

# Energy Optimization for a Simple Household Microgrid Based on Demand Response

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**Abstract.** Energy management in demand side is an important way to realize demand response in smart grid, in this paper we established a household microgrid model which includes PV, battery and home appliances, and then we proposed an intelligent coordination control scheme for home energy management. For the thermostatically controlled appliances (TCA), an optimization based on Nash bargaining solution of game theory is applied to make a tradeoff between operating cost and satisfaction. For other control variables, like the charge and discharge of battery and the energy trading among external grid, these can be calculated through optimization aim at minimizing the total cost of the household photovoltaic microgrids system. Finally, in the numerical example, we calculate the related variables and the optimization results; moreover, a comparison between traditional optimization shows that the household electricity optimization in our scheme is effective and humanize for reducing the operation cost and achieving demand response.

## Introduction

The increase of social energy consumption and the diminishing of fossil fuel have made it necessary to improve the efficiency of energy consumption for home. The smart household photovoltaic microgrids which benefit from the rapid development of household photovoltaic technology, are flexible, reliable, and have a strong ability to accept photovoltaic energy. It not only can solve the photovoltaic inherent instability, but also can realize the shift of microgrids from passive to active network through the integration of management of the distributed generation (DG) and loads. Meanwhile, the flexibility of the control of heating, ventilation, and air conditioner (HVAC) systems make the optimal control of HVAC become an important means to realize demand response<sup>[1]</sup>. These features all can be helpful in maintaining a balance between power supply and demand<sup>[2]</sup>. Therefore, integrating household photovoltaic system and carrying on effective control could be a new way to improve the efficiency of electricity consumption in demand side, then the residential intelligent electricity consumption can be achieved.

In this paper, we presented a coordination control scheme for household photovoltaic microgrids, which was aimed at laying down operating rules for home appliances by analyzing the game relationship between economics and satisfaction for the use of TCA, and calculated the solution that satisfied Nash Equilibrium. Then, through dispatching storage battery and the energy between microgrid and external grid, household photovoltaic microgrids' economic optimal operation can be realized.

## Household Photovoltaic Microgrids System

### Home Energy Management System for Household

The household photovoltaic microgrid in this paper includes DG (photovoltaic arrays), energy storage device (storage battery), external grid, smart home appliances and the relevant intelligent controller. Based on intelligent interaction technology and AMI, the energy management can integrate local control system, smart meter and smart interactive terminals and then achieve the control and management of the system. As shown in Fig. 1.

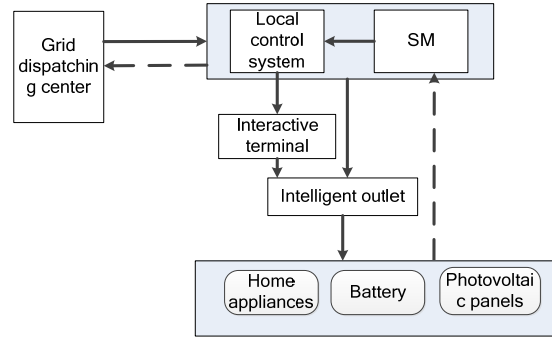


Fig. 1 The energy management system for household photovoltaic microgrid.

## Loads Modeling

For loads, depending on the consumption patterns and user requirements, general electrical load can be divided into fixed load, schedulable load and transferable load. Among them, the fixed load changes naturally according to the needs of life and work, with mandatory and randomness, cannot participate in demand response; the schedulable load, especially the TCA, which usually runs for a long time, affected by temperature and climatic factors greatly and its power is adjustable even can be intermittently interrupted; the transferable load whose power is not easy to adjust, but its work time interval can be scheduled within a certain time range. So it can be used as the active load to participate in the system's demand response<sup>[3]</sup>. In this paper we mainly discussed HVAC system which contained air conditioner and water heater as schedulable load. In fact, the battery also can be considered as a kind of transferable load, due to its similar characteristics as electric vehicle (EV).

For air conditioner, because human's temperature sensitivity is limited, as long as temperatures within a certain range, users' feeling will not be affected greatly, so air conditioner belongs to schedulable load. We assume the air conditioner is inverter air conditioner, so its power can be varied within a certain range. The power of air conditioner is associated with the set temperature, outside temperature and housing area and other factors, in order to simplify the model, the relation between the indoor temperature and the power of air conditioner can be expressed as the following linear dynamical mode<sup>[4]</sup>:

$$T_{in}(h) = T_{in}(h-1) + \lambda_1(T_{out}(h) - T_{in}(h-1)) + \lambda_2 P_{ac}(h) \quad (1)$$

where  $\lambda_1$  and  $\lambda_2$  are parameters that specify the thermal characteristics of the appliance and the environment in which it operates;  $P_{ac}(h)$  is the power of air conditioner;  $T_{in}(h)$  is the indoor temperature controlled by air conditioner;  $T_{out}(h)$  is the outdoor temperature.

