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Safety Analysis of Lateral Interval between Military Training Airspace and Civil Route

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Abstract. in order to reasonably optimize the military training airspace and ensure the safety interval with civil aviation routes, the paper adopts the improved ellipsoid Event model to evaluate the lateral separation between military training airspace and civil aviation routes. After the lateral position deviation probability model for the military and civil aircraft is established, the paper carries out analysis and research from three aspects which are respectively are initial circling points of fighter, the speed and the artificial operation error of slope. And then it estimates the deviation probability of the lateral position for the military aircraft. Combined with the deviation probability model of civil aviation, finally, the safety interval of 10 kilometers between military training airspace and enroute boundary is assessed. The simulation results show that the improved model is effective and can provide the theoretical basis for the calculation of the safety interval for military and civil aviation.

Introduction

In recent years, with the rapid development of civil aviation, the air traffic flow is increasing rapidly. Moreover, with the renewal of weapons and equipment of the Air Force, constantly innovation of the training subjects and the demand for training airspace being continuously increased, the contradiction of demand for airspace resources of military and civil aviation is worsened continuously. On the premise that it is in line with the standard of military and civil aviation collision, reducing the range of training airspace of individual subjects is an effective way to solve the contradiction between the civil and military aviation. In 1960s, *Reich* proposed the REICH collision risk model[1] for the first time, which was used to analyze the safety of air traffic route structure. In 2003, *Peter Brooker* proposed the Event model[2] which is used to evaluate the risk of lateral collision. With the basic model of a variety of collision risks being proposed, many scholars at home and abroad have conducted in-depth research and put forward a variety of improved models[3][4]. However, the above research models only involves the collision probability of civil aircraft and there are few literature about the collision probability between the military aircraft and civil aviation. This paper will use the Event model to build the corresponding models and calculate the risk of collision between the civil aircraft and the military aircraft in the adjacent air route.

Principle and improvement of ellipsoid Event model

The collision box defined in the Event model is too conservative. In fact, as the body and wingspan are of different lengths, when the two aircrafts are approaching with each other, they will generally avoid collision by adjusting the vertical height difference. The space formed by the transverse deflection of the aircraft wings is closer to the ellipsoid as shown in the following figure.

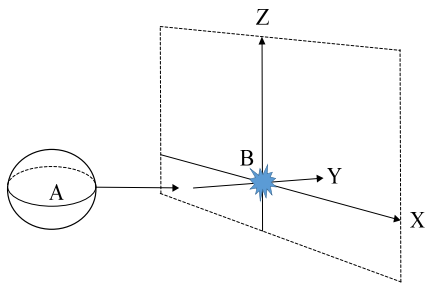

Fig.1 The schematic diagram of improved collision box

The aircraft A is the center; *a* is the long half shaft, *b* is the short half shaft, *h* is the polar radius (a, b, h are respectively the aircraft's fuselage length, wingspan and body height). When the aircraft B is just on the ellipsoid edge of the aircraft A, the two aircrafts have body contact, which is equivalent to that there is absolute collision between aircraft B and A. The collision probability of the same height layer in the paralleled aviation route is:

$$N_{ay} = GERh\frac{E(S)}{2L}\left(4a+\frac{u4b}{v}\right)P_z(0)\left(1+\frac{w4b}{v4h}\right) \tag{1}$$

$GERh$ is frequency of losing the lateral spacing within each hour, L is longitudinal spacing standard, $E(S)$ is the number of aircrafts which are flying in the same direction within $2L$ distance, u , v , w are the relative velocity in the longitudinal, lateral and vertical direction of the interval time when aircraft A goes through the aircraft B in the same flying direction. $P_z(0)$ is the vertical overlap probability for two aircrafts with the same height layer.

The frequency of the aircraft A when going through the lateral interval layer is related to the performance navigation accuracy and other factors while it has nothing to do with the shape of the collision box. And the probability of the aircraft B in the extended collision box is proportional to the area of the extended collision box. As the collision risk is the product of frequency for aircraft A to go through the interval layer and the probability for aircraft B which is within the extended box collision, the improved collision probability can be obtained by calculating the proportion of the projected area of the collision box on the interval plate before and after the improvement.

