

Determination of 3-Ary α -Resolution in Lattice-valued Propositional Logic LP(X)

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Abstract

One of key issues for α - $n(t)$ ary resolution automated reasoning based on lattice-valued logic with truth-value in a lattice implication algebra is to investigate the α - $n(t)$ ary resolution of some generalized literals. In this article, the determination of α -resolution of any 3-ary generalized literals which include the implication operators not more than 2 in LP(X). It not only lay the foundation for practical implementation of automated reasoning algorithm in LP(X), but also provides the strong support for α - $n(t)$ ary resolution automated reasoning approaches.

Keywords: Incomparability; lattice implication algebra; Lattice-valued logic; automated reasoning; α – $n(t)$ ary resolution

1. Introduction

As is known to all, one significant function of artificial intelligence is to make computer simulate human being in dealing with uncertain information. And logic establishes the foundation for it. However, certain information process is based on the classic logic. Non-classical logics consist of these logics handling a wide variety of uncertainties (such as fuzziness, randomness, and so on) and fuzzy reasoning. Therefore, non-classical logic has been proved to be a formal and useful technique for computer science to deal with fuzzy and uncertain information. Many-valued logic, as the extension and development of classical logic, has always been

a crucial direction in non-classical logic. Lattice-valued logic, an important many-valued logic, has two prominent roles: One is to extend the chain-type truth-valued field of the current logics to some relatively general lattices. The other is that the incompletely comparable property of truth value characterized by the general lattice can more effectively reflect the uncertainty of human being's thinking, judging and decision. Hence, lattice-valued logic has become a research field and strongly influenced the development of algebraic logic, computer science and artificial intelligent technology. In order to investigate a many-valued logical system whose propositional value is given in a lattice, in 1993, Xu first established the lattice implication algebra by

combining lattice and implication algebra, and explored many useful structures^{9,13}.

As the use of non-classical logics becomes increasingly important in computer science, AI and logic programming, the developing efficient automated theorem proving based on non-classical logic is also an active area of research (e.g., for fuzzy logic and many-valued logic, among others). The essential idea in many of those methods is to transform the resolution algorithm into fuzzy logic and many-valued logic to that of classical logic. To the best of our knowledge, proof theory for lattice-valued logic has so far not been extensively developed. There has also been investigations of resolution-based automated reasoning in lattice-valued logic based on LIA (e.g., among others,^{1,2,3,4,5,6,7,8,12,13,14,15}). The aim of dealing with incomparability leads to the complexity of logical formula in LIA based lattice-valued logic. Correspondingly, the resolution methods in LIA based lattice-valued logic have new features such as (a) resolution is based on generalized literals, which contain constants and implication connectives; (b) resolution is proceeded at a different truth-valued level α chosen from the truth-valued field LIA and the number of resolution generalized literals is fixed at 2 in each resolution in α -resolution deduction. So, the α -resolution is also called α -2 ary resolution; (c) it is not easy to judge directly if two generalized literals are α -resolvent or not, because the structure of generalized literal is very complex. Due to these new features, it is not feasible to apply directly the resolution-based automated reasoning theory and methods in classical logic and in many chain-type many-valued logics into that of lattice-valued logic with incomparability. Hence, an α -2 ary resolution principle for a lattice-valued propositional logic LP(X) has been proposed in^{12,13}, which can be used to prove whether a lattice-valued logical formula in LP(X) is false at a truth-value level α (i.e., α -false) or not, and the theorems of soundness and completeness for the α -2 ary resolution principle were also proved. In addition, the work in¹³ extends the α -2 ary resolution principle for LP(X) to the corresponding lattice-valued first-order logic LF(X).

With the development of research, it shows that

α -2 ary resolution automated reasoning based on lattice-valued logic aiming at processing uncertain information with incomparability is scientific and effective. But there are limitations in α -2 ary resolution automated reasoning in two aspects: (1) α -2 ary resolution can only process the resolution of 2-ary generalized literals; (2) the number of resolution generalized literals is fixed at 2 in each resolution in α -2 ary resolution deduction. These limitations make the α -2 ary resolution automated reasoning theory and applications are limited, and also directly affect the efficiency of α -2 ary resolution automated reasoning. The complexity of lattice-valued logic systems based on LIAs and the logical formulae, will limit the efficiency of α -2 ary resolution automated reasoning. Therefore, it is necessary to study resolution automated reasoning theory, methods, algorithms and procedures which improve the resolution automated reasoning efficiency under the premise of keeping the depict ability in complexity problems. To resolve these limitations, Xu¹⁸ extended the number of resolution generalized literal from 2 to n , and proposed the general form of α -resolution, and the soundness and completeness are also built. In α - $n(t)$ ary resolution, the number $n(t)$ of resolution generalized literals is not fixed at some number, but it will be different in the each resolution, where $n(t)$ means the number of resolution generalized literals in the t th resolution.

In order to study the α - $n(t)$ ary resolution automated reasoning, it is very important to determine if many generalized literals group are α -resolvent or not, it also effect the reasoning process. So, we will especially focus on how to determine if generalized literals group are α -resolvent (i.e., α -resolution) or not.

In this paper, we mainly discuss the α -solution of 3-ary generalized literals which include not more than 2 implication operators. It will be of great use to provide foundation to study α -resolvent of many generalized literals group. Thus, it will be further to lay the foundation on researching α - $n(t)$ ary resolution automated reasoning.

2. Preliminaries

Definition 1.¹⁰ Let (L, \vee, \wedge, O, I) be a bounded lattice with an order-reversing involution $'$, the greatest element I and the smallest element O , and

$$\rightarrow: L \times L \longrightarrow L$$

be a mapping. $\mathcal{L} = (L, \vee, \wedge, ', \rightarrow, O, I)$ is called a lattice implication algebra if the following conditions hold for any $x, y, z \in L$:

- (I₁) $x \rightarrow (y \rightarrow z) = y \rightarrow (x \rightarrow z)$;
- (I₂) $x \rightarrow x = I$;
- (I₃) $x \rightarrow y = y' \rightarrow x'$;
- (I₄) $x \rightarrow y = y \rightarrow x = I$ implies $x = y$;
- (I₅) $(x \rightarrow y) \rightarrow y = (y \rightarrow x) \rightarrow x$;
- (I₁) $(x \vee y) \rightarrow z = (x \rightarrow z) \wedge (y \rightarrow z)$;
- (I₂) $(x \wedge y) \rightarrow z = (x \rightarrow z) \vee (y \rightarrow z)$.

In this paper, we denote \mathcal{L} as a lattice implication algebra $(L, \vee, \wedge, ', \rightarrow, O, I)$.

We list some basic properties of lattice implication algebras. It is useful to develop these topics in other sections.

Theorem 1.¹⁴ Let \mathcal{L} be a lattice implication algebra. Then for any $x, y, z \in L$, the following conclusions hold:

- (1) if $I \rightarrow x = I$, then $x = I$;
- (2) $I \rightarrow x = x$ and $x \rightarrow O = x'$;
- (3) $O \rightarrow x = I$ and $x \rightarrow I = I$;
- (4) $(x \rightarrow y) \rightarrow ((y \rightarrow z) \rightarrow (x \rightarrow z)) = I$;
- (5) $(x \rightarrow y) \vee (y \rightarrow x) = I$;
- (6) if $x \leq y$, then $x \rightarrow z \geq y \rightarrow z$ and $z \rightarrow x \leq z \rightarrow y$;
- (7) $x \leq y$ if and only if $x \rightarrow y = I$;
- (8) $(z \rightarrow x) \rightarrow (z \rightarrow y) = (x \wedge z) \rightarrow y = (x \rightarrow z) \rightarrow (x \rightarrow y)$;
- (9) $x \rightarrow (y \vee z) = (y \rightarrow z) \rightarrow (x \rightarrow z)$;
- (10) $x \rightarrow (y \rightarrow z) = (x \vee y) \rightarrow z$ if and only if $x \rightarrow (y \rightarrow z) = x \rightarrow z = y \rightarrow z$;
- (11) $z \leq y \rightarrow x$ if and only if $y \leq z \rightarrow x$.

