

## DOR Signal Simulating Method

Yong-Biao Shi, Wei-Min Zheng, Li Tong, Yin Yu

Shanghai Astronomical Observatory, Chinese

Academy of Sciences

E-mail: shiyb@shao.ac.cn

Wei-Min Zheng, Li Tong

Key Laboratory of Radio Astronomy, Chinese Academy

of Sciences,

Shanghai, China

Wei-Min Zheng

Shanghai Key Laboratory of Space Navigation and Positioning Techniques, Shanghai 200030

Shanghai, China

**Abstract**—The delta-differential one-way ranging (Delta-DOR) technology is proposed as the higher accuracy method used in the deep-space probes for VLBI observation. The accuracy of about 2 nrad in Ka band was achieved in the NASA Mars Reconnaissance Orbiter (MRO), 2005. However, only a few researches have studied on how to simulate/extract the DOR signal in baseband (or local DOR). In this paper, we present a method the down-link DOR signals simulation. Compared with other method, it generates 2 or 3 orders of magnitude less data with much higher speed. A case was examined to verify this method.

**Keywords**—Delta-DOR; VLBI; simulate

### I. INTRODUCTION

Very long baseline interferometry (VLBI) is the radio interferometry technology that developed from the end of 1960s with high resolution, high measurement accuracy, and it is widely used in the field of astrophysics, astrometry and space exploration [1]. To improve the onboard beacon efficiency, the Jet Propulsion Laboratory (JPL) of National Aeronautics and Space Administration (NASA), presented a special VLBI technology -- delta-DOR, which has even higher accuracy than that of normal VLBI. Delta-DOR was first used in the 1979 for the orbit determination of Voyager [2],[3]. The VLBI technology was first used in the China's Lunar Exploration Project in 2007, the delta-DOR was first formally used in the Chang'E-3 in 2012, and the group delay accuracy is better than 1ns [4],[5].

Many excellent results were obtained with DOR technology during the last 5 years [6],[7].

A typical down-link DOR signal of emission of probes, is [8]:

$$s(t) = \sqrt{2P_T} \sin(2\pi f_c t + \theta d(t)) + m_1 \sin(2\pi f_1 t) + m_2 \sin(2\pi f_2 t) \quad (1)$$

In which  $P_T$  is the total power of the emission signals,  $f_c$  is the carrier frequency,  $d(t)$  is the telemetry and  $\theta$  is the modulation index. The DOR tones will have two pairs of frequencies  $f_1$  and  $f_2$ , and modulation indices are  $m_1$  and  $m_2$ . Sometimes, there are only the first pair DOR tones.

The down-link DOR Signal must be firstly transformed to the intermediate frequency (IF) as the signal is modulated by a high carrier frequency (about 8.4 GHz in X band for instance), and then transformed to baseband. There are many works to be done during this process, for instance: converting the frequency to baseband, amplifying the signal and filtering the signal. It is not difficult to finish these works with the help of the development of hardware equipment, without considering the budget. It is difficult to complete the simulation in X band directly because it will produce a large amount of data during the procedure mentioned above. So, to simulate the baseband signal is more realistic and convenient.

Researchers have studied how to obtain the DOR signal with software[9], and there are some improvement [10],[11]. It was assumed the typical down-link DOR signal was already processed by frequency conversion, filtered and sampled, to acquire the sequence  $s(n)$  [9]. Two FFT transforms are required to find the corresponding frequency used in the method. However, there are three problems with this method. First, the down-link DOR signal must be frequency converted, filtered and sampled with hardware or other method; otherwise this method does not work. Secondly, in this method, it is difficult finding the precise frequency to make the spectrum of the down-link DOR shifting, especially during the second FFT transform, quite precise frequency is required to take the DOR signal.

In our new method, the processes of frequency conversion, filtering and sampling are disregarded for simulating the DOR signal, since it will simulate all the components in the baseband. Compared with the previous method, the down-link DOR signal can be processed directly, without spectrum shifting, neither the frequency conversion, filtering.

Meanwhile, the generated data can be 2 or 3 orders of magnitude less than that of the previous methods, the operating speed and the efficiency can be highly improved. To simulate a down-link DOR signal with the bandwidth of 40 MHz of 5 seconds operating at X band, it only takes 30 seconds, which is much faster than method used at the moment, which takes one hour to simulate the down-link DOR signal with the same bandwidth and the same operating time.

