nonlinear production possibility frontier and show that comparative advantage can make total profits increase, cooperation win-win and prices mutual beneficial. Finally we point out several problems that should be paid attention to the comparative advantage.

II. Ricardian Model Under a Linear Product Possibility Frontier

In Ricardian model there are two counties, two kinds of goods and one element. Assume that country A and country B both produce goods X and Y. And the labor rates of X and Y in Country A are a_X , a_Y respectively, while the labor rates of X and Y in country B are b_X , b_Y respectively. Ricardo shows that if $\frac{a_X}{b_X} > \frac{a_Y}{b_Y}$ then on goods X, country A has a comparative advantage while on goods Y country B has a comparative advantage.

A. Comparative Advantage Makes Total Profits Increasing

On the product possibility boundary, if the product of X is $x_A \in [0, a_x]$, then the total product in country A is

$$Q_A = x_A + a_y - \frac{a_y}{a_x} x_A ,$$

And the total product in country B is

$$Q_B = x_B + b_y - \frac{b_y}{b_x} x_B , \text{ for } x_B \in [0, b_x] .$$

Then the total is

$$Q = a_y + b_y + x_A + x_B - \frac{a_y}{a_x} x_A - \frac{b_y}{b_x} x_B .$$

Differentiating it, we get

$$dQ = \left(1 - \frac{a_y}{a_x}\right) dx_A + \left(1 - \frac{b_y}{b_x}\right) dx_B .$$

It is easy to see that

(1) if $\frac{a_y}{a_x} < \frac{b_y}{b_x}$, then one unite change of x_A will cause a larger change Q of than that of x_A,that is , country A has a comparative advantage on X. And if $x_A = a_x$ and $x_B = 0$, then Q reaches maximum, $Q_{\max} = a_x + b_y$, where H is the optimal in Figure 1 below.

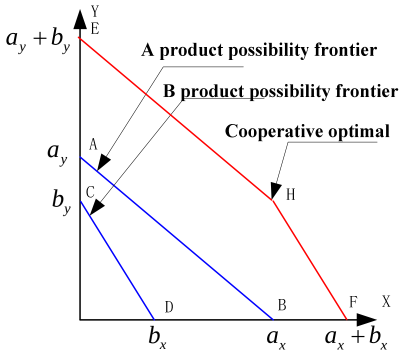

Fig.1. County A has a comparative advantage on goods Y

In Fig.1, line A is the product possibility frontier of country A, line CD is the product possibility frontier of country B, and broken line EHF is the product possibility frontier of the two countries A and B when cooperation.

(2) if $\frac{a_y}{a_x}=\frac{b_y}{b_x}$,then a unite change of x_A cause the same change of Q as that of y_B, that is there is no comparative advantage between country A and B.

B. Comparative Advantage Makes Both Win

As we know, in a closed system, for country A, the unit of X to exchange Y is equivalent to the labor exchange rate a_y/a_x ; for country B, the unit of Y to exchange X is equivalent to the labor exchange rate b_x/b_y . So if $\frac{a_y}{a_x}<\frac{b_y}{b_x}$, there is a comparative advantage, and country A should exchange X to goods Y of country B while country B should exchange Y to goods X of country A. And for country A, the quantity M_A of Y exchanging to a unite X is bigger than a_y/a_x , and country A would like to exchange Y to X for the increasing total profits, and vice versa.

For country A, the product equilibrium in closed system is $(x_A,\ f(x_A))$ with total product $Q_A=x_A+f(x_A)$, which is $Q_{AF}=\frac{a_y}{a_x}\cdot x_A+f(x_A)$ if Y is considered as the standard value. And in an open system, the total product is $Q_A=a_x$, which is $Q_{Ak}=M_A\cdot a_x$ if Y is as the standard value.

So the increment

$$\Delta Q_A=Q_{Ak}-Q_{AF}=M_A\cdot a_x-\left(\frac{a_y}{a_x}\cdot x_A+f(x_A)\right)$$

Or

$$\Delta Q_A==x_A\cdot\left(M_A-\frac{a_y}{a_x}\right)+\left((M_A\cdot a_x-x_A)-f(x_A)\right)$$

Then, if $M_A>\frac{a_y}{a_x}$,we have

$$(M_A\cdot a_x-x_A)-f(x_A)>0,$$

That is $\Delta Q_A=Q_{Ak}-Q_{AF}>0$.