The rectangular EGIK in the figure is the original rectangular extension collision box and the extended collision box after improvement is the diagram enclosed by *CMNSRQPD* (as shown in the shadow area). *R(O)* is the ratio between the area of the expanded collision box S_2 after improvement and the area of the expanded collision box before the improvement.

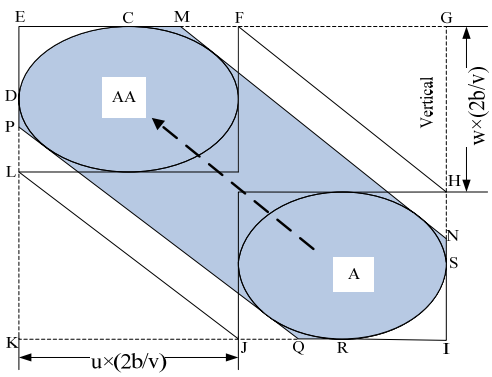

Fig.2. The expend collision box

The area of the collision box before improvement is:

$$S_1 = S_{EGIK} = KI \times GI = (u\times\frac{2b}{v}+2a)\times(w\times\frac{2b}{v}+2b) \tag{2}$$

The area of the collision box after improvement is:

$$\begin{aligned} S_2 &= S_{CMNSRQPD} = S_1 - S_{\Delta MGN} - S_{\Delta PQK} - S_{ECD} - S_{RSI} \\ &= \frac{3(abw+b^2u)}{v} + (\frac{1}{2}\pi+\frac{7}{4})ab \end{aligned} \tag{3}$$

According to the literature[5], the ratio of the area of the extended collision box after improvement is:

$$R(S/O)=\frac{S_2}{S_1}=\frac{\frac{3(abw+b^2u)}{v}+(\frac{1}{2}\pi+\frac{7}{4})ab}{4(\frac{uwb^2}{v^2}+\frac{abw+b^2u}{v}+ab)} \tag{4}$$

Based on the conclusions in the above content, the collision probability N'_{ay} of the improved Event model after improvement by using the *R (S/O)* to multiply the formula (1).

$$\begin{aligned} N'_{ay} &= N_{ay}\times R(S/O) \\ &= GERh\frac{E(S)}{2L}\left(4a+\frac{u4b}{v}\right)P_z(0)\left(1+\frac{w4b}{v4h}\right) \\ &\times\frac{\frac{3(abw+b^2u)}{v}+(\frac{1}{2}\pi+\frac{7}{4})ab}{4(\frac{uwb^2}{v^2}+\frac{abw+b^2u}{v}+ab)} \end{aligned} \tag{5}$$

Problem description

In the Event model, *GERh* is the frequency of the collision box passing through the interval spacer. In civil aviation operation, it can be used to replace the statistical frequency lost by the lateral interval within the unit time. However, there is no corresponding statistical data on the collision risk between military aircraft and civil aviation. However, it can be estimated by the lateral position deviation probability model and the lateral position deviation probability model of the civil aircraft.

Hypothesis description

In equation (5), parameter $GERh$ is the statistical data while such work is a lack between the military and civil aviation. And because in the same height of the parallel aviation route, the aviation direction of the aircraft is the same while direction of military aircraft flight in the same height of training course is not fixed, the Event model can not be directly used to calculate the collision risk of civil and military aviation. According to the flight characteristics of military and civil aviation, Event must be transformed.

Firstly, the prerequisite conditions of the model are explained:

(1) The airspace conditions of the study are that the aviation route and the training airspace are set up in parallel; the height of training airspace center is the same as the route center line and the training space and the route meet the 10km interval standard.

(2) Military aircrafts and civil aircrafts are independent of each other in terms of their positions.

(3) The probabilities for the military and civil aviation aircrafts flying in the same direction and the opposite direction are the same.

(4) Military aircrafts fly along the edge of the airspace as it mainly considers the spiral on the slope 45 degrees.