Moreover, in this paper, the error analysis is conducted. Consequently, the DOR signal can be simulated according to the required accuracy, and more flexible.

## II. THE MATHEMATICAL PRINCIPLE OF SIMULATING THE DOR SIGNALS

A typical down-link DOR signal is shown in (1). As we are more interested in DOR signal itself, the DOR signals ( $DOR_1$  and  $DOR_2$ ) without the subcarrier or the telemetry are shown in (2):

$$s(t) = \sqrt{2P_T} \sin(2\pi f_c t + m_1 \sin(2\pi f_1 t) + m_2 \sin(2\pi f_2 t)) \quad (2)$$

$m_1$  and  $m_2$  are the modulation factors of the  $DOR_1$  and  $DOR_2$  respectively. And  $f_1$  and  $f_2$  are the frequencies of  $DOR_1$  and  $DOR_2$ , respectively. According to Euler formula, and  $\omega = 2\pi f$ , (2) can be written as:

$$\begin{aligned} s(t) &= \sqrt{2P_T} \frac{e^{j(\omega_c t + \sum_{i=1}^2 \sin(\omega_i t))} - e^{-j(\omega_c t + \sum_{i=1}^2 \sin(\omega_i t))}}{2j} \\ &= \frac{\sqrt{2P_T}}{2j} (e^{j\omega_c t} e^{jm_1 \sin \omega_1 t} e^{jm_2 \sin \omega_2 t} - e^{-j\omega_c t} e^{-jm_1 \sin \omega_1 t} e^{-jm_2 \sin \omega_2 t}) \end{aligned} \quad (3)$$

Because the negative frequency domain is equivalent to the positive frequency domain, it is sufficient to analyze the positive frequency domain. According to Taylor expansion, the positive frequency domain of (3) can be expanded as:

$$\begin{aligned} s_1(t) &= \frac{\sqrt{2P_T}}{2j} (e^{j\omega_c t} e^{jm_1 \sin \omega_1 t} e^{jm_2 \sin \omega_2 t}) \\ &= \frac{\sqrt{2P_T}}{2j} \left( e^{j\omega_c t} \prod_{i=1}^2 \left( \sum_{k=0}^{\infty} \frac{(jm_i \sin \omega_i t)^k}{k!} \right) \right) \end{aligned} \quad (4)$$

In (4), it gets the baseband and the harmonics of the two DOR signals. To recover a signal from its spectrum, it is enough to analyze the zero frequency and the baseband components. It is known that while  $k = 2M$  ( $M = 0, 1, 2, 3, \dots$ ), or even power, the zero and even harmonics are included, while  $k = 2M+1$  ( $M = 0, 1, 2, 3, \dots$ ), or odd power, the baseband and the odd harmonics are included. So (4) can also be written as:

$$s_1(t) = \frac{\sqrt{2P_T}}{2j} \left( e^{j\omega_c t} \prod_{i=1}^2 \left( \sum_{k=0}^{\infty} a_{ik} \sin k \omega_i t \right) \right) \quad (5)$$

$$\text{Where } a_{i0} = \sum_{M=0}^{\infty} \frac{m_i^{(2M)}}{(2M)!} j^{2M} \sum_{i=1}^M \left( \frac{1}{2^{\lfloor \frac{M}{2} + i \rfloor}} \right), \text{ ignoring}$$

the high harmonic components, (5) can be written as:

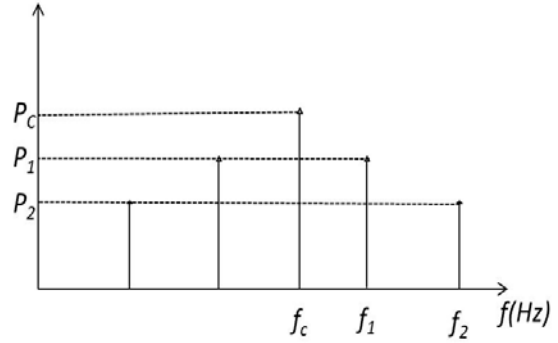


Figure 1. Spectrum of downlink DOR signals. In the figure, it is assumed that only two DOR signals are included, while the higher frequency harmonic and the differential components are ignored.