Similarly, we have

$$\Delta Q_B=Q_{Bk}-Q_{BF}>0,$$

Where Q_{Bk} is the total product when country B is in development status, and Q_{BF} is the total product when country B is in closed status.

Now the indifference curve is as figure 2 below.

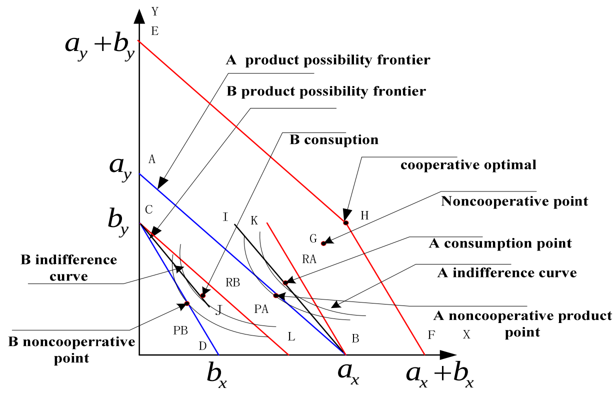

Fig.2. Comparative advantage makes both win

In Fig.2, point PA /PB is the product point of country A/B in closed system (i.e. the tangent point between the product possibility frontier and the indifference curve in country A?B) , point RA/RB is the consumption point (i.e., the tangent point between the allocation line and the indifference curve), which shows the increasing of warfare in both country.

C. Mutual Trading Prices

(1) Relation of trading prices

Clearly, the quantity of X put by country A is equal to that of Y called by B, while the quantity of Y put by country B is equal to that of X called by A, that is

$$M_A\cdot\Delta x_A=\Delta y_B,\ M_B\cdot\Delta y_B=\Delta x_A.$$

So $M_A=\frac{1}{M_B}$.

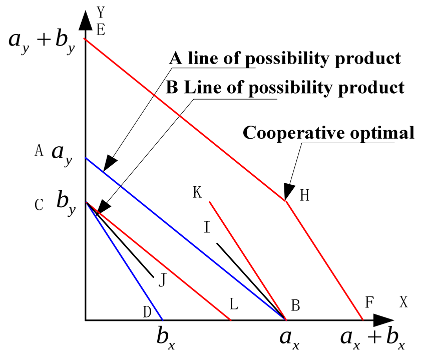

Fig.3. Relation of trading prices and mutual beneficial prices

That is to say, the price M_A of one unit X in country A to exchange goods Y in country B is equal to the inverse of the price M_B of one unit Y in country B to exchange goods X in country A, according to the pricing of Y.

From Fig.3, the relation of trading price means that the allocation line BI of country A is parallel to the allocation line CJ of country B.

(2) Range of mutual beneficial prices

If $M_A > \frac{a_y}{a_x}$ and $M_B > \frac{b_x}{b_y}$, then

$$\frac{a_y}{a_x} < M_A < \frac{b_y}{b_x}, \text{ or } \frac{b_x}{b_y} < M_B < \frac{a_x}{a_y}.$$

That is, if $\frac{a_y}{a_x} < M_A < \frac{b_y}{b_x}$ ($\frac{b_x}{b_y} < M_B < \frac{a_x}{a_y}$), then both countries have mutual beneficial trading, and we get the range of mutual prices before.

From Fig.3, the range of mutual prices reflects that the allocation line BI of country A is taken shape by the optimal allocation line BK and product possibility line AB, while the allocation line CJ of country B is taken shape by the optimal allocation line CL and product possibility line CD.

III. Ricardian Model Under a Nonlinear Product Possibility Frontier

In general there are three classical shapes of product possibility: (1) convex, (2) concave and (3) line, which corresponding to increasing, decreasing and unchanging of the economical size profit

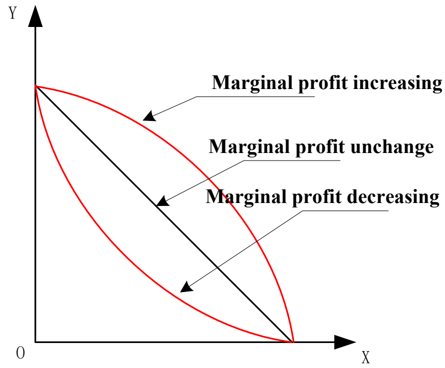

Fig. 4. Curves of product possibility

(1) If the size profit is increasing, i.e. the opportunity cost is decreasing, then the marginal rate of transfer MRT is decreasing, or, $M_{xy} = -\frac{dy}{dx}$ is decreasing, so $\frac{d^2y}{dx^2} > 0$.