Definition 2.¹¹ Let X be a set of propositional variables, $T = L \cup \{', \rightarrow\}$ be a type with $\text{ar}(')=1$, $\text{ar}(\rightarrow)=2$ and $\text{ar}(\alpha)=0$ for any $\alpha \in L$. The propositional algebra of the lattice-valued propositional calculus on the set X of propositional variables is the free T algebra on X is denoted by $LP(X)$.

Theorem 2.¹⁴ $LP(X)$ is the minimal set Y which satisfies:

- (1) $X \cup L \subseteq Y$.
- (2) if $p, q \in Y$, then $p', p \rightarrow q \in Y$.

Definition 3.¹⁴ A valuation of $LP(X)$ is a propositional algebra homomorphism $v: LP(X) \rightarrow L$.

Definition 4.¹⁴ Let $p \in LP(X)$, $\alpha \in L$. If there exists a valuation v of $LP(X)$ such that $v(p) \geq \alpha$, p is satisfiable by a truth-value level α , in short, α -satisfiable; If $v(p) \geq \alpha$ for every valuation v , p is valid by the truth-value level α , in short, α -valid. If $\alpha = I$, then p is valid simply.

Definition 5.¹¹ Let $p \in LP(X)$. If $v(p) \leq \alpha$ for any valuation v of $LP(X)$, p is always false by the truth-valued level α , in short, α -false. If $\alpha = O$, then p is valid.

Definition 6.¹¹ A lattice-valued propositional logical formula f is called an extremely simple form, in short, ESF, if a lattice-valued propositional logical formula f^* obtained by deleting any constant or literal or implication item appearing in f is not equivalent to f .

Definition 7.¹¹ A lattice-valued propositional logical formula f is called an indecomposable extremely simple form, in short, IESF, if:

- (1) f is an ESF containing connective \rightarrow and $'$.
- (2) for any $g \in LP(X)$, if $g \in \overline{f}$ in $LP(X)$, then g is an ESF containing connective \rightarrow and $'$ at most, where

$\overline{LP(X)} = (LP(X) / \equiv, \vee, \wedge, ', \rightarrow)$ is a lattice implication algebra.

$$LP(X) / \equiv = \{\overline{p} | p \in LP(X)\}, \overline{p} = \{q \in LP(X) | q = p\}.$$

Definition 8.¹⁴ All the constants, literals and IESFs are generalized literals.

Definition 9.¹⁴ Let $\alpha \in L$, and G_1, G_2 be two generalized clauses of the form:

$$G_1 = g_1 \vee \cdots \vee g_i \vee \cdots \vee g_n,$$

$$G_2 = h_1 \vee \cdots \vee h_j \vee \cdots \vee h_n$$

If $g_i \wedge h_j \leq \alpha$, then

$$G = g_1 \vee \cdots \vee g_{i-1} \vee g_{i+1} \vee \cdots \vee g_n \vee h_1 \vee \cdots \vee h_{j-1} \vee h_{j+1} \vee \cdots \vee h_n$$

is an α -resolvent of G_1 and G_2 , denoted by $G = R_\alpha(G_1, G_2)$, and g_i and h_j form an α -resolution pair, denoted by $(g_i, h_j) - \alpha$.

Let g be a generalized literal in $LP(X)$:

$D_\alpha(g) = \{h | (g, h) - \alpha, h \text{ is a generalized literal in } LP(X)\}$. $D_\alpha(g)$ is the α -resolution field of g .

Definition 10.¹⁸ Let $C_i = p_{i1} \vee \cdots \vee p_{im_i}$ be generalized clauses of $LP(X)$, $H_i = \{p_{i1}, \cdots, p_{im_i}\}$ the set of all disjuncts occurring in $C_i, i = 1, 2, \cdots, m, \alpha \in L$. For any $i \in \{1, 2, \cdots, m\}$, if there exist generalized literals $x_i \in H_i$ such that $x_1 \wedge x_2 \wedge \cdots \wedge x_m \leq \alpha$, then

$$C_1(x_1 = \alpha) \vee C_2(x_2 = \alpha) \vee \cdots \vee C_m(x_m = \alpha)$$

is called an α -resolvent of C_1, C_2, \cdots, C_m , denoted by $R_{p(g-\alpha)}(C_1(x_1), C_2(x_2), \cdots, C_m(x_m))$, x_1, x_2, \cdots, x_m are called an α -resolution group.

The α -resolution group x_1, x_2, \cdots, x_m , denoted by $(x_1, x_2, \cdots, x_m) - \alpha$.

Definition 11. Let g be a generalized literal in $LP(X)$ and $\alpha \in L$. Denote

$$D_\alpha^m(g) = \{(h_1, h_2, \cdots, h_m) | (g, h_1, h_2, \cdots, h_m) - \alpha\},$$

where h_1, h_2, \cdots, h_m are generalized literals in $LP(X)$. $D_\alpha^m(g)$ is the $(m+1)$ -ary α -resolution field of g .

From the Definition 10 and Definition 11, we can obtain the following result, easily.

Theorem 3. Let g, h_1, h_2, \cdots, h_m are generalized literals in $LP(X)$.

$$(1) (h_1, h_2, \cdots, h_m) \in D_\alpha^m(g) \Leftrightarrow$$

$$(g, h_1, h_2, \cdots, h_{i-1}, h_{i+1}, \cdots, h_m) \in D_\alpha^m(h_i).$$

(2) If $g = \alpha \in L$, then $D_\alpha^m(g)$ is the set of any generalized literals group in $LP(X)$.

(3) If g is a constant and $g \not\leq \alpha$, then $(h_1, h_2, \cdots, h_m) \in D_\alpha^m(g)$ if and only if $h_1 \wedge h_2 \wedge \cdots \wedge h_m \leq \alpha$.

In this paper, we always assume that α satisfies the condition:

(1) α is a dual numerator and

(2) $\alpha' \wedge (\alpha' \rightarrow \alpha) \leq \alpha$ and $\bigvee_{\beta \in L} (\beta \wedge \beta') \leq \alpha$.

We mainly discuss the α -solvent of 3-ary generalized literals which include the number of implication operators not more than 2.

3. Determination of 3-ary α -resolution in $LP(X)$

In α -3 ary resolution, it is very important to judge the α -solvent of 3-ary generalized literals. As the complexity of generalized literals, so it is difficult to discuss the α -solvent of any three generalized literals. In this section, we mainly discuss the α -solvent of 3-ary generalized literals which include not more than 2 implication operators. The determination of α -solvent of 3-ary generalized literals considering all the elements in the following sets in $LP(X)$.

$w_1 = \{f | f \text{ is a generalized literal in } LP(X), \text{ and there exist } p, q \in \mathcal{L}, p \neq q, \text{ such that } f \leq p \rightarrow q\};$

$w_2 = \{f | f \text{ is a generalized literal in } LP(X), \text{ and there exist } p \in \mathcal{L}, a \in L, \text{ such that } f \leq p \rightarrow a\};$

$w_3 = \{f | f \text{ is a generalized literal in } LP(X), \text{ and there exist } p, q \in \mathcal{L}, p \neq q, \text{ such that } f \leq (p \rightarrow q)'\};$

$w_4 = \{f | f \text{ is a generalized literal in } LP(X), \text{ and there exist } p \in \mathcal{L}, a \in L, a' \not\leq \alpha, \text{ such that } f \leq (p \rightarrow a)'\}.$

$w = L \cup \mathcal{L} \cup_i^4 w_i$, where L is the set of constants and \mathcal{L} is the set of all literals in $LP(X)$. Let $h_i \in w, i = 1, 2, 3$, if $h_i \leq \alpha$ for some $i \in \{1, 2, 3\}$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$. Hence, the topic of this paper will be discussed under the condition $h_i \not\leq \alpha$ for any $i = 1, 2, 3$.

