Soft E-inversive Semigroup

Bingshen Lan[1, a*]
[1]The Engineering and Technical College of Chengdu University of Technology, Leshan, China
[a]512316180@qq.com

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Abstract. In this paper, the concepts of Soft E-inversive semigroup, soft normal semigroup on E-inversive semigroup will be defined. Some results will be obtained. Finally, the concept of soft group congruence on Soft E-inversive semigroup is given, and the connections among soft group congruence and soft normal semigroup on E-inversive semigroup are revealed.

Introduction

1999, Molodstov gives soft set theory which is new mathematical tools deal with uncertainty problem in many fields [1]. After, soft set theory has been extended in various ways by many authors. Aktas H, et al give the concept soft group.[2], Ali discusses soft binary relationon and soft equivalence on set[3], Sezgin A, Atagün A O give the concept normalistic soft group[4] .

If $F: A \to P(U)$, then (F, A) is called soft set on U.[1]

If (ρ, A) is soft set on $U \times U$, then (ρ, A) is called soft binary relation on $U \times U$.[3]

A Semigroup S with set E(S) of idempotents, if for any $s \in S$ suah that $\exists x \in S, sx \in E(S)$, then S will be called E-inversive semigroup.[5]

Let S be a semigroup, for any $s \in S$, if $xsx = x, x \in S$, then x is called a weak inverse of s. let $W(s) = \{x \in s : xsx = x\}$

denote all weak inverses of s. [6]

A Subsemigroup T of a E-inversive semigroup S is said to be unitary if for any $x, y \in S$,

(1) if $x+y \in S$, $x \in S$, then $y \in S$; and

(2) if $x+y \in S$, $y \in S$, then $x \in S$. [6]

A subsemigroup T of E-inversive semigroup S is said to be self-conjugate if for any $x \in S$, such that $xSx' \in S$, $x'Sx \in S$, $x' \in W(x)$ [6].

A subsemigroup T of E-inversive semigroup S is said to be noamal If T such that the following three conditions.

(1)T contains all the idempotents of S, $E(S) \subseteq T$;

(2)T is unitary;

(3)T is self-conjugate.[6]

A binary relation ρ on a set X (that is a subset of $X \times X$) is called a equivalence if

$(x, x) \in \rho$ for all x in X, that is ρ reflexive;

$\forall x, y \in X, (x, y) \in \rho \Rightarrow (y, x) \in \rho$, that is ρ symmetric;

(3) $\forall x, y, z \in X, (x, y) \in \rho, (y, z) \in \rho \Rightarrow (x, z) \in \rho$, that is ρ transitive. [7]

Let S be a semigroup. A binary relation ρ on the set S is called left compatible (with the operation on S) if

$\forall s,t,a \in S,(s,t) \in \rho \Rightarrow (as,at) \in \rho$,

And right compatible if

$\forall s,t,a \in S,(s,t) \in \rho \Rightarrow (sa,ta) \in \rho$.

It is called compatible if

$\forall s,t,a,b \in S,(s,t) \in \rho,(a,b) \in \rho \Rightarrow (sa,tb) \in \rho$.

A left (right) compatible equivalence is called a left (right) congruence. A compatible equivalence is called a congruence. [7]

Soft E-Inversive Semigroup

Definition 2.1 Let *(F, A)* be a soft set on E-inversive semigroup *S*, for any *a* of *A*, *F(a)* is a E-inversive subsemigroup of *S*, then *(F,A)* is called Soft E-inversive semigroup on *S*.

Example2.1 Let S = $\{e, f, a, b\}$ be the E-inversive semigroup with multiplication table

	e	f	a	b
e	f	e	e	e
f	e	f	f	f
a	e	f	a	e
b	f	f	b	e

Let *A*=$\{\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{4}\}$, and let

F(1)= $\{f\}$,F(2)= $\{f,a\}$,F(3)= $\{e,f,a\}$,F(4)= $\{e,f,a,b\}$

It is easy to see that

F(1)= $\{f\}$, F(2)= $\{f,a\}$,F(3)= $\{e,f,a\}$,F(4)= $\{e,f,a,b\}$

are E-inversive subsemigroups of *S*. So, *(F, A)* is Soft E-inversive semigroup on *S*.

Proposition2.1 If *(F, A)*, (*H, A)* be Soft E-inversive semigroup on E-inversive semigroup *S*, then $(F,A)\bigcap(H,A)$ is Soft E-inversive semigroup on *S*.

Proof. Let *(U, C)* = $(F,A)\bigcap(H,A)$, *C*=*A*, for any *a* of *A*, *U (a)* =*F(x)*, or *H(x)*, hence $(F,A)\bigcap(H,A)$ is Soft E-inversive semigroup as required.

Proposition 2.2 If *(F, A)*, *(H, B)* be Soft E-inversive semigroup on E-inversive semigroup *S*, $A\bigcap B=\Phi$, then $(F,A)\bigcup(H,B)$ is Soft E-inversive semigroup on *S*.