(2) If the size profit is not changed, i.e. the opportunity cost is not changed then the marginal rate of transfer MRT is not changed, or, $M_{xy} = -\frac{dy}{dx} = c$, then $\frac{d\left(M_{xy}\right)}{dx} = -\frac{d^2y}{dx^2} = 0$, or $\frac{d^2y}{dx^2} = 0$.

Now the mathematical analysis of comparative advantages as follows when the product possibility frontier is nonlinear.

A. Comparative Advantage Makes Total Profits Increasing

Assume that the product possibility frontier in country A (country B) is a continuous second differential nonlinear function $y = f(x)$ ($y = g(x)$).

Then the quantity of country A on product X is

$$Q_A = x_A + f(x_A) \quad x_A \in [0, a_x],$$

And the quantity of country B on product Y is

$$Q_B = x_B + g(x_B) \quad x_B \in [0, b_x].$$

So the total is

$$Q = x_A + x_B + f(x_A) + g(x_B).$$

Differentiating it, we have

$$\frac{\partial Q}{\partial x_A} = 1 + \frac{df(x_A)}{dx_A},$$

$$\frac{\partial Q}{\partial x_B} = 1 + \frac{dg(x_B)}{dx_B}.$$

(1) If $\frac{\partial Q}{\partial x_A}=1+\frac{df\left(x_A\right)}{dx_A}>\frac{\partial Q}{\partial x_B}=1+\frac{dg\left(x_B\right)}{dx_B}$,or $\frac{df\left(x_A\right)}{dx_A}>\frac{d\left(x_B\right)}{dx_B}$,then one unite change of x_A will cause a larger change Q of than that of x_B,that is , country A has a comparative advantage on X while country B has a comparative advantage on Y.

(2) if $\frac{\partial Q}{\partial x_A}=1+\frac{df\left(x_A\right)}{dx_A}<\frac{\partial Q}{\partial y_B}=1+\frac{dg\left(x_B\right)}{dx_B}$,or $\frac{df\left(x_A\right)}{dx_A}<\frac{d\left(x_B\right)}{dx_B}$ then both unit changes of x_A and make the same change of Q, so there is no comparative advantage between them.

So by Ferma maximum theory, the necessary maximum condition on Q is

$$\begin{cases}1+\frac{df\left(x_A\right)}{dx_A}=0\\1+\frac{dg\left(x_B\right)}{dx_B}=0\end{cases}\Rightarrow\begin{cases}\frac{df\left(x_A\right)}{dx_A}=-1\\\frac{dg\left(x_B\right)}{dx_B}=-1\end{cases}\Rightarrow\begin{cases}\left.\frac{df\left(x\right)}{dx}\right|_{x_A}=-1\\\left.\frac{dg\left(x\right)}{dx}\right|_{x_B}=-1\end{cases}.$$

That is, Q reaches maximum when $\left.\frac{df\left(x\right)}{dx}\right|_{x_A}=\left.\frac{dg\left(x\right)}{dx}\right|_{x_B}=-1$.

When the size profit is increasing, or the opportunity cost is decreasing the marginal rate of transfer $M_{xy}=-\frac{dy}{dx}$ is decreasing if the product possibility curve is concave, and then $\frac{d^2y}{dx^2}<0$, so Q has a maximum. Of course, the results in linear case are the same as in Ricardian model.

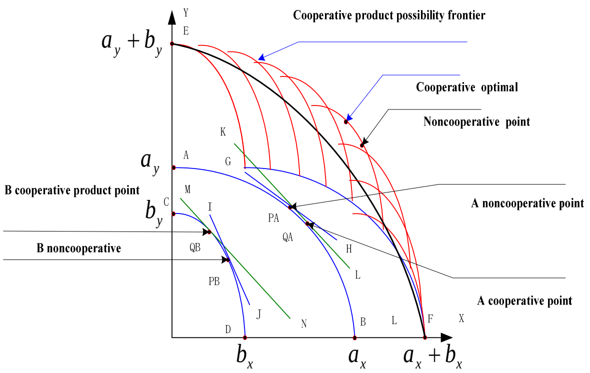

Fig.5. Comparative advantage makes total profits increasing

In Fig.5,the curve AB is the product possibility frontier of country A, the curve CD is the product possibility frontier of country B, and the curve EF is the product possibility frontier of cooperation between A and . The point PA is the equilibrium of country A in a closed system. The line KL satisfies $\left.\frac{df\left(x\right)}{dx}\right|_{x_A}=-1$, being tangent to the curve AB with tangent point QA, which is the optimal of country A when both product, while the line MN satisfies $\left.\frac{dg\left(x\right)}{dx}\right|_{x_B}=-1$ being tangent to the curve CD with tangent point QB, which is the optimal of country B when both product.