3.1. The Structure of $D_\alpha^2(g)$ when $h_1 = g \in L$

If $g \leq \alpha$, obviously, $h_1 \wedge h_2 \wedge h_3 \leq \alpha$ for any generalized literals h_2, h_3 in $LP(X)$. So the following discussions under the condition $g \not\leq \alpha$, the different cases are presented in table 1.

Table 1. Different Cases of Structure of $D_\alpha^2(g)$ when $h_1 = g \in L$.

		h_2					
		L	\mathcal{L}	w_1	w_2	w_3	w_4
h_3	L	A1					
	\mathcal{L}	A2	B1				
	w_1	A3	B2	C1			
	w_2	A4	B3	C2	D1		
	w_3	A5	B4	C3	D2	E1	
	w_4	A6	B5	C4	D4	E2	F1

A. Assume $h_2 = g_2 \in L$.

A1. If $h_3 \in L$, then $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

A2. If $h_3 \in \mathcal{L}$, then $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

A3. If $h_3 \in w_1$, then $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

A4. If $h_3 \in w_2$, then $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

A5. If $h_3 \in w_3$, then $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

A6. If $h_3 \in w_4$, then $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

B. Assume $h_2 = x_2 \in \mathcal{L}$.

B1. If $h_3 = x_3 \in \mathcal{L}$ and $x_2 = x'_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

B2. If $h_3 \in w_1$ and $h_3 \leq (x_3 \rightarrow y_3)$,

B21. If $h_3 = ((x_3 \rightarrow y_3) \rightarrow z_3)'$ and $x_2 = z_3$, then $h_1 \wedge h_2 \wedge h_3 \leq g \wedge x_2 \wedge z'_3 \leq \alpha$.

B22. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

B3. When $h_3 \in w_2$ and $h_3 \leq (x_3 \rightarrow \alpha_3)$

B31. If $x_2 = x_3$ and $\alpha' \rightarrow \alpha_3 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$. In fact, if $v(x_2) \leq \alpha$ for any valuation v , we have we have $h_1 \wedge h_2 \wedge h_3 \leq \alpha$. If $v(x_2) \not\leq \alpha$, as $v(x_2) \wedge v(x_2)' \leq \alpha$ and α is a dual numerator, it follows that $v(x_2)' \leq \alpha$, and so $v(x_2) \geq \alpha'$. We have $v(x_2) \rightarrow \alpha_3 \leq \alpha' \rightarrow \alpha_3 \leq \alpha$. Therefore $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

B32. If $h_3 = ((x_3 \rightarrow \alpha_3) \rightarrow y_3)'$, then $h_1 \wedge h_2 \wedge h_3 \leq g \wedge x_2 \wedge y'_3$. And so, when $x_2 = y_3$, we have $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

B4. If $h_3 \in w_3$, $h_3 \leq (x_3 \rightarrow y_3)'$, and $x_2 = x'_3$ or $x_2 = y_3$, we have $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

B5. If $h_3 \in w_4$ and $h_3 \leq (x_3 \rightarrow \alpha_3)'$, then $h_1 \wedge h_2 \wedge h_3 \leq g \wedge x_2 \wedge x_3$. Thus, when $x_2 = x'_3$, it follows that $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

C. Assume $h_2 \in w_1$ and $h_2 \leq x_2 \rightarrow y_2$.

C1. If $h_3 \in w_1$, and $h_3 \leq x_3 \rightarrow y_3$

C11. When $h_2 = x_2 \rightarrow y_2$, $h_3 = ((x_3 \rightarrow y_3) \rightarrow \alpha_3)'$ and $\alpha'_3 \leq \alpha$, $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

C12. When $h_2 = ((x_2 \rightarrow y_2) \rightarrow \alpha_2)'$, $h_3 = ((x_3 \rightarrow y_3) \rightarrow \alpha_3)'$ and $\alpha'_2 \leq \alpha$ or $\alpha'_3 \leq \alpha$, $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

C13. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

C2. If $h_3 \in w_2$ and $h_3 \leq x_3 \rightarrow \alpha_3$, then $h_1 \wedge h_2 \wedge h_3 \leq h_1 \wedge (x_2 \rightarrow y_2) \wedge (x_3 \rightarrow \alpha_3)$.

C21. When $h_2 = x_2 \rightarrow y_2$ and $h_3 = ((x_3 \rightarrow y_3) \rightarrow \alpha_3)'$, and $x_2 \rightarrow y_2 = x_3 \rightarrow y_3$, $\alpha' \rightarrow \alpha_3 \leq \alpha$, $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

C22. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

C4. If $h_3 \in w_3$ and $h_3 \leq (x_3 \rightarrow y_3)'$, $x_2 = x_3$ and $y_2 = y_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

C4. If $h_3 \in w_4$ and $h_3 \leq (x_3 \rightarrow \alpha_3)'$ and $\alpha'_3 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

C31. When $h_2 = ((x_2 \rightarrow y_2) \rightarrow z_2)'$ and $h_3 = ((x_3 \rightarrow (y_3 \rightarrow \alpha_3)))'$, then $h_1 \wedge h_2 \wedge h_3 \leq g \wedge z_2 \wedge x_3$ or $h_1 \wedge h_2 \wedge h_3 \leq g \wedge z_2 \wedge y_3$. If $z'_2 = x_3$ or $z'_2 = y_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

C32. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

D. Assume $h_2 \in w_2$ and $h_2 \leq x_2 \rightarrow \alpha_2$.

D1. If $h_3 \in w_2$ and $h_3 \leq x_3 \rightarrow \alpha_3$, then $h_1 \wedge h_2 \wedge h_3 \leq h_1 \wedge (x_2 \rightarrow \alpha_2) \wedge (x_3 \rightarrow \alpha_3)$.

D11. When $x_2 = x'_3$ and $\alpha'_3 \rightarrow (\alpha' \rightarrow \alpha_2) \leq \alpha$, $h_1 \wedge h_2 \wedge h_3 \leq \alpha$. In fact, if $v(x_2 \rightarrow \alpha_2) \leq \alpha$ for any valuation v in LP(X), we have $h_1 \wedge h_2 \wedge h_3 \leq \alpha$. If $v(x_2 \rightarrow \alpha_2) \not\leq \alpha$, then $v(x_2 \rightarrow \alpha_2)' \leq \alpha$, that is $v(x_2 \rightarrow \alpha_2) \geq \alpha'$, hence $v(x_2) \leq \alpha' \rightarrow \alpha_2$. Consequently, $v(x_3 \rightarrow \alpha_3) = v(x_3) \rightarrow \alpha_3 = \alpha'_3 \rightarrow v(x_3)' \leq \alpha'_3 \rightarrow (\alpha' \rightarrow \alpha_2) \leq \alpha$, so $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

D12. When $h_2 = ((x_2 \rightarrow \alpha_2) \rightarrow z_2)'$, then $h_1 \wedge h_2 \wedge h_3 \leq g \wedge z'_2 \wedge (x_3 \rightarrow \alpha_3)$. If $z'_2 = x_3$ and $\alpha' \rightarrow \alpha_3 \leq \alpha$, we have $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

D13. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

D2. If $h_3 \in w_3$ and $h_3 \leq (x_3 \rightarrow y_3)'$, then $h_1 \wedge h_2 \wedge h_3 \leq g \wedge (x_2 \rightarrow \alpha_2) \wedge (x_3 \rightarrow y_3)'$, and so

D21. If $x_3 = x_2$ and $\alpha' \rightarrow \alpha_2 \leq \alpha$, $h_1 \wedge h_2 \wedge h_3 \leq \alpha$. In fact, it is similar to the proof of 3.1 (B31).

D22. If $y'_3 = x_2$ and $\alpha' \rightarrow \alpha_2 \leq \alpha$, $h_1 \wedge h_2 \wedge h_3 \leq \alpha$. In fact, it is similar to the proof of 3.1 (B31).