Reference coordinate system

In the ideal state, the center coordinates of track for the hovering training is the coordinate center O. The positive direction of X axis after passing O point will point to the side of the aviation route which is vertical to the training airspace boundary where the rectangular coordinate system is established. At this point, the boundary equations of the airspace is $x=r_0$. At this point, the the equation of the civil aviation route center is $x=r_0+20km$.

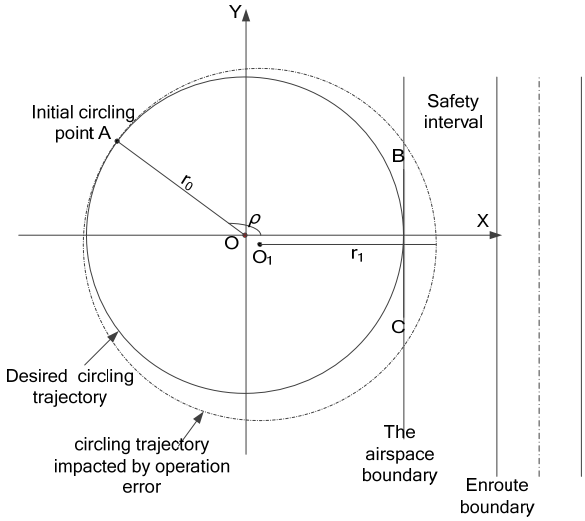

Fig.3. the schematic diagram of lateral safety interval between military training airspace and enroute boundary

The military aircraft A is defined as a rectangular collision box and the long and short half shafts and polar radius are respectively: $\frac{\lambda_{mx}+\lambda_{jx}}{2}$, $\frac{\lambda_{my}+\lambda_{jy}}{2}$, $\frac{\lambda_{mz}+\lambda_{jz}}{2}$.Among them, λ_{mx}、λ_{my}、λ_{mz} and λ_{jx}、λ_{jy}、λ_{jz} are respectively fuselage length, wingspan and body height of civil aviation and military aviation aircraft. In this way, the civil aviation aircraft B can be regarded as a point. When the point A contact with the collision box B, you can think that A and B have collided with each other. Among them:

B is regarded as the starting point to establish the rectangular coordinate system, the X axis is flight direction of B along the route, Y is the horizontal aircraft perpendicular to the X axis and the Z axis is perpendicular to the XOY aircraft being upper. The spacer is defined by a B which regards B as the starting point and it is the aircraft determined in the longitudinal and vertical directions. An extended collision box is extended which is the position which is passed through by the interval spacer.

In this way, the Event model can be rewritten as:

$$\begin{aligned} N'_{qy} &= GERh\frac{\alpha 2E(S)}{2L}\left(2\lambda_x+\frac{u_s 2\lambda_y}{v_s}\right)P_z(0)\left(1+\frac{w_s 2\lambda_y}{v_s 2\lambda_z}\right)\times R(S) \\ &+GERh\frac{(1-\alpha)2E(O)}{2L}\left(2\lambda_x+\frac{u_o 2\lambda_y}{v_o}\right)P_z(0)\left(1+\frac{w_o 2\lambda_y}{v_o 2\lambda_z}\right)\times R(O) \end{aligned} \tag{6}$$

Among them, α is the probability of the military aircraft and civil aircraft flying in the same direction; $1-\alpha$ is the probability of the military aircraft and civil aircraft flying in the opposite direction. $2E(S)$ is the number of the civil aviation aircraft within the 2L distance. u_s, v_s, w_s are the relative speeds in longitudinal, lateral and vertical directions of the two aircrafts when A flies through the interval spacer of B when they are flying the same direction. u_o, v_o, w_o are the relative speeds in longitudinal, lateral and vertical directions of the two aircrafts when A flies through the interval spacer of B when they are flying the opposite direction. Reasonable assumptions can be made: $v_s = v_o$, $w_s = w_o$

Some of the parameters in the formula have small degree of changing and the impact risk of relative collision is limited in the safety assessment of each area. The values of these parameters are referred to the relevant literature. This paper focuses on the calculation method of the lateral overlap probability and the length of the aircraft.