For electric water heaters, it is acceptable as long as the water temperature is not too much deviation from the optimum temperature, so electric heaters are also adjustable load, and because the water has the ability to maintain temperature in a certain degree, so the water heater also have the characteristics of transferable load<sup>[5]</sup>. According to the law of conservation of energy and thermal knowledge, in consideration of indoor temperature, hourly water volume demand and temperature demand, the model can be expressed as Eq.(2)-(3):

$$c\rho V_0 T(h+1) = (1 - \beta_w) \{ \psi P_{eh}(h) \Delta h + c\rho [(V_0 - F_{eh}(h))T(h) + F_{eh}(h)T_r] \} \quad (2)$$

$$F_{a.eh}(h) = F_{eh}(h) \cdot \min \left\{ \frac{T_{s.eh}(h)}{T(h)}, 1 \right\} \quad (3)$$

Where  $c$  is the specific heat of water;  $\rho$  is the density of water;  $V_0$  denotes the volume of tank;  $\beta_w$  is the heat loss coefficient;  $\psi$  is the conversion factor between electric energy and heat;  $P_{eh}(h)$  is the power of water heater;  $T_r$  is the inlet water temperature;  $T_{s,eh}(h)$  and  $F_{eh}(h)$  are the empirical value of temperature and volume of the water used in every hour;  $T(h)$  and  $F_{a,eh}(h)$  are the actual value of temperature and volume of the water.

In actual use, when the water temperature is higher than the demand temperature, users will adjust the water temperature by mixing cold water, which will reduce the demand of hot water, so we make amendment for the volume demand of water as Eq.(3).

In the system, battery is also a important controllable equipment. The energy stored in battery is related to its value in previous time, when charging the model can be expressed as:

$$E_b(h+1) = (1-\tau)E_b(h) + P_{b,eh}(h)\eta_{ch}\Delta t \quad (4)$$

When discharging the model can be expressed as:

$$E_b(h+1) = (1-\tau)E_b(h) - \frac{P_{b,dis}(h)}{\eta_{dis}}\Delta t \quad (5)$$

Where  $E_b(h)$  refers to the energy stored in the battery at time period  $h$ ;  $P_{ch}(h)$  and  $P_{dis}(h)$  to charge and discharge power, respectively;  $\eta_{ch}$  and  $\eta_{dis}$  to charge and discharge efficiencies, respectively; and  $\tau$  to the hourly self-discharge decay<sup>[6]</sup>.

For the photovoltaic equipment, its model is not described in this paper and a set of empirical power data in a typical summer day is used to characterize the fluctuant photovoltaic power.

## Optimization Model

### Active Loads' Economic Model and Satisfaction Model

For the use of air conditioner, in considering the real-time pricing, we build economic index  $C_{ac}$ :

$$C_{ac} = \sum_{h=1}^{24} eb(h) \times P_{ac}(h) \quad (6)$$

Because the indoor temperature is user's direct experience of the use of air conditioner, so the user's satisfaction model  $S_{ac}$  should be a function about temperature<sup>[7]</sup>:

$$S_{ac} = \sum_{h=1}^{24} f_{ac}(T(h) - T_{ac,com}) \quad (7)$$

$$f_{ac}(\Delta T_{ac}(h)) = \begin{cases} a_1(\Delta T_{ac}(h))^2 + b_1 & |\Delta T_{ac}(h)| \leq 5 \\ 0 & otherwise \end{cases} \quad (8)$$

Constraints:

$$0 \leq P_{ac}(h) \leq P_{ac,max} \quad (9)$$

$$T_{ac,com} - 5 \leq T(h) \leq T_{ac,com} + 5 \quad (10)$$

Where  $eb(h)$  is the price of purchasing electricity;  $P_{ac,max}$  is the maximum power of air conditioner;  $T_{ac,com}$  denotes the most comfortable indoor temperature; the function  $f_{ac}(\Delta T_{ac})$  is a quadratic function about temperature as Eq.(8); when the temperature difference is 0, the function value reaches its maximum ( $b_1$ ) and when the temperature reaches beyond the constraints ( $5^\circ\text{C}$  higher than  $T_{ac,com}$  or  $5^\circ\text{C}$  lower than  $T_{ac,com}$ ), the function value is zero.