D23. When $h_2 = ((x_2 \rightarrow \alpha_2) \rightarrow z_2)'$, then $h_1 \wedge h_2 \wedge h_3 \leq g \wedge z'_2 \wedge (x_3 \rightarrow \alpha_3)$. If $z'_2 = x_3$ and $\alpha' \rightarrow \alpha_3 \leq \alpha$, we have $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

D24. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

D3. If $h_3 \in w_4$ and $h_3 \leq (x_3 \rightarrow \alpha_3)'$, then $h_1 \wedge h_2 \wedge h_3 \leq g \wedge (x_2 \rightarrow \alpha_2) \wedge (x_3 \rightarrow \alpha_3)'$.

D31. When $x_2 = x'_3$ and $\alpha' \rightarrow \alpha_2 \leq \alpha$, $h_1 \wedge h_2 \wedge h_3 \leq \alpha$. In fact, it is similar to the proof of 3.1 (B31).

D32. When $h_2 = ((x_2 \rightarrow \alpha_2) \rightarrow z_2)'$, then $h_1 \wedge h_2 \wedge h_3 \leq g \wedge z'_2 \wedge x_3$. If $z'_2 = x_3$, we have $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

D33. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

E. Assume $h_2 \in w_3$ and $h_2 \leq (x_2 \rightarrow y_2)'$.

E1. If $h_3 \in w_3$ and $h_3 \leq (x_3 \rightarrow y_3)'$, then $h_1 \wedge h_2 \wedge h_3 \leq h_1 \wedge (x_2 \rightarrow y_2)' \wedge (x_3 \rightarrow y_3)'$,

E11. If $x_2 = x'_3$ or $x_2 = y_3$ or $y_2 = x_2$ or $y_2 = y'_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

E12. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

E2. If $h_3 \in w_4$ and $h_3 \leq (x_3 \rightarrow \alpha_3)'$, then $h_1 \wedge h_2 \wedge h_3 \leq h_1 \wedge (x_2 \rightarrow y_2)' \wedge (x_3 \rightarrow \alpha_3)' \leq h_1 \wedge x_2 \wedge x_3$ or $h_1 \wedge h_2 \wedge h_3 \leq h_1 \wedge y_2' \wedge x_3$.

E21. If $x_2 = x_3'$ or $y_2 = x_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

E22. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

F. Assume $h_2 \in w_4$ and $h_2 \leq (x_2 \rightarrow \alpha_2)'$.

F1. If $h_3 \in w_4$ and $h_3 \leq (x_3 \rightarrow \alpha_3)'$, then $h_1 \wedge h_2 \wedge h_3 \leq h_1 \wedge x_2 \wedge x_3'$ or $h_1 \wedge h_2 \wedge h_3 \leq h_1 \wedge \alpha_2 \wedge \alpha_3$,

F11. If $x_2 = x_3'$ or $x_2 = y_3$ or $\alpha_2 \wedge \alpha_3 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

F12. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

3.2. The Structure of $D_\alpha^2(g)$ when $h_1 = x_1 \in \mathcal{L}$

In this section, the different cases need to be discussed in the table 2.

Table 2. Different Cases of Structure of $D_\alpha^2(g)$ when $h_1 = x_1 \in \mathcal{L}$.

		$h_2 \in$				
		\mathcal{L}	w_1	w_2	w_3	w_4
$h_3 \in$	\mathcal{L}	A1				
	w_1	A2	B1			
	w_2	A3	B2	C1		
	w_3	A4	B3	C2	D1	
	w_4	A5	B4	C3	D2	E1

A. Assume $h_2 = x_2 \in \mathcal{L}$.

In this case, if $x_1 = x_2'$, then $h_1 \wedge h_2 \wedge h_2 \leq \alpha$ for any generalized literal h_3 . So, the following cases will be discussed under the condition $x_1 \neq x_2'$:

A1. If $h_3 \in \mathcal{L}$ and $h_3 = x_3$, then $h_1 \wedge h_2 \wedge h_3 = x_1 \wedge x_2 \wedge x_3$.

A11. If $x_2 = x_3'$ or $x_1 = x_3'$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

A12. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

A2. If $h_3 \in w_1$ and $h_3 \leq x_3 \rightarrow y_3$, then $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

A3. If $h_3 \in w_2$ and $h_3 \leq x_3 \rightarrow \alpha_3$, then $h_1 \wedge h_2 \wedge h_3 \leq x_1 \wedge x_2 \wedge (x_3 \rightarrow \alpha_3)$.

A31. If $x_1 = x_3$ and $\alpha' \rightarrow \alpha_3 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$. In fact, it is similar to the proof of 3.1 (B31).

A32. If $x_2 = x_3$ and $\alpha' \rightarrow \alpha_3 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$. In fact, it is similar to the proof of 3.1 (B31).

A33. If $h_3 = ((x_3 \rightarrow \alpha_3) \rightarrow y_3)'$, then $h_1 \wedge h_2 \wedge h_3 \leq x_1 \wedge x_2 \wedge y_3'$. And so, when $x_2 = y_3$ or $x_1 = y_3$, we have $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

A34. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

A4. If $h_3 \in w_3$ and $h_3 \leq (x_3 \rightarrow y_3)'$, then $h_1 \wedge h_2 \wedge h_3 \leq x_1 \wedge x_2 \wedge (x_3 \rightarrow y_3)'$.

A41. If $x_1 = x_3'$ or $x_2 = x_3'$ or $x_1 = y_3$ or $x_2 = y_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

A42. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

A5. If $h_3 \in w_4$ and $h_3 \leq (x_3 \rightarrow \alpha_3)' \leq x_3$, then $h_1 \wedge h_2 \wedge h_3 \leq x_1 \wedge x_2 \wedge (x_3 \rightarrow \alpha_3)' \leq x_1 \wedge x_2 \wedge x_3$.

A51. If $x_1 = x_3'$ or $x_2 = x_3'$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

A52. When $h_3 = (x_3 \rightarrow (y_3 \rightarrow z_3))'$ or $(x_3 \rightarrow (\alpha_3 \rightarrow z_3))'$, we have $h_1 \wedge h_2 \wedge h_3 \leq x_1 \wedge x_2 \wedge z_3'$. If $x_1 = z_3$ or $x_2 = z_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

A53. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

B. Assume $h_2 \in w_1$ and $h_2 \leq x_2 \rightarrow y_2$.

B1. If $h_3 \in w_1$ and $h_3 \leq x_3 \rightarrow y_3$, then $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

B2. If $h_3 \in w_2$ and $h_3 \leq x_3 \rightarrow \alpha_3$, then $h_1 \wedge h_2 \wedge h_3 \leq x_1 \wedge (x_2 \rightarrow y_2) \wedge (x_3 \rightarrow \alpha_3)$.

B21. If $x_1 = x_3$ and $\alpha' \rightarrow \alpha_3 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$. In fact, it is similar to the proof of 3.1 (B31).

B22. When $h_3 = (x_3 \rightarrow y_3) \rightarrow \alpha_3$, if $x_2 = x_3, y_2 = y_3$ and $\alpha' \rightarrow \alpha_3 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

B23. When $h_3 = ((x_3 \rightarrow \alpha_3) \rightarrow z_3)'$, if $x_1 = z_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

B24. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

B3. If $h_3 \in w_3$ and $h_3 \leq (x_3 \rightarrow y_3)'$, then $h_1 \wedge h_2 \wedge h_3 \leq x_1 \wedge (x_2 \rightarrow y_2) \wedge (x_3 \rightarrow y_3)'$.