Probability model of lateral position deviation of civil aircraft

The probability density function of lateral yaw error is $f_y(y_1)$. y_1 is the distance from the route center line of aircraft. Yaw probability density function is $f_{norm\_y}(y_1)$. That is to say:

$$f_{norm\_y}(y_1) = \frac{1}{2a_1}\exp(-\left|\frac{r_0 + 20 - y_1}{a_1}\right|) \tag{7}$$

a_1 is the corresponding parameters of the probability density function for the general yaw error (caused by navigation accuracy). This parameter can be determined by the RNP value. N of the RNP value refers to that the aircraft flies in n sea miles range onthe both side of the route with 95% of the probability. According to different RNP values, the parameters a_1 can be calculated in the corresponding navigation conditions. According to the calculation of the reference literature [6], when the RNP value is 4 and a_1 is 1.33.

Probability model of lateral position deviation of military aircraft

Calculation of probability density function of turning radius

The movement error of the pilots in the military training flight follows the normal distribution[7]:

$$\gamma(M_{last}) = \frac{1}{\sqrt{2\pi}\sigma}\exp(-\frac{(M_{last} - M_{should})^2}{2\sigma^2}) \tag{8}$$

$$M_{last} = M_{should} + \triangle M_{error}$$

Among them: M_{should} is the expected movement, M_{last} is the final movement, $\triangle M_{error}$ is the movement error, σ is the standard error.

The corresponding relationship between the pilot's final action and the expected action is in line with the normal distribution.

This chapter uses hovering as the example to calculate the training flight deviation. In the process of hovering, the factors affecting the lateral deviation of the aircraft are mainly speed, flight direction, turning gradient γ, lateral wind M, the starting point of hovering $A(x_A, y_A)$ and its positioning accuracy, positioning accuracy GPS. As shown in figure 1, assuming the aircraft starts hovering from point A in a clockwise direction, the expected hovering trajectories are shown as solid lines. Under the influence of the movement error and the direction of the wind, the hovering trajectory is shown as the two spotted lines. The center of the circle *O(0,0)* of the expected hovering trajectory is used as the center to build the coordinate system, X axis is perpendicular to the spatial outward side of the boundary. The relationship between the radius of the turning and the speed of the aircraft is calculated according to the formula of turning radius:

$$r_0 = \frac{v^2}{g\tan\gamma} \tag{9}$$

The relationship between the spiral starting point (x_A, y_A) and r_0 is :

$$\begin{cases} x_A = K_1 \times r_0 \\ y_A = K_2 \times \sqrt{r_0^2 - x_A^2} \end{cases} \tag{10}$$

Among them, $K_1 \in [-1,1]$, K_2 is 1 which is the positive half axis of Y axis and when it is -1, it is negative half axis of Y axis.

Assuming that the expected speed of hovering of the aircraft is *v=900km/h*, the expected turning gradient is 45°, and the slope error is $\triangle\gamma$, both of them are affected by the pilot's action error, so it is in line with the normal distribution given by equation (8). Under the influence of the speed error and turn gradient error, the turning radius is:

$$r_1 = \frac{(v + \triangle v)^2}{g\cdot\tan(\gamma + \triangle\gamma)} \tag{11}$$

It is assumed that the speed error and the turning gradient are independent random variables

In this way, the probability density functions of turning radius can be expressed as:

$$f_{R_1}(r_1) = \frac{1}{\sqrt{2\pi}\sigma_{\Delta\gamma}}\int_0^{\frac{\pi}{2}}\left\{\sqrt{\pi\left[1 - e^{-\left[\frac{r_1\cdot\tan\Gamma - 250}{\sqrt{2}\sigma_{\Delta V}}\right]^2}\right]} - \sqrt{\pi\left[1 - e^{-\left[\frac{250}{\sqrt{2}\sigma_{\Delta V}}\right]^2}\right]}\right\}\cdot e^{-\frac{\left(\Gamma - \frac{\pi}{4}\right)^2}{2\sigma_{\Delta\gamma}}}d\Gamma \tag{12}$$