B. Comparative Advantage Makes Both Win

Assume that in a closed system the product equilibrium of country A (country B) is $\left(x_A,\ f\left(x_A\right)\right)$ $\left(x_B,\ g\left(x_B\right)\right)$, then the equivalent labor rate of exchange X to Y is $-\left.\frac{df\left(x\right)}{dx}\right|_{x_A}$ $\left(-1\Big/\left.\frac{dg\left(x\right)}{dx}\right|_{x_B}\right)$. So if country A has a comparative advantage on X and country B has a comparative advantage on Y, then A products X more and gives up producing Y and then exchange X to Y, and vice versa. Since for country A, the quantity M_A of the unit of X exchanging to Y in country B is bigger than $-\left.\frac{df\left(x\right)}{dx}\right|_{x_A}$, country A would like to exchange X to Y and make total profits increasing, and vice versa.

For country A in a closed system the total quantity is $Q_A=x_A+f\left(x_A\right)$ at the product equilibrium $\left(x_A,\ f\left(x_A\right)\right)$, which in Y's value standard is

$$Q_{AF}=-\left.\frac{df\left(x\right)}{dx}\right|_{x_A}\cdot x_A+f\left(x_A\right).$$

In an open status, the total quantity is $Q_A=x_A+\Delta x+f\left(x_A+\Delta x\right)$, which in Y's value standard is $Q_{Ak}=M_A\cdot\left(x_A+\Delta x\right)+f\left(x_A+\Delta x\right)$. So the increment is

$$\Delta Q_A=M_A\cdot\left(x_A+\Delta x\right)+f\left(x_A+\Delta x\right)+\left.\frac{df\left(x\right)}{dx}\right|_{x_A}\cdot x_A-f\left(x_A\right),$$

or

$$\Delta Q_A=x_A\cdot\left(M_A+\left.\frac{df\left(x\right)}{dx}\right|_{x_A}\right)+\left(\left(f\left(x_A+\Delta x\right)-f\left(x_A\right)\right)-M_A\cdot\Delta x\right).$$

Since at $\left(x_A, f\left(x_A\right)\right)$, country A has a comparative advantage, then if $M_A > -\left.\frac{df(x)}{dx}\right|_{x_A}$ we have

$$M_A + \left.\frac{df(x)}{dx}\right|_{x_A} > 0 \text{, or}$$

$$M_A < \frac{f(x+\Delta x) - f(x)}{\Delta x} \Rightarrow \left(f(x+\Delta x) - f(x)\right) - M_A \cdot \Delta x > 0$$

Thus, $\Delta Q_A = Q_{Ak} - Q_{AF} > 0$.

Similarly, we have $\Delta Q_B = Q_{Bk} - Q_{BF} > 0$.

The indifference curve is as in Fig.6.

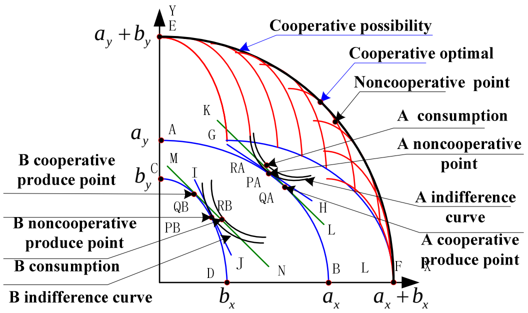

Fig. 6. Comparative advantage makes both win

C. *Mutually beneficial Trading Prices*

(1) Relation of trading prices

Clearly, the quantity of X put by country A is equal to that of Y called by B, while the quantity of Y put by country B is equal to that of X called by A, that is

$$M_A \cdot \Delta x_A = \Delta y_B,\ M_B \cdot \Delta y_B = \Delta x_A.$$

Then,

$$M_A = \frac{1}{M_B}.$$

That is to say, the price M_A of one unit X in country A to exchange goods Y in country B is equal to the inverse of the price M_B of one unit Y in country B to exchange goods X in country A, according to the pricing of Y. the relation of trading price means that the allocation line KL of country A is parallel to the allocation line MN of country B.