B31. If $x_2 = x_3$ and $y_2 = y_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

B32. If $x_1 = x_3$ or $x_2 = y_3'$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

B33. When $h_3 = ((x_3 \rightarrow \alpha_3) \rightarrow z_3)'$, if $x_1 = z_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

B34. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

B4. If $h_3 \in w_4$ and $h_3 \leq (x_3 \rightarrow \alpha_3)' \leq x_3$, then $h_1 \wedge h_2 \wedge h_3 \leq x_1 \wedge (x_2 \rightarrow y_2) \wedge (x_3 \rightarrow \alpha_3)' \leq x_1 \wedge (x_2 \rightarrow y_2) \wedge x_3$.

B41. If $x_1 = x_3'$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

B42. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

C. Assume $h_2 \in w_2$ and $h_2 \leq x_2 \rightarrow \alpha_2$.

In this case, if $x_1 = x_2$ and $\alpha' \rightarrow \alpha_2 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_2 \leq \alpha$ for any generalized literal h_3 . So, the following cases will be discussed under the condition $x_1 \neq x_2$:

C1. If $h_3 \in w_2$ and $h_3 \leq x_3 \rightarrow \alpha_3$, then $h_1 \wedge h_2 \wedge h_3 \leq h_1 \wedge (x_2 \rightarrow \alpha_2) \wedge (x_3 \rightarrow \alpha_3)$.

C11. If $x_1 = x_3$ and $\alpha' \rightarrow \alpha_3 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_2 \leq \alpha$. In fact, it is similar to the proof of 3.1 (B31).

C12. If $x_2 = x'_3$ and $\alpha'_3 \rightarrow (\alpha' \rightarrow \alpha_2) \leq \alpha$, then $h_1 \wedge h_2 \wedge h_2 \leq \alpha$. In fact, it is similar to the proof of 3.1 (D11).

C13. When $h_2 = ((x_2 \rightarrow \alpha_2) \rightarrow z_2)'$, then $h_1 \wedge h_2 \wedge h_2 \leq x_1 \wedge z'_2 \wedge (x_3 \rightarrow \alpha_3)$:

(1) If $x_1 = z_2$, then $h_1 \wedge h_2 \wedge h_2 \leq \alpha$.

(2) If $z_2 = x'_3$ and $\alpha'_3 \rightarrow (\alpha' \rightarrow \alpha_2) \leq \alpha$, then $h_1 \wedge h_2 \wedge h_2 \leq \alpha$. In fact, it is similar to the proof of 3.1 (B31).

C14. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

C2. If $h_3 \in w_3$ and $h_3 \leq (x_3 \rightarrow y_3)'$, then $h_1 \wedge h_2 \wedge h_3 \leq x_1 \wedge (x_2 \rightarrow \alpha_2) \wedge (x_3 \rightarrow y_3)'$.

C21. If $x_1 = x'_3$ or $x_1 = y_3$, then $h_1 \wedge h_2 \wedge h_2 \leq \alpha$.

C22. If $x_2 = x_3$ or $x_2 = y'_3$ and $\alpha' \rightarrow \alpha_2 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_2 \leq \alpha$.

C23. When $h_2 = ((x_2 \rightarrow \alpha_2) \rightarrow z_2)'$, then $h_1 \wedge h_2 \wedge h_2 \leq x_1 \wedge z'_2 \wedge (x_3 \rightarrow y_3)'$:

(1) If $x_1 = z_2$, then $h_1 \wedge h_2 \wedge h_2 \leq \alpha$.

(2) If $z_2 = x_3$ or $z_2 = y'_3$, then $h_1 \wedge h_2 \wedge h_2 \leq \alpha$.

C24. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

C3. If $h_3 \in w_4$ and $h_3 \leq (x_3 \rightarrow \alpha_3)' \leq x_3$, then $h_1 \wedge h_2 \wedge h_3 \leq x_1 \wedge (x_2 \rightarrow \alpha_2) \wedge (\alpha_3 \rightarrow x_3)' \leq x_1 \wedge (x_2 \rightarrow \alpha_2) \wedge x_3$.

C31. If $x_1 = x'_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

C32. If $x_2 = x_3$ and $\alpha' \rightarrow \alpha_2 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$. In fact, it is similar to the proof of 3.1 (B31).

C33. When $h_2 = ((x_2 \rightarrow \alpha_2) \rightarrow z_2)'$, then $h_1 \wedge h_2 \wedge h_2 \leq x_1 \wedge z'_2 \wedge x_3$:

(1) If $x_1 = z_2$, then $h_1 \wedge h_2 \wedge h_2 \leq \alpha$.

(2) If $z_2 = x_3$, then $h_1 \wedge h_2 \wedge h_2 \leq \alpha$.

C33. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

D. Assume $h_2 \in w_3$ and $h_2 \leq (x_2 \rightarrow y_2)'$.

D1. If $h_3 \in w_3$ and $h_3 \leq (x_3 \rightarrow y_3)'$, then $h_1 \wedge h_2 \wedge h_3 \leq x_1 \wedge (x_2 \rightarrow y_2)' \wedge (x_3 \rightarrow y_3)'$:

D11. If $x_1 = x'_2$ or $x_2 = x'_3$ or $x_1 = x'_3$ or $x_1 = y_2$ or $x_3 = y_2$ or $x_1 = y_3$ or $x_2 = y_3$, $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

D12. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

D3. If $h_3 \in w_4$ and $h_3 \leq (x_3 \rightarrow \alpha_3)'$, then $h_1 \wedge h_2 \wedge h_3 \leq x_1 \wedge (x_2 \rightarrow y_2)' \wedge (x_3 \rightarrow \alpha_3)'$.

D31. If $x_1 = x'_3$ or $x_2 = x'_3$ or $y_2 = x_3$ or $x_1 = x'_2$ or $x_1 = y_2$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

D32. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

E. Assume $h_2 \in w_4$ and $h_2 \leq (x_2 \rightarrow \alpha_2)'$.

E1. If $h_3 \in w_4$ and $h_3 \leq (x_3 \rightarrow \alpha_3)'$, then $h_1 \wedge h_2 \wedge h_3 \leq x_1 \wedge x_2 \wedge x_3$. When $x_1 = x'_2$ or $x_2 = x'_3$ or $x_1 = x'_3$, we have $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

3.3. The Structure of $D_\alpha^2(g)$ when $g = h_1 \in w_1$

In this section, the different cases need to be discussed in the table 3.

Table 3. Different Cases of Structure of $D_\alpha^2(g)$ when $g = h_1 \in w_1$.

		$h_2 \in$			
		w_1	w_2	w_3	w_4
$h_3 \in$	w_1	A1			
	w_2	A2			
	w_3	A3	B2	C1	
	w_4	A4	B3	C2	D1

A. Assume $h_2 \in w_1$ and $h_2 \leq x_2 \rightarrow y_2$.

If $h_1 = (x_1 \rightarrow y_1) \rightarrow z_1$, $h_2 = ((x_2 \rightarrow y_2) \rightarrow z_2)'$ or $h_1 = ((x_1 \rightarrow y_1) \rightarrow z_1)'$, $h_2 = ((x_2 \rightarrow y_2) \rightarrow z_2)$ and $x_1 = x_2, y_1 = y_2, z_1 = z_2$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$. Therefore, the following discussions will be made under the condition that these cases do not occur.

A1. If $h_3 \in w_1$, then $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

A11. If $h_1 = (x_1 \rightarrow y_1) \rightarrow z_1$, $h_2 = ((x_2 \rightarrow y_2) \rightarrow z_2)'$, $h_3 = ((x_3 \rightarrow y_3) \rightarrow z_3)'$ or $h_1 = ((x_1 \rightarrow y_1) \rightarrow z_1)'$, $h_2 = ((x_2 \rightarrow y_2) \rightarrow z_2)$, $h_3 = (x_3 \rightarrow y_3) \rightarrow z_3$ and $x_1 = x_3, y_1 = y_3, z_1 = z_3$, or $x_2 = x_3, y_2 = y_3, z_2 = z_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

A12. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

A2. If $h_3 \in w_2$ and $h_3 \leq x_3 \rightarrow \alpha_3$, then $h_1 \wedge h_2 \wedge h_3 \leq (x_1 \rightarrow y_1) \wedge (x_2 \rightarrow y_2) \wedge (x_3 \rightarrow \alpha_3)$.