Proof Let *(U,C)*= $(F,A)\bigcup(H,B)$, for any *a* of *C*, then $a \in A-B$ or $a \in B-A$, If $a \in A-B$,U(a)=F(x), or $a \in B-A$,*U(a)*=*H(x)*, hence $(F,A)\bigcup(H,B)$ is Soft E-inversive semigroup as required.

Definition 2.2 Let *(F, A)* be a soft set on E-inversive semigroup *S*, for any *a* of *A*, *F(a)* is a normal subsemigroup of *S*, then *(F,A)* is called Soft normal semigroup on *S*.

Proposition2.3 If *(F, A)* is Soft normal semigroup on E-inversive semigroup S, then *(F, A)* is Soft E-inversive semigroup on *S*.

Proof For any a of *A*, *F(a)* is normal subsemigroup of *S*, then $E(S) \subseteq F(a)$, *F(a)* contains all the idempotents of *S*, and for any *s* of *F(a)*, if $sx \in E(S), x \in S$, and *F(a)* is unitary, it is clear that $x \in F(a)$, so *F(a)* is E-inversive subsemigroup of *S*. hence (F,A) is soft E-inversive semigroup on *S*.

Proposition2.4 If *(F, A)*, *(H, B)* are soft normal semigroup on E-inversive semigroup *S*, then $(F,A)\wedge(H,B)$ is Soft normal semigroup on S.

Proof Let *(U,C)*= $(F,A)\wedge(H,B)$, C= $A\times B$, $U(\alpha,\beta)=F(\alpha)\cap H(\beta)$, for any $\alpha\in A,\beta\in B$, $E(S)\subseteq F(a)$, $E(S)\subseteq H(\beta), E(S)\subseteq F(a)\bigcap H(\beta)$; and $F(\alpha)$, $H(\beta)$ are self-conjugate, unitary, $U(\alpha,\beta)=F(\alpha)\cap H(\beta)$ is self-conjugate, unitary; Hence $U(\alpha,\beta)=F(\alpha)\cap H(\beta)$, is Soft normal semigroup as required.

Soft Group Congruences on E-Inversive Semigroup

If ρ is a congruence on semigroup *S*, then we can define a binary operation on the quotient set S/ρ as follows:

$(a\rho)(b\rho)=(ab)\rho$.[7]

If ρ is a congruence on E-inversive semigroup *S*, S/ρ is a group, then ρ is saided a group congruence on E-inversive semigroup *S*.

Definition3.1 S is E-inversive semigroup, let (ρ, A) be soft binary relation on $S\times S$, for any $\alpha\in A$, $\rho(\alpha)$ is a group congruence on a E-inversive semigroup *S*, then (ρ, A) is called soft group congruence on E-inversive semigroup *S*.

Proposition3.1 let *N* be normal subsemigroup of E-inversive semigroup *S*, then

$$\rho_N=\{(a,b)\in S\times S|(\exists p,q\in N)\,pa=bq\}$$

is a group congruence on *S*. Conversely, let ρ be a a group congruence on E-inversive semigroup *S*, then

$$N_\rho=\left\{a\in S|\ a\rho=1_{S/\rho}\right\}$$

is a normal subsemigroup of E-inversive semigroup S[8].

Proposition3.2 let（*F*，A）is soft normal semigroup of E-inversive semigroup *S*, for any $a\in A$, let

$$\rho(a)\doteq\rho_{F(a)}=\{(a,b)\in S\times S|(\exists p,q\in F(a))\,pa=bq\}$$

Then（ρ，A）is soft group congruence on E-inversive semigroup *S*.

Conversely, let（ρ，B）is soft group congruence on E-inversive semigroup *S*, then, for any $b\in B$, let

$$F(b)=N_{\rho(b)}=\left\{s\in S|\ s\rho=1_{S/\rho(b)}\right\}$$

Then (F, B) is soft normal semigroup of E-inversive semigroup S.

Proof If（*F*，A）is soft normal semigroup of E-inversive semigroup *S* , so for any $a\in A$, *F(a)* is normal subsemigroup of E-inversive semigroup S,

Let, $\rho(a)\doteq\rho_{F(a)}=\{(a,b)\in S\times S|(\exists p,q\in F(a))\,pa=bq\}$

From Proposition3.1, $\rho(a)\doteq\rho_{F(a)}=\{(a,b)\in S\times S|(\exists p,q\in F(a))\,pa=bq\}$ is a group congruence on *S*. hence,（ρ，A）is soft group congruence on E-inversive semigroup *S*.

Conversely, if（ρ，B）is soft group congruence on E-inversive semigroup *S*, so any $b\in B$, ρ(b) is group congruence on S, let

$$F(b)=N_{\rho(b)}=\left\{s\in S|\ s\rho=1_{S/\rho(b)}\right\}$$

From Proposition3.1,

$$F(b)=N_{\rho(b)}=\left\{s\in S|\ s\rho=1_{S/\rho(b)}\right\}$$

is a normal subsemigroup of E-inversive semigroup *S*. hence, *(F, B)* is soft normal semigroup of E-inversive semigroup *S*.

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