Similarly, we can get the economic index and satisfaction index for electric water heater:

$$C_{eh} = \sum_{h=1}^{24} eb(h) \times P_{eh}(h) \quad (11)$$

$$S_{eh} = \sum_{h=1}^{24} f_{eh}(T_{s.eh}(h) - T_{eh}(h)) \quad (12)$$

$$f(\Delta T_{eh}(h)) = \begin{cases} a_2(\Delta T_{eh}(h))^2 + b_2 & T_{eh}(h) \leq T_{s.eh}(h) \\ b_2 & T_{eh}(h) > T_{s.eh}(h) \end{cases} \quad (13)$$

Constraints:

$$0 \leq P_{eh}(h) \leq P_{eh.max} \quad (14)$$

$$T_{s.eh}(h) - 10 \leq T_{eh}(h) \leq T_{eh.max} \quad (15)$$

Where  $P_{ac.max}$  is the maximum power of water heater;  $T_{eh.max}$  denotes the water heater's maximum desired temperature;  $T_{s.eh}(h)$  is the user's demand temperature for hot water at time period  $h$ ;  $f_{eh}(\Delta T_{eh})$  is a quadratic function on the difference between actual temperature and demand temperature as Eq.(13). In the range of constraints when the actual temperature reaches (or exceeds) the needs of the water temperature, the value of function reaches its maximum ( $b_2$ ), and as the actual temperature dropped, the function value decreases and when it drops to the minimum temperature requirement the function value reaches the minimum (zero).

**Economic Optimization Model for Whole System.** In addition to the home appliance load, there are also photovoltaic equipment and energy storage device in household photovoltaic microgrids. In order to take full advantage of energy storage to allocate energy supply rationally, and allow users to obtain more economic benefits, our goal should be minimizing the operate cost for users, and regarding energy balance and power limits for DG and transmission lines as constrains.

To achieve the economical optimization of whole system, the objective function of optimal control for our system should be the operation cost which includes the cost of the electricity purchased from external grid, the earnings achieved through selling the electricity to external grid, the maintenance cost and depreciation loss of storage battery:

$$C_{sys} = \sum_{h=1}^{24} eb(h)P_{gb}(h) - es(h)P_{gs}(h) + \sum_{h=1}^{24} \beta_b(P_{b.ch}(h) + P_{b.dis}(h)) \quad (16)$$

Where  $P_{gb}(h)$  and  $P_{gs}(h)$  are the power purchased from external grid and sold to external grid respectively;  $es(h)$  is the price of selling electricity;  $\beta_b$  is the coefficient of depreciation for battery's operation<sup>[8][9]</sup>.

Constrains for battery and external grid:

1)Power balance for the system:

$$P_{gb}(h) - P_{gs}(h) + P_{b.dis}(h) - P_{b.ch}(h) = P_l(h) \quad (17)$$

$$P_l(h) = P_{eh}(h) + P_{ac}(h)$$

2)Constraints for battery's capacity and power:

$$E_{b.min} \leq E_b(h) \leq E_{b.max} \quad (18)$$

$$0 \leq P_{b.dis}, P_{b.ch} \leq P_{b.max} \quad (19)$$

Where  $E_{b.max}$  and  $E_{b.min}$  refer to the maximum and minimum capacity of the storage battery;  $P_{b.max}$  represent the maximum value of the charge and discharge power.

3)Power constraints for transmission lines:

$$0 \leq P_{gb}(h) \leq P_{gb.max} \quad (20)$$

$$0 \leq P_{gs}(h) \leq P_{gs.max} \quad (21)$$

Where  $P_{gb.max}$  and  $P_{gs.max}$  refer to the input and output power limitation of the transformer which connects the external grid.

## Problem Formulation

### Multi-objective Optimization Based on Bargaining Game

For the household appliances with characteristics of active loads, their economic index and satisfaction index affect each other. In most literature, when dealing with such multi-objective optimization problem, they often use weighted method to integrate multi-objective into one objective problem, but since the weight coefficient is set by user artificially, the method has subjectivity. However, sometimes it is needed to know how much satisfaction is compromised in order to improve economic index, or vice versa. Thus it can offer further detailed data support for the use of home appliances. So it is necessary to establish quantitative analysis and dynamic analysis for the household appliances management. In this paper, we regard these two optimization goals as two bargaining players, and use the bargaining game theory to solve the multi-objective optimization problem.