A21. When $h_3 = (x_3 \rightarrow y_3) \rightarrow \alpha_3$, if $x_1 = x_3, y_1 = y_3$ or $x_2 = x_3, y_2 = y_3$ and $\alpha' \rightarrow \alpha_3 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

A22. When $h_1 = ((x_1 \rightarrow y_1) \rightarrow z_1)'$ or $h_2 = ((x_2 \rightarrow y_2) \rightarrow z_2)'$, then $h_1 \wedge h_2 \wedge h_3 \leq z_1' \wedge (x_2 \rightarrow y_2) \wedge (x_3 \rightarrow \alpha_3)$ or $h_1 \wedge h_2 \wedge h_3 \leq z_1' \wedge z_2' \wedge (x_3 \rightarrow \alpha_3)$:

(1). If $z_1' = x_3$ and $\alpha' \rightarrow \alpha_3 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$. In fact, it is similar to the proof of 3.1 (B31).

(2). If $z_2' = x_3$ and $\alpha' \rightarrow \alpha_3 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$. In fact, it is similar to the proof of 3.1 (B31).

(3). When $h_3 = ((x_3 \rightarrow \alpha_3) \rightarrow z_3)'$, if $z_1 = z_3'$ or $z_2 = z_3'$ or $z_1 = z_2'$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

A23. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

A3. If $h_3 \in w_3$ and $h_3 \leq (x_3 \rightarrow y_3)'$, then $h_1 \wedge h_2 \wedge h_3 \leq (x_1 \rightarrow y_1) \wedge (x_2 \rightarrow y_2) \wedge (x_3 \rightarrow y_3)'$.

A31. If $x_1 = x_3$ and $y_1 = y_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

A32. If $x_2 = x_3$ and $y_2 = y_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

A33. When $h_1 = ((x_1 \rightarrow y_1) \rightarrow z_1)'$, $h_2 = ((x_2 \rightarrow y_2) \rightarrow z_2)'$, $h_3 = ((x_3 \rightarrow \alpha_3) \rightarrow z_3)'$, if $z_1 = z_3'$ or $z_2 = z_3'$ or $z_1 = z_2'$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

A34. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

A4. When $h_3 \in w_4$ and $h_3 \leq (x_3 \rightarrow \alpha_3)' \leq x_3$,

A41. If $h_1 = ((x_1 \rightarrow y_1) \rightarrow z_1)' \leq z_1'$ and $z_1 = x_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

A42. If $h_2 = ((x_2 \rightarrow y_2) \rightarrow z_2)' \leq z_2'$ and $z_2 = x_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

A43. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

B. Assume $h_2 \in w_2$ and $h_2 \leq x_2 \rightarrow \alpha_2$.

B1. If $h_3 \in w_2$ and $h_3 \leq x_3 \rightarrow \alpha_3$, then $h_1 \wedge h_2 \wedge h_3 \leq (x_1 \rightarrow y_1) \wedge (x_2 \rightarrow \alpha_2) \wedge (x_3 \rightarrow \alpha_3)$.

B11. If $x_2 = x_3'$ and $\alpha_3' \rightarrow (\alpha' \rightarrow \alpha_2) \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$. In fact, it is similar to the proof of 3.1 (D11).

B12. When $h_2 = (x_2 \rightarrow y_2) \rightarrow \alpha_2$ or $h_3 = (x_3 \rightarrow y_3) \rightarrow \alpha_3$, if $x_1 = x_2, y_1 = y_2$ or $x_1 = x_3, y_1 = y_3$ and $\alpha' \rightarrow \alpha_3 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

B13. When $h_1 = ((x_1 \rightarrow y_1) \rightarrow z_1)'$, $h_1 \wedge h_2 \wedge h_3 \leq z_1' \wedge (x_2 \rightarrow \alpha_2) \wedge (x_3 \rightarrow \alpha_3)$.

(1) If $z_1 = x_2'$ and $\alpha' \rightarrow \alpha_2' \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$. In fact, it is similar to the proof of 3.1 (B31).

(2) If $z_1 = x_3'$ and $\alpha' \rightarrow \alpha_3' \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$. In fact, it is similar to the proof of 3.1 (B31).

(3) When $h_2 = ((x_2 \rightarrow \beta_2) \rightarrow z_2)'$, $h_1 \wedge h_2 \wedge h_3 \leq z_1' \wedge z_2' \wedge (x_3 \rightarrow \alpha_3)$.

(I) If $z_2 = x_3'$ and $\alpha' \rightarrow \alpha_3 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

(II) If $z_2 = z_1'$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

B14. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

B2. If $h_3 \in w_3$ and $h_3 \leq (x_3 \rightarrow y_3)'$, then $h_1 \wedge h_2 \wedge h_3 \leq (x_1 \rightarrow y_1) \wedge (x_2 \rightarrow \alpha_2) \wedge (x_3 \rightarrow y_3)'$.

B21. If $x_1 = x_3$ and $x_1 = y_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

B22. If $x_2 = x_3$ or $x_2 = y_3'$ and $\alpha' \rightarrow \alpha_2 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

B23. When $h_1 = ((x_1 \rightarrow y_1) \rightarrow z_1)'$ or $h_2 = ((x_2 \rightarrow \alpha_2) \rightarrow z_2)'$, $h_1 \wedge h_2 \wedge h_3 \leq z_1' \wedge (x_2 \rightarrow \alpha_2) \wedge x_3$ or $h_1 \wedge h_2 \wedge h_3 \leq z_1' \wedge (x_2 \rightarrow \alpha_2) \wedge y_3'$ or $h_1 \wedge h_2 \wedge h_3 \leq z_1' \wedge z_2' \wedge x_3$.

(1). If $z_1 = x_2'$ and $\alpha' \rightarrow \alpha_2 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

(2). If $z_1 = x_3$ or $z_1 = y_3'$ or $z_2 = x_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

B24. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

B3. If $h_3 \in w_4$ and $h_3 \leq (x_3 \rightarrow \alpha_3)' \leq x_3$, then $h_1 \wedge h_2 \wedge h_3 \leq (x_1 \rightarrow y_1) \wedge (x_2 \rightarrow \alpha_2) \wedge (x_3 \rightarrow \alpha_3)' \leq (x_1 \rightarrow y_1) \wedge (x_2 \rightarrow \alpha_2) \wedge x_3$.

B31. If $x_2 = x_3$ and $\alpha' \rightarrow \alpha_2 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

B32. When $h_1 = ((x_1 \rightarrow y_1) \rightarrow z_1)'$ or $h_2 = ((x_2 \rightarrow \alpha_2) \rightarrow z_2)'$, $h_1 \wedge h_2 \wedge h_3 \leq z_1' \wedge (x_2 \rightarrow \alpha_2) \wedge x_3$ or $h_1 \wedge h_2 \wedge h_3 \leq z_1' \wedge z_2' \wedge x_3$.

(1). If $z_1 = x_2'$ and $\alpha' \rightarrow \alpha_2 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

(2). If $z_1 = x_3$ or $z_2 = x_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

B33. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

C. Assume $h_2 \in w_3$ and $h_2 \leq (x_2 \rightarrow y_2)'$.

C1. If $h_3 \in w_3$ and $h_3 \leq (x_3 \rightarrow y_3)'$, then $h_1 \wedge h_2 \wedge h_3 \leq (x_1 \rightarrow y_1) \wedge (x_2 \rightarrow y_2)' \wedge (x_3 \rightarrow y_3)'$:

C11. If $x_1 = x_2$ and $y_1 = y_2$, $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

C12. If $x_1 = x_3$ and $y_1 = y_3$, $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

C13. If $x_2 = y_3$ or $y_2 = x_3$ or $x_2 = x_3'$ or $y_2 = y_3'$, $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

C14. When $h_1 = ((x_1 \rightarrow y_1) \rightarrow z_1)'$, then $h_1 \wedge h_2 \wedge h_3 \leq z_1' \wedge (x_2 \rightarrow y_2)' \wedge (x_3 \rightarrow y_3)'$, if $z_1 = x_2$ or $z_1 = x_3$ or $z_1 = y_2'$ or $z_1 = y_3'$, we have $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

C15. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

C3. If $h_3 \in w_4$ and $h_3 \leq (x_3 \rightarrow \alpha_3)'$, then $h_1 \wedge h_2 \wedge h_3 \leq (x_1 \rightarrow y_1) \wedge (x_2 \rightarrow y_2)' \wedge (x_3 \rightarrow \alpha_3)'$.