$$\begin{aligned} s_1(t) &\approx \frac{\sqrt{2P_T}}{2j} \left( e^{j\omega_c t} \prod_{i=1}^2 (a_{i0} + a_{i1} \sin \omega_i t) \right) \\ &= \frac{\sqrt{2P_T}}{2j} \left( e^{j\omega_c t} \left( a_0 + \sum_{i=1}^N a_{i1} \sin \omega_i t \right) \right) \end{aligned} \quad (6)$$

Where  $a_0, a_{i1}$  can be obtained by:

$$a_0 = \prod_{i=0}^2 a_{i0}, \quad a_{i1} = a_{i1} \prod_{j=1}^2 a_{j0}, \quad j \neq i \quad (7)$$

The spectrum of (5) is shown in Fig. 1 In it, the harmonic and differential components are ignored. The strengths of the carrier,  $DOR_1$  and  $DOR_2$  are  $P_c, P_1$  and  $P_2$ , respectively. The two DOR signals  $s_b(t)$  in baseband can be simulated based on the same signal power as that shown in Fig. 1:

$$s_b(t) = \sum_{i=1}^2 a_i \sin \omega_i t \quad (8)$$

The spectrum of the simulated baseband DOR signal is shown in Fig. 2. Since the power of signals shown in Fig. 2 should be the same as that shown in Fig. 1, the coefficient of  $a_i$  can be calculated:

$$a_i = \frac{\left| \frac{\sqrt{2P_T}}{2j} \left( \sum_{i=1}^2 a_{i1} \right) \right|}{2} \quad (9)$$

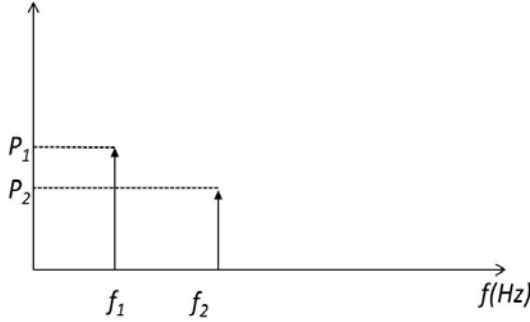


Figure 2. Spectrum of simulated DOR signal. The strengths of the two signals are  $P_1$  and  $P_2$ , respectively.

### III. ERROR ANALYSIS

Errors will be introduced during the Taylor expansion, disregarding the high harmonics. The Taylor expansion error is:

$$R_M = \left| \prod_{i=1}^2 \frac{(jm_i \sin \omega_i t)^M}{M!} e^{jm_i \sin \omega_i t} \right| \quad (10)$$

$$\leq \left| \left( \frac{1}{M!} \right)^2 \right|$$

Meanwhile, in (6), there will be error due to the finite value of  $k$ :

$$R'_M = R_{Mi} \prod_{j=1}^2 (a_{i0} + a_{i1} \sin(\omega_i t)) \quad (11)$$

$$< \sum_{i=1}^2 R_{Mi} \prod_{j=1}^2 (a_{i0} + a_{i1}), i \neq j$$

In which  $R_{Mi}$  is the expansion error of each signal, and is given by:

$$R_{Mi} = \frac{1}{2^{\frac{M}{2}-1}} \quad (12)$$

The total error would be:

$$R_{MT} = R_M + R'_M$$

$$< \left( \frac{1}{M!} \right)^2 + \frac{1}{2^{\frac{M}{2}-1}} \prod_{i=1}^2 (m_i)^2 \left| \prod_{j=1}^2 (a_{i0} + a_{i1}) \right| \quad (13)$$

$$i \neq j$$

The comparison result of the calculated error and the real error calculated with MATLAB program is shown in TABLE I. It can be seen that the ideal accuracy is highly improved with the increase of  $k$ . However, the real calculated error does not change after  $k$  increases more than 5. One possible reason can be due to performance limitation of the computer.

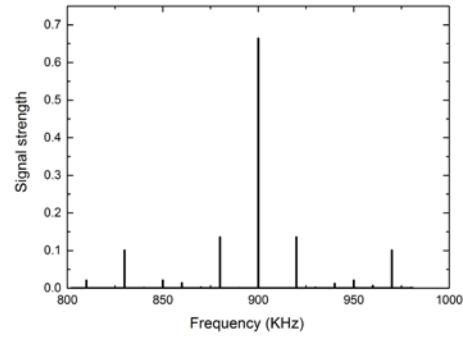


Figure 3. The positive domain spectrum of down-link DOR signal. The power is 1,  $f_c$ ,  $f_1$  and  $f_2$  are 900 kHz, 20 kHz and 50 kHz, respectively.