The Nash bargaining game problems can be described as the following maximization problem:

$$\max(f_1(x) - d_1)(f_2(x) - d_2) \quad (22)$$

Where  $f_1$  and  $f_2$  are utility functions of two players,  $d_1$  and  $d_2$  denote the possible worst payoff of each player. According to the above Nash axioms, we can know that the negotiate solution should be as far away from the worst payoff, and it must on the Pareto frontier, as Fig. 2.

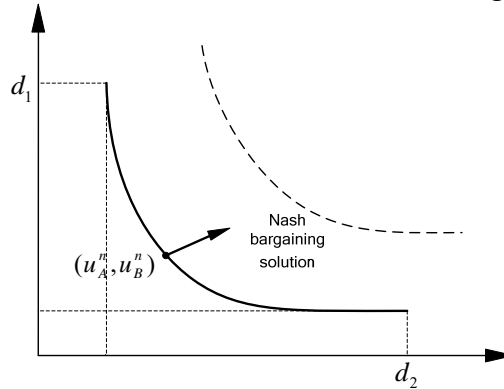


Fig. 2 Relationship between two players in the bargaining game and the Nash bargaining solution.

### Optimization for TCA Considering Satisfaction and Economy

In order to balance the economic index and satisfaction index, the bargaining game model was used to find the Nash equilibrium solution. It is equivalent to solving the following quadratic optimization problem (used the water heater as example):

$$\max(C - C_{eh.\max})(S - S_{eh.\min}) \quad (23)$$

Where  $C_{eh.\max}$  and  $S_{eh.\min}$  denote the maximum economic index and minimum satisfaction index respectively. These two values can be calculated through maximizing the economic index ( $C_{eh}$ ) in Eq. (11) and minimizing the satisfaction index ( $S_{eh}$ ) in Eq.(12). Eq.(14) and Eq.(15) are the constraints of optimization.

For the quadratic optimization problem about Eq.(20), when there are lots of variables, it is difficult to find the optimal solution, but due to the optimal solution must be on the Pareto frontier, we can use the method in the literature [11] to solve the optimization problem. First, see the economic index as objective function to minimize it. After obtaining the minimum economic index value, this optimization index could be regard as a constraint and gradually relax it by a certain step. Used the Genetic Algorithm (GA), the maximum satisfaction values under different economic indexes could be obtained. It can be described as follows:

$$S = \max S_{eh} \quad (24)$$

$$0 \leq P_{eh}(h) \leq P_{eh.max} \quad (25)$$

$$T_{eh}^s(h) - 10 \leq T_{eh}^s(h) \leq T_{eh.max}^s \quad (26)$$

$$\sum_{h=1}^{24} eb(h) \times P_{eh}(h) \leq (1 + \alpha) C_{eh.min} \quad (27)$$

Where  $\alpha$  is the restraint coefficient of economic index. In each time period the value of  $\alpha$  changes, a new maximum satisfaction value will be obtained. When we get enough points to represent the Pareto front of this multi-objective optimization, the point that satisfies Nash equilibrium (Eq. (23)) could be found and the powers correspond to this point are the control variables for the optimization.

### Optimization for the Operation of Battery

After get the operation information of TCA in household microgrid, the control variables in household photovoltaic microgrid would just have the powers of battery and the powers between microgrid and external grid. By minimizing the objective function for whole system (Eq.(16)), combining with the required operational constraints (Eq.(17~21)), we can get the scheduling information about battery and external grid. For the reason of there are switch variables which can characterize the state of charging and discharging of battery and the direction of power flow between microgrid and external grid, the Mixed Integer Linear Programming (MILP) algorithm<sup>[12]</sup> is used to find a solution.

### Simulation Results

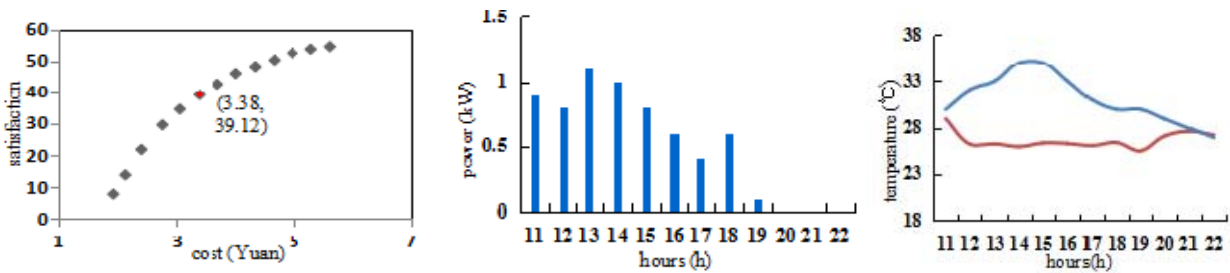


Fig. 3 Analysis and related results about the control of air conditioner.