C31. If $x_1 = x'_3$ or $x_2 = x'_3$ or $y_2 = x_3$ or $x_1 = x'_2$ or $x_1 = y_2$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

C32. If $x_1 = x'_3$ or $x_2 = x'_3$ or $y_2 = x_3$ or $x_1 = x'_2$ or $x_1 = y_2$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

C33. When $h_1 = ((x_1 \rightarrow y_1) \rightarrow z_1)'$, then $h_1 \wedge h_2 \wedge h_3 \leq z'_1 \wedge (x_2 \rightarrow y_2)' \wedge (x_3 \rightarrow \alpha_3)'$, if $z_1 = x_2$ or $z_1 = x_3$ or $z_1 = y'_2$, we have $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

C34. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

D. Assume $h_2 \in w_4$ and $h_2 \leq (x_2 \rightarrow \alpha_2)'$.

D1. If $h_3 \in w_4$ and $h_3 \leq (x_3 \rightarrow \alpha_3)'$, then $h_1 \wedge h_2 \wedge h_3 \leq (x_1 \rightarrow y_1) \wedge (x_2 \rightarrow \alpha_2)' \wedge (x_3 \rightarrow \alpha_3)'$.

D11. If $x_2 = x'_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

D12. When $h_1 = ((x_1 \rightarrow y_1) \rightarrow z_1)'$, we have $h_1 \wedge h_2 \wedge h_3 \leq z'_1 \wedge x_2 \wedge x_3$. If $z_1 = x_2$ or $z_1 = x_3$ or $x_2 = x'_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

D13. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

3.4. The Structure of $D_\alpha^2(g)$ when $g = h_2 \in w_2$

In this section, the different cases need to be discussed in the table 4.

Table 4. Different Cases of Structure of $D_\alpha^2(g)$ when $g = h_2 \in w_2$.

		$h_2 \in$			
		w_2	w_3	w_4	
$h_3 \in$	w_2	A1			
	w_3	A2	B1		
	w_4	A3	B2	C1	

A. Assume $h_2 \in w_2$ and $h_2 \leq x_2 \rightarrow \alpha_2$.

A1. If $h_3 \in w_2$ and $h_3 \leq x_3 \rightarrow \alpha_3$, then $h_1 \wedge h_2 \wedge h_3 \leq (x_1 \rightarrow \alpha_1) \wedge (x_2 \rightarrow \alpha_2) \wedge (x_3 \rightarrow \alpha_3)$.

A11. If $x_1 = x'_2$ and $\alpha'_2 \rightarrow (\alpha' \rightarrow \alpha_1) \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$. In fact, it is similar to the proof of 3.1 (D11).

A12. If $x_1 = x'_3$ and $\alpha'_3 \rightarrow (\alpha' \rightarrow \alpha_1) \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

A13. If $x_2 = x'_3$ and $\alpha'_3 \rightarrow (\alpha' \rightarrow \alpha_2) \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

A14. When $h_1 = ((x_1 \rightarrow \alpha_1) \rightarrow z_1)'$, then $h_1 \wedge h_2 \wedge h_3 \leq z'_1 \wedge (x_2 \rightarrow \alpha_2) \wedge (x_3 \rightarrow \alpha_3)$.

(1) If $z_1 = x'_2$ and $\alpha' \rightarrow \alpha_2 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

(2) If $z_1 = x'_3$ and $\alpha' \rightarrow \alpha_3 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

A15. When $h_2 = ((x_2 \rightarrow \alpha_2) \rightarrow z_2)'$ or $h_3 = ((x_3 \rightarrow \alpha_3) \rightarrow z_3)'$, the discussions is analogous to the A14.

A16. When $h_1 = ((x_1 \rightarrow \alpha_1) \rightarrow z_1)'$, $h_2 = ((x_2 \rightarrow \alpha_2) \rightarrow z_2)'$ and $h_3 = ((x_3 \rightarrow \alpha_3) \rightarrow z_3)'$, then $h_1 \wedge h_2 \wedge h_3 \leq z'_1 \wedge z'_2 \wedge z'_3$. If $z_1 = z'_2$ or $z_1 = z'_3$ or $z_2 = z'_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

A17. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

A2. If $h_3 \in w_3$ and $h_3 \leq (x_3 \rightarrow y_3)'$, then $h_1 \wedge h_2 \wedge h_3 \leq (x_1 \rightarrow \alpha_1) \wedge (x_2 \rightarrow \alpha_2) \wedge (x_3 \rightarrow y_3)' \leq (x_1 \rightarrow \alpha_1) \wedge (x_2 \rightarrow \alpha_2) \wedge y'_3$ or $h_1 \wedge h_2 \wedge h_3 \leq (x_1 \rightarrow \alpha_1) \wedge (x_2 \rightarrow \alpha_2) \wedge x_3$.

A21. If $x_1 = x_3$ or $x_1 = y'_3$ and $\alpha' \rightarrow \alpha_1 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

A22. If $x_2 = x_3$ or $x_2 = y'_3$ and $\alpha' \rightarrow \alpha_2 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

A23. When $h_1 = ((x_1 \rightarrow \alpha_1) \rightarrow z_1)'$ or $h_2 = ((x_2 \rightarrow \alpha_2) \rightarrow z_2)'$, $h_1 \wedge h_2 \wedge h_3 \leq z'_1 \wedge (x_2 \rightarrow \alpha_2) \wedge x_3$ or $h_1 \wedge h_2 \wedge h_3 \leq z'_1 \wedge (x_2 \rightarrow \alpha_2) \wedge y'_3$ or $h_1 \wedge h_2 \wedge h_3 \leq z'_1 \wedge z'_2 \wedge x_3$.

(1). If $z_1 = x'_2$ or $x_3 = x_2$ or $y_3 = x'_2$ and $\alpha' \rightarrow \alpha_2 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

(2). If $z_2 = x'_1$ or $x_3 = x_1$ or $y_3 = x'_1$ and $\alpha' \rightarrow \alpha_1 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

(3). If $z_1 = x_3$ or $z_1 = y'_3$ or $z_2 = x_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

A24. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

A3. If $h_3 \in w_4$ and $h_3 \leq (x_3 \rightarrow \alpha_3)' \leq x_3$, then $h_1 \wedge h_2 \wedge h_3 \leq (x_1 \rightarrow \alpha_1) \wedge (x_2 \rightarrow \alpha_2) \wedge (x_3 \rightarrow \alpha_3)' \leq (x_1 \rightarrow \alpha_1) \wedge (x_2 \rightarrow \alpha_2) \wedge x_3$.

A31. If $x_2 = x_3$ and $\alpha' \rightarrow \alpha_2 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$. In fact, it is similar to the proof of 3.1 (B31).

A32. If $x_1 = x_3$ and $\alpha' \rightarrow \alpha_1 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$. In fact, it is similar to the proof of 3.1 (B31).

A33. When $h_1 = ((x_1 \rightarrow \alpha_1) \rightarrow z_1)'$ or $h_2 = ((x_2 \rightarrow \alpha_2) \rightarrow z_2)'$, $h_1 \wedge h_2 \wedge h_3 \leq z'_1 \wedge (x_2 \rightarrow \alpha_2) \wedge x_3$ or $h_1 \wedge h_2 \wedge h_3 \leq z'_1 \wedge z'_2 \wedge x_3$.