Circle center calculation of the turning trajectory

Suppose that the center of the trajectory is $O1(x_1, y_1)$, aviation angle is θ, so the following formula is shown:

$$\begin{cases} x_1 = x_A + r_1 \cos\theta \\ y_1 = y_A - r_1 \sin\theta \end{cases} \tag{13}$$

Assuming that the deviation error of the flight direction is jointly caused by the navigation precision, the error of the airborne navigation equipment and the pilot operation error, it follows the normal distribution in terms of its angle. Based on the conservative number given in the 8168 document by the International Civil Aviation Organization, the standard deviation is: σ_θ=2.6°. According to the results given in the 8168 document, the course error follows normal distribution.

$$f_{norm\_y}(\theta') = \frac{1}{\sqrt{2\pi}\sigma_\theta} \exp\left(-\frac{(\theta'-\theta)^2}{2\sigma_\theta^2}\right) \tag{14}$$

The expressions for the aviation direction angle under ideal conditions are:

$$\theta = \begin{cases} \pi - \arctan\left(\frac{y_A}{x_A}\right) & x_A > 0, y_A \neq 0 \\ 2\pi - \arctan\left(\frac{y_A}{x_A}\right) & x_A < 0, y_A \neq 0 \\ \frac{\pi}{2} & x_A = 0, y_A = r_0 \\ -\frac{\pi}{2} & x_A = 0, y_A = -r_0 \\ \pi & x_A = r_0, y_A = 0 \\ 0 & x_A = -r_0, y_A = 0 \end{cases} \tag{15}$$

The original flight path value ΔL of the military aviation deviation is

$$\Delta L = x_1 + r_1 - r_0 \tag{16}$$

The horizontal position of military aircraft is j

$$j = x_1 + r_1 \tag{17}$$

Probability model of lateral position deviation of military and civil aviation

The horizontal coordinate of the departure point of the military aviation flight is compared to the coordinate value of the deviation point of civil aviation. If it is less than zero, the lateral collision occurs, and then the percentage of the points which are less than zero taking the total data is calculated, the error probability *GERh* of the lateral deviation of the civil and military aviation can be obtained.

$$\begin{aligned} \Delta L' &= c - j \\ &= c - (x_1 + r_1) \end{aligned} \tag{18}$$

The horizontal position of civil aircraft is c

Test results

In order to verify the effectiveness of the calculation method of the lateral interval between the civil and military aircraft, the value of *GERh* is simulated,

The starting turning point is set up, the values of K1 and K2 are respectively:

① $\begin{cases} K_1 = 1 \\ K_2 = 1 \end{cases}$ ② $\begin{cases} K_1 = \frac{\sqrt{2}}{2} \\ K_2 = 1 \end{cases}$ ③ $\begin{cases} K_1 = 0 \\ K_2 = 1 \end{cases}$ ④ $\begin{cases} K_1 = -\frac{\sqrt{2}}{2} \\ K_2 = 1 \end{cases}$ ⑤ $\begin{cases} K_1 = -1 \\ K_2 = 1 \end{cases}$ ⑥ $\begin{cases} K_1 = -\frac{\sqrt{2}}{2} \\ K_2 = -1 \end{cases}$ ⑦ $\begin{cases} K_1 = 0 \\ K_2 = -1 \end{cases}$ ⑧ $\begin{cases} K_1 = \frac{\sqrt{2}}{2} \\ K_2 = -1 \end{cases}$

Eight sets of data respectively represent the data points in four quadrant and two coordinate axis.