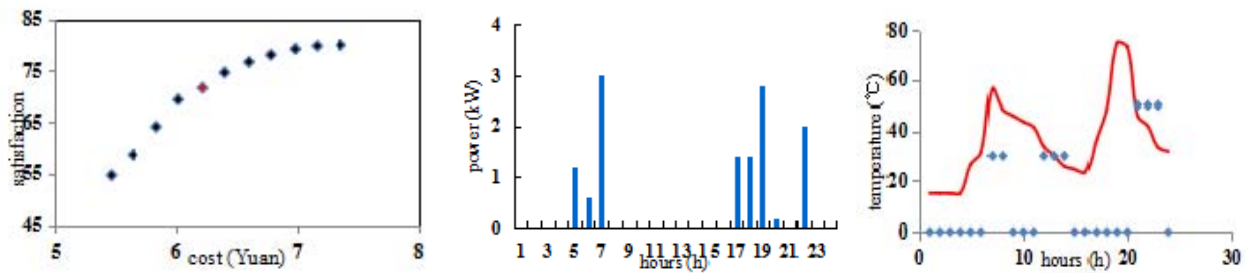


Fig. 4 Analysis and related results about the control of water heater.

In Fig. 3, there are three sub-graphs about the data of the air conditioner's operation. The left panel shows the analysis of the relationship between cost and satisfaction, the red point is the Nash bargaining solution that satisfies Eq. (20), and the power that correspond to the red point are shown in the mid panel; in the right panel, it shows the outdoor temperature (blue line) and the indoor temperature (red line) after optimal control, from the graph of temperature, it shows that the electricity cost reduced through sacrifice a part of satisfaction. So the indoor temperature does not



reach the optimum temperature of 24 °C, but the overall control effect is good, it remained at the national guidance recommended summer indoor temperature 26 °C.

In Fig. 4, the left panel shows the analysis of the relationship between cost and satisfaction about the operation of water heater; the power of water heater is shown in the mid panel; in the right panel, it shows the water temperature (red line) and the set temperature (blue point) in each hour, we can see that after optimization control, the electric water heater consumes less power during the high price periods in day, in the evening when water consumption and the water temperature demands are relatively high, the electric water heater consume power consumption is relatively more, the water temperature is controlled in line with the user's needs.

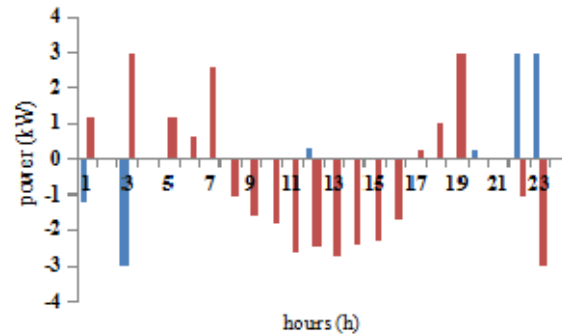


Fig. 5 Power about battery and external grid.

Fig. 5 depicts the charge and discharge power of the storage battery (the blue bar) within 24 hours after the economical optimization as well as the power between home microgrid and grid (the red bar). As shown in the figure, under the premise to meet the load demand, in low price periods system purchase more electricity from the grid, in high price periods battery supply to the load more. Because in the case of considering loss of depreciation, in periods of higher purchasing electricity price, battery discharge can get more revenue. For instance, if the battery discharges 1kWh to loads at low price periods, it can save 0.15 Yuan, but if the battery discharges 1kWh to loads at high price periods, it can save 0.75 Yuan. So it can be seen that in our scheme the storage battery is controlled reasonably to avoid the frequent charge-discharge thereby taking into account the battery life. According to calculation, for our system the user can earn 7.393 Yuan one day.

Compared to other optimization problem for home energy management, the most obvious difference is that we adopt game theory to solve the multi-objective optimization, so that we can make quantitative and dynamic analysis for the management of household appliances.