(2) Range of mutual beneficial prices

If $M_A > -\left.\frac{df(x)}{dx}\right|_{x_A}$ and $M_B > -1\Big/\left.\frac{dg(x)}{dx}\right|_{x_B}$

Then $-\left.\frac{df(x)}{dx}\right|_{x_A} < M_A < -\left.\frac{dg(x)}{dx}\right|_{x_B}$,

or $-1\Big/\left.\frac{dg(x)}{dx}\right|_{x_B} < M_B < -1\Big/\left.\frac{df(x)}{dx}\right|_{x_A}$.

That is to say, under the above conditions, both country win within the range of mutual beneficial trading prices.

When both countries are in the optimal, then

$$M_A = -\left.\frac{df(x)}{dx}\right|_{x_A} = -\left.\frac{dg(x)}{dx}\right|_{x_B} = 1,$$

$$M_B = -1\Big/\left.\frac{df(x)}{dx}\right|_{x_A} = -1\Big/\left.\frac{dg(x)}{dx}\right|_{x_B} = 1.$$

However, if both of the comparative advantage is not enough or over, then both countries will lose, for the line of formulating allocation is not parallel to the line of best optimal allocation, and the rang of mutual beneficial trading makes the tangent of allocation line within the tangents at the product equilibrium in closed system, as in Fig.7 and 8,

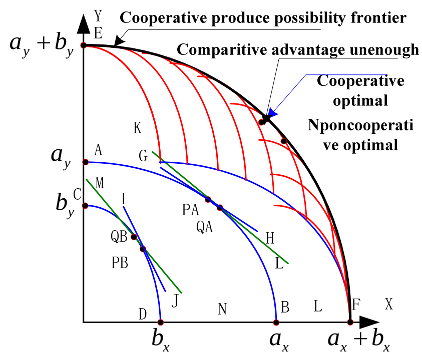

Fig.7. Comparative advantage is not enough

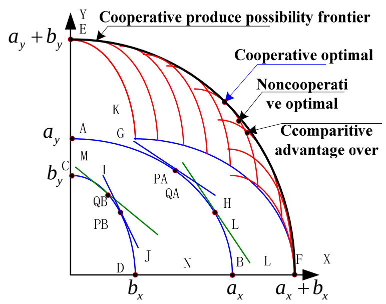

Fig.8. Comparative advantage is over

IV. CONCLUSIONS

By comparing our model under nonlinear product possibility frontier with Ricardian model in linear case, we can draw the following results:

(1) Ricardian model is a special case of our model;

(2) If there is a linear product frontier, the condition of comparative advantage is the tangent of product possibility frontier, and has nothing to do with labor more or less, which makes unified between Smith model and Ricardian model.

(3) Comparative advantage makes total profits increasing.

(4) Comparative advantage makes both countries win.

(5) In our nonlinear model the optimal of comparative advantage is that both countries produce a pair of the two goods, and there are three cases of comparative advantage, not enough, sufficiency or over, while in Ricardian model under a linear product possibility frontier, the optimal is that both countries produce only one different goods, and there are only two cases of comparative advantage, not enough or sufficiency.

And we can learn from the above that

(1) We should enhance the whole level of product possibility frontier to avoid falling in the strap "comparative advantage";

(2) We should look comparative advantage in a dynamic view, and foster our key competitive ability, especially when we are in a weaker status;

(3) We should develop our comparative advantage properly, prevent from being not enough or over.

REFERENCES

[1] D. Ricardo. On the Principles of Political Economy and Taxation.Bejing, The commercial Press , 1976. (In Chinese)

[2] Zhiming Zhang. Evolution of comparative advantage and Industry development . Xiamen Unversity Docoral dissertation,2002. (In Chinese)

[3] Jianhong Lin, Yuankang Xu. Comparative study of comparative advantage and competitive advantage. Business Study , 2004. (In Chinese)

[4] Shijun Wang. Comprehensive comparitive advantage thoery with empirical study. Zhejiang Unversity Docoral dissertation,2008. (In Chinese)

[5] Michael P. Todaro, Stephen C. Smith. Economic Development Addison Wesley, 2005.

[6] N. Gregory Mankiw. Principles of Economics. Beijing, Peking University Press, 2006. (In Chinese)