(1). If $z_1 = x'_2$ or $x_3 = x_2$ and $\alpha' \rightarrow \alpha_2 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

(2). If $z_2 = x'_1$ or $x_3 = x_1$ and $\alpha' \rightarrow \alpha_1 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

(3). If $z_1 = x_3$ or $z_1 = z'_3$ or $z_2 = x_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

A34. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

B. Assume $h_2 \in w_3$ and $h_2 \leq (x_2 \rightarrow y_2)'$.

B1. If $h_3 \in w_3$ and $h_3 \leq (x_3 \rightarrow y_3)'$, then $h_1 \wedge h_2 \wedge h_3 \leq (x_1 \rightarrow \alpha_1) \wedge (x_2 \rightarrow y_2)' \wedge (x_3 \rightarrow y_3)'$:

B11. If $x_1 = x_2$ or $x_1 = y'_2$ or $x_1 = x_3$ or $x_1 = y'_3$, and $\alpha' \rightarrow \alpha_1 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

B12. If $x_2 = y_3$ or $y_2 = x_3$ or $x_2 = x'_3$ or $y_2 = y'_3$, $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

B13. When $h_1 = ((x_1 \rightarrow \alpha_1) \rightarrow z_1)'$, then $h_1 \wedge h_2 \wedge h_3 \leq z'_1 \wedge (x_2 \rightarrow y_2)' \wedge (x_3 \rightarrow y_3)'$. If $z_1 = x'_2$ or $z_1 = y'_2$ or $z_1 = x'_3$ or $z_1 = y'_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

B14. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

B2. If $h_3 \in w_4$ and $h_3 \leq (x_3 \rightarrow \alpha_3)'$, then $h_1 \wedge h_2 \wedge h_3 \leq (x_1 \rightarrow \alpha_1) \wedge (x_2 \rightarrow y_2)' \wedge (x_3 \rightarrow \alpha_3)'$.

B21. If $x_1 = x_2$ or $x_1 = y'_2$ or $x_1 = x_3$, and $\alpha' \rightarrow \alpha_1 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

B22. If $x_2 = x'_3$ or $y_2 = x_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

B23. When $h_1 = ((x_1 \rightarrow \alpha_1) \rightarrow z_1)'$, then $h_1 \wedge h_2 \wedge h_3 \leq z'_1 \wedge (x_2 \rightarrow y_2)' \wedge (x_3 \rightarrow \alpha_3)'$. If $z_1 = x'_2$ or $z_1 = y'_2$ or $z_1 = x'_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

B23. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

C. Assume $h_2 \in w_4$ and $h_2 \leq (x_2 \rightarrow \alpha_2)'$.

C1. If $h_3 \in w_4$ and $h_3 \leq (x_3 \rightarrow \alpha_3)'$, then $h_1 \wedge h_2 \wedge h_3 \leq (x_1 \rightarrow \alpha_1) \wedge (x_2 \rightarrow \alpha_2)' \wedge (x_3 \rightarrow \alpha_3)'$.

C11. If $x_2 = x'_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

C12. If $x_1 = x_3$ or $x_1 = x_2$, and $\alpha' \rightarrow \alpha_1 \leq \alpha$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

C13. When $h_1 = ((x_1 \rightarrow \alpha_1) \rightarrow z_1)'$, we have $h_1 \wedge h_2 \wedge h_3 \leq z'_1 \wedge x_2 \wedge x_3$. If $z_1 = x_2$ or $z_1 = x_3$ or $x_2 = x'_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

C14. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

3.5. The Structure of $D^2_\alpha(g)$ when $g = h_1 \in w_3$

In this section, the different cases need to be discussed in the table 5.

Table 5. Different Cases of Structure of $D^2_\alpha(g)$ when $g = h_2 \in w_3$.

		$h_2 \in$	
		w_3	w_4
$h_3 \in$	w_3	A1	
	w_4	A2	B1

A. Assume $h_2 \in w_3$ and $h_2 \leq (x_2 \rightarrow y_2)'$.

A1. If $h_3 \in w_3$ and $h_3 \leq (x_3 \rightarrow y_3)'$, then $h_1 \wedge h_2 \wedge h_3 \leq (x_1 \rightarrow y_1)' \wedge (x_2 \rightarrow y_2)' \wedge (x_3 \rightarrow y_3)'$:

A11. If $x_1 = x'_2$ or $x_1 = y_2$ or $x_1 = x'_3$ or $x_1 = y_3$ or $x_2 = y_3$ or $y_2 = x_3$ or $x_2 = x'_3$ or $y_2 = y'_3$, $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

A12. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

A2. If $h_3 \in w_4$ and $h_3 \leq (x_3 \rightarrow \alpha_3)'$, then $h_1 \wedge h_2 \wedge h_3 \leq (x_1 \rightarrow y_1)' \wedge (x_2 \rightarrow y_2)' \wedge (x_3 \rightarrow \alpha_3)'$.

A21. If $x_1 = x'_2$ or $x_1 = y_2$ or $x_1 = x'_3$ or $y_1 = x_2$ or $y_1 = y'_2$ or $y_1 = x_3$ or $x_2 = x'_3$ or $y_2 = x_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

A22. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

B. Assume $h_2 \in w_4$ and $h_2 \leq (x_2 \rightarrow \alpha_2)'$.

B1. If $h_3 \in w_4$ and $h_3 \leq (x_3 \rightarrow \alpha_3)'$, then $h_1 \wedge h_2 \wedge h_3 \leq (x_1 \rightarrow y_1)' \wedge (x_2 \rightarrow \alpha_2)' \wedge (x_3 \rightarrow \alpha_3)'$.

B11. If $x_1 = x'_2$ or $x_1 = x'_3$ or $x_2 = x'_3$ or $y_1 = x_2$ or $y_1 = x_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

B12. When $h_1 = (x_1 \rightarrow (y_1 \rightarrow z_1))'$ or $h_1 = (x_1 \rightarrow (\alpha_1 \rightarrow z_1))'$, then $h_1 \wedge h_2 \wedge h_3 \leq z'_1 \wedge (x_2 \rightarrow \alpha_2)' \wedge (x_3 \rightarrow \alpha_3)'$. If $z_1 = x_2$ or $z_1 = x_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

B13. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

3.6. The Structure of $D^2_\alpha(g)$ when $g = h_1 \in w_4$

A. Assume $h_2 \in w_4$ and $h_2 \leq (x_2 \rightarrow \alpha_2)'$.

A1. If $h_3 \in w_4$ and $h_3 \leq (x_3 \rightarrow \alpha_3)'$, then $h_1 \wedge h_2 \wedge h_3 \leq (x_2 \rightarrow \alpha_2)' \wedge (x_2 \rightarrow \alpha_2)' \wedge (x_3 \rightarrow \alpha_3)'$.

A11. If $x_1 = x'_2$ or $x_1 = x'_3$ or $x_2 = x'_3$, then $h_1 \wedge h_2 \wedge h_3 \leq \alpha$.

A12. Otherwise, $h_1 \wedge h_2 \wedge h_3 \not\leq \alpha$.

4. Conclusions

In this paper, we have mainly discussed the determination of α -3 ary resolution generalized literals which include not more than 2 implication operators not more than 2 in lattice-valued logical system LP(X) with truth-value in a lattice implication algebra. The structure of $D^2_\alpha(g)$ is investigated, where g is a generalized literal. It not only lay the foundation for practical implementation of automated reasoning algorithm in LP(X), but also provide the strong support for α - $n(t)$ ary resolution automated reasoning approaches.

Further research will be focused on the algebraic structure of resolution field, which is generated by some α -resolution generalized literals. And it will be used for construction automated reasoning algorithm and designing practical automated reasoning program.

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