In order to ensure the reliability of the results in the calculation, the United States F16 fighter and civil aviation A380 aircraft are used as the research objects and the number ④ is used as an

example to carry out calculation. When $\begin{cases} K_1 = -\frac{\sqrt{2}}{2} \\ K_2 = 1 \end{cases}$ according to the literature[8][9] and combining with the above analysis, the relevant empirical data is shown in the following table:

Table.1. The table of simulation parameters

Parameter	Value	Parameter	Value
$E(S)$	0.61	λ_{my}(m)	79.8
$E(O)$	0.01	λ_{mz}(m)	24.1
v_{TAS}/(m/s)	250	λ_{jx}(m)	15.6
γ（°）	45	λ_{jy}(m)	9.96
$\sigma_{\Delta V}$	15	λ_{jz}(m)	4.88
$\sigma_{\Delta\gamma}$	2	u_s（m/s）	3
$P_z(0)$	0.5	u_0（m/s）	497
L(km)	222	v_s/v_0（m/s）	5.23
λ_{mx}(m)	72.8	w_s/w_0（m/s）	0.58

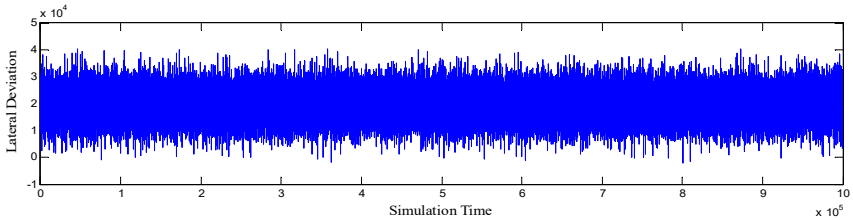

Fig.4. The simulation diagram of lateral deviaton between military and civil aviation aircrafts

We can get the result: $GERh = 3.5070\times10^{-5}$, $R(S) = 0.7681$, $R(O) = 0.6767$ .we can get $N'_{ay} = 7.365\times10^{-9}$ (the time per hour of collision between military and civil aviation aircrafts) by equation (6).

In the same principle, the frequency of losing the lateral interval and the number of collision in each hour corresponded in the rest seven sets of data are respectively:

① $\begin{cases} \text{GERh} = 1.3460\times10^{-5} \\ N'_{ay} = 2.827\times10^{-9} \end{cases}$ ② $\begin{cases} \text{GERh} = 1.3930\times10^{-5} \\ N'_{ay} = 2.927\times10^{-9} \end{cases}$ ③ $\begin{cases} \text{GERh} = 1.8760\times10^{-5} \\ N'_{ay} = 3.940\times10^{-9} \end{cases}$ ⑤ $\begin{cases} \text{GERh} = 5.1490\times10^{-5} \\ N'_{ay} = 1.08\times10^{-8} \end{cases}$

⑥ $\begin{cases} \text{GERh} = 3.4880\times10^{-5} \\ N'_{ay} = 7.325\times10^{-9} \end{cases}$ ⑦ $\begin{cases} \text{GERh} = 1.8470\times10^{-5} \\ N'_{ay} = 3.879\times10^{-9} \end{cases}$ ⑧ $\begin{cases} \text{GERh} = 1.3910\times10^{-5} \\ N'_{ay} = 2.921\times10^{-9} \end{cases}$

Conclusion

1) The loss probability of the lateral interval is only related to the horizontal coordinate of the start turning point A(x_A, y_A). The further the starting turning point to the aviation route is, the greater the deviation will be.

2) ICAO specifies that the target security level is （5×10^{-9} time per hour).Every data of group is in line except the number ⑥. For the training subjects with hovering, maintaining 10 km between the military training airspace and civil aviation routes as the safety interval does not guarantee absolute safety.

Suggestions

1) Because in the training subject of hovering, the probability of getting out of the training airspace and collision with the civil aviation routes is greater, so the personnel should avoid arranging the hovering subjects near the boundary area of the training airspace.

2) In this paper, the speed and slope standard deviation selected in the paper is the standard for

general pilot, so for the pilots who have just flied alone or the pilots whose flying techniques is not good, they should try to avoid a hovering maneuver in the airspace adjacent to the civil aviation routes.

3) For the hovering maneuver for tactical requirements, the hovering starting point circled the starting point should be selected in the area of X positive axis on the side adjacent to the civil aviation route.

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