### IV. CASE ANALYSIS

An example of the simulated DOR signals from the down-link DOR signal with this method was studied in order to verify this method. The down-link DOR signal is constructed by MATLAB with  $f_c$ ,  $f_1$  and  $f_2$  are set to 900 kHz, 20 kHz and 50 kHz, respectively, in (5), and the power index is assumed as 1. And the spectrum of this signal is shown in Fig.3. According to (9), a baseband signal can be simulated

TABLE I. THE COMPARISON OF THE IDEAL ERROR AND THE REAL ERROR WITH DIFFERENT EXPAND ITEMS

<b>k value</b>	<b>ideal error (%)</b>	<b>real error (%)</b>
<b>3</b>	<b>0.96</b>	<b>1</b>
<b>5</b>	<b>0.00017</b>	<b>0.0065</b>
<b>7</b>	<b>0.00000014</b>	<b>0.0065</b>
<b>9</b>	<b><math>1.96 \times 10^{-13}</math></b>	<b>0.0065</b>

$$s_b(t) = a_1 \sin(2\pi f_1 t) + a_2 \sin(2\pi f_2 t) \quad (14)$$

The coefficient  $a_1$  and  $a_2$  can be calculated by (9). The spectrum of this baseband signal is shown in Fig. 4, which takes the value of 5.

To give a better comparison, the center frequency or the baseline of down-link DOR signal spectrum was shifted to 0, as shown in Fig. 5. It is clear that the two curves agree well (less than 0.0065% shown in TABLE I), which means that our method is correct.

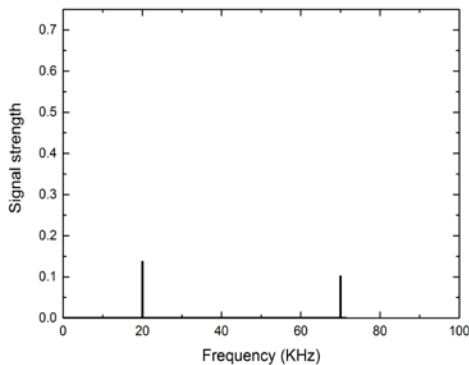


Figure 3. The positive spectrum of simulation DOR signal. The coefficient of the simulation DOR signal is acquired by (14).

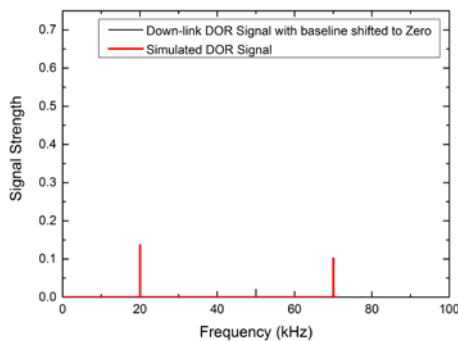


Figure 4. Comparison of the down-link DOR signal and the simulation DOR signal, the black curve is the down-link DOR signal with its baseline shifted to Zero. The red curve is the simulated DOR signal.

## V. DISCUSSION

Normally, the telemetry, subcarrier and even the remote residue are included in the downlink signal; anyhow, this method is still applicable for simulating the DOR signal in baseband from down-link DOR signal if they have the same form of sinusoidal signal. However the accuracy may be lower with the same order of Taylor expansion. The telemetry, for instance, the spectrum of the telemetry has a bandwidth of 2 KHz, and can be treated as series of sinusoidal signals, it is required to have higher Taylor

expansion to acquire the same accuracy compared with the only DOR included down-link signal with this method.

More generally, noise is also included in the downlink signal received by the station:

$$v(t) = s(t) + n(t) \quad (15)$$

The components of the noise  $n(t)$  are quite complex, which include the background noise of the space, the noise of the antenna, the noise of the transmission path and so on. However, all the noise can be regarded as Gauss distribution, which means the noise and signal can be processed separately. Even in this case, the method is still applicable.

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