In order to reflect the advantages of our scheme, we establish another traditional optimization model which minimizes the single-objective function Eq. (21), it contains the cost of system and dissatisfaction of appliances' operation (we see the negative number of satisfaction as dissatisfaction). Then, the genetic algorithm (GA) is applied to solve this model.

$$\begin{aligned} \max \quad & \alpha C_{sys} - \beta S_{eh} - \gamma S_{ac} \\ \text{subject to} \quad & \text{operating constrains} \end{aligned} \quad (26)$$

Where,  $\alpha$ ,  $\beta$  and  $\gamma$  are weight coefficients, in this calculation, all their values equal to 1.

TABLE 1 Comparison between two scheme

	Earn	Sac	Seh
Our scheme	7.393	39.12	71.73
Traditional scheme	14.172	3.77	77.83

In Table 1, results of two schemes are compared. It could be find that, our scheme can balance the economy and the satisfaction on the active loads. Although the revenue in single-objective

optimization is more than the revenue in our scheme, it is obvious that the satisfaction of air conditioner is very low, that is not reasonable for any user. Theoretically, it can make the results in single-objective optimization more reasonable by changing the coefficient, but it is difficult to choose the specific values of them, and in some cases it can't find the correct solutions because of the complex constraints.

## Conclusions

In this paper, we consider the household photovoltaic microgrid system as our study objective, in order to make full use of home appliances to the demand response, game theory is used to analyze the relationship between the economy and satisfaction of the active loads' operation, thus based on the above analysis we can make home appliances' operate plans. Then, in order to achieve the economical optimization of whole system, we see operation cost as objective function, and use the mixed integer linear programming method to calculate the control variables on the operation of DGs and external grid. In the numerical example, it can be shown that the energy dispatching of all components is reasonable under our scheme, and compared to traditional method, our scheme can balance the economic and the satisfaction, and is more close to the practical application, and make it possible to achieve the personalization of the home appliances management.

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## References

- [1] Aswani, A., et al. "Reducing Transient and Steady State Electricity Consumption in HVAC Using Learning-Based Model-Predictive Control." *Proceedings of the IEEE* 100.1(2012):240-253.
- [2] Pourmousavi, S. A., and M. H. Nehrir. "Demand response for smart microgrid: Initial results." *Innovative Smart Grid Technologies (ISGT), 2011 IEEE PESIEEE*, 2011:1 - 6.
- [3] Choi S, Park S, Kang D J, et al. A microgrid energy management system for inducing optimal demand response// *Smart Grid Communications (SmartGridComm), 2011 IEEE International Conference on IEEE*, 2011:19 - 24.
- [4] Li, Na, L. Chen, and S. H. Low. "Optimal demand response based on utility maximization in power networks." *Power and Energy Society General Meeting, 2011 IEEEIEEE*, 2011:1-8.
- [5] Nehrir, M. H., B. J. Lameres, and V. Gerez. "A customer-interactive electric water heater demand-side management strategy using fuzzy logic." *Power Engineering Society 1999 Winter Meeting, IEEEIEEE*, 1999:433-436 vol.1.
- [6] Chen, S. X, H. B. Gooi, and M. Q. Wang. "Sizing of Energy Storage for Microgrids." *Smart Grid IEEE Transactions on* 3.1(2012):142-151.
- [7] Tsui, K. M., and S. C. Chan. "Demand Response Optimization for Smart Home Scheduling Under Real-Time Pricing." *IEEE Transactions on Smart Grid* 3.4(2012):1812-1821.
- [8] Lennard, Mitchell, and A. Date. "Two stage stochastic optimisation of highly distributed PV/Battery microgrids with grid connection." *Smart Grid Technologies - Asia (ISGT ASIA), 2015 IEEE Innovative IEEE*, 2015.
- [9] Lemaire-Potteau, Elisabeth, et al. "Assessment of storage ageing in different types of PV systems: technical and economical aspects." *23rd European Photovoltaic Solar Energy Conference (Valencia, Spain, 2008). 2008*.
- [10] Montet, Christian, and Daniel Serra. *Game theory and economics*. Basingstoke, UK: Palgrave macmillan, 2003.



- [11] Chen, Liang, et al. "Multi-objective long-term generation scheduling based on two person-zero sum games." Control Conference (CCC), 2012 31st ChineseIEEE, 2012:6958-6962
- [12] Hu C, Luo S, Li Z, et al. Energy Coordinative Optimization of Wind-Storage-Load Microgrids Based on Short-Term Prediction. Energies, 2015, 8(2):1505-1528.