

## **Portfolio Optimization From a Set of Preference Ordered Projects Using an Ant Colony Based Multi-objective Approach**

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### **Abstract**

In this paper, a good portfolio is found through an ant colony algorithm (including a local search) that approximates the Pareto front regarding some kind of project categorization, cardinalities, discrepancies with priorities given by the ranking, and the average rank of supported projects; this approach is an improvement towards a proper modeling of preferences. The available information is only projects' ranking and costs, and usually, resource allocation follows the ranking priorities until they are depleted. Results show that our proposal outperforms previous approaches.

**Keywords:** Project portfolio, multi-objective optimization, ant-colony meta-heuristic, Multi-Criteria Decision.

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## 1. Introduction

As was stated by Kleinmuntz,<sup>1</sup> resource allocation problems are ubiquitous in business and government organizations. Usually, there are more good ideas for projects or programs than resources (funds, capacity, time, etc.) to support them. According to Salo et al.,<sup>2</sup> manufacturing enterprises recognize that success depends on the selection of research and development project portfolios, expecting that these projects permit them to develop new products that generate growing benefits. Local governments allocate public funds to projects and programs that improve social and educational services. Environmental regulations and alternative policy measures attempt to mitigate harmful consequences of human activity. To fight poverty, governments in underdeveloped countries fund many helpful social programs.

Portfolio consequences are usually described by multiple attributes related to the organizational goals and objectives (e.g. Ref. 3). A vector  $z(x) = \langle z_1(x), z_2(x), \dots, z_p(x) \rangle$  is associated with the consequences of a portfolio  $x$  considering  $p$  criteria. This is a vector representation of the portfolio's impact. In the simplest case,  $z(x)$  is obtained from the cumulative sum of the benefits of the selected projects. Without loss of generality, we can assume that higher criterion values are preferred to lower values. The best portfolio is obtained by solving the following problem:

$$\max_{x \in R_F} \left\{ \langle z_1(x), z_2(x), \dots, z_p(x) \rangle \right\}, \quad (1)$$

where  $R_F$  is the space of feasible portfolios, and is usually determined by the available budget, and by constraints to the kind of projects, social roles and geographic zones. This problem has been approached by many papers in the scientific literature (e.g. Refs. 3-8).

However, sometimes in the process of selecting a project portfolio, a *Decision Maker* (DM) usually starts with limited information about projects and portfolios. This is the case of the *Portfolio Selection Problem on a Set of Ordered Projects* (PSPSOP) where the only information available is the project ranking, and the project cost.<sup>9</sup> In many situations the DM feels more comfortable employing simple decision procedures, because of the lack of available information, lack of time, aversion to more elaborate decision methods, and even because of his/her fondness for established organizational practices. Cooper et al.<sup>10</sup> argue the

popularity of scoring and ranking methods in *R&D* project management in most American enterprises. Methods of scoring and ranking are used by most of the government organizations that fund *R&D* projects. Sometimes there is no information about individual project's contribution to the organizational objectives in Problem 1 (for instance, as a professional soccer team chooses new players to enrich its staff). As was stated in Ref. 9, other important scenarios are those derived from exercises of *participatory budgeting*, which can be defined as a public space in which the government and the society agree how to adapt the priorities of the citizenship to the public policy agenda. The utility of these participatory exercises is that the government obtains information about the priorities of the participating social sectors, and thus can perhaps identify programs that are of consensual benefit. However, the challenge that is still not approached by participatory budgeting is how to translate priorities and policies stemming from the exercise into a system of concrete social action projects, each with well-defined resources and falling within a frame of an approved general budget. Although the priorities of the citizenship have been identified, and are likely to be reflected in a ranking of public policy actions, it is still necessary to solve a portfolio problem of public projects that should implement the agreed agenda.

The usual method of composing portfolios following the priorities given by the project ranking is a simple but rigid process that has been questioned by several authors (e.g. Refs. 11 and 12). As was stated by Fernandez et al. in Ref. 13, the best decision concerning the maximization of the portfolio impact is not guaranteed by strict use of the ranking information. Selecting the final portfolio by following the ranking guarantees that the best projects will be supported, but this does not necessarily compose the best portfolio. If the decision maker distrusts to some extent the quality of the information provided by the ranking, or he/she is put off by costly projects, then he/she would be reluctant concerning assignments arising from strictly rank positions; mainly when the ranking suggests supporting projects whose costs noticeably exceed the average.<sup>13</sup> Therefore, concerning PSPSOP, in order to measure the impact of a given portfolio (this is the natural criterion to choose the best one), it is necessary to balance the quality and the number of supported projects.

This work proposes a multi-objective optimization based approach to solve the PSPSOP. The multi-objective optimization model regards some kind of project categorization, cardinalities of different categories, discrepancies with priorities given by the project ranking, and the average rank of supported projects. Each one of the previous issues is considered a different dimension of the multi-objective problem. The optimization problem is solved by an ant colony algorithm. The paper is structured as follows: a criticism of previous related approaches is given in Section 2. The new optimization model is detailed in Section 3. Section 4 shows the ant colony metaheuristic method used to solve the PSPSOP. Section 5 presents the experiments and results derived from the research. Finally, we draw some conclusions in Section 6.

## 2. A brief outline of previous approaches

The information provided by a simple project ranking is not enough for portfolio optimization purposes. It is necessary to compare the quality of the feasible portfolios in order to find the best one. Thus, the problem of searching for the best project-portfolio can be reduced to finding a method for assessing project returns, or at least a comparative way to analyze alternative portfolio proposals. Some papers have suggested making a representation of the quality of portfolios using a vector of proxy variables. This idea was suggested firstly by Fernandez et al.<sup>13</sup> and Fernandez and Olmedo.<sup>9</sup> If no reliable information about project returns is available, the impact of a given portfolio may be represented by a set of proxy variables measuring different criteria of portfolio quality. Following,<sup>9</sup> let us denote by  $\{v_1, \dots, v_K\}$  such a set. Without loss of generality, we suppose that the DM's preferences are increasing with values of  $v_i$  :

**Definition 1:** A vector  $V^* = \langle v_1, \dots, v_K \rangle^*$  is Pareto dominated by  $V^{**}$  if and only if  $v_i^{**} > v_i^*$  for some  $i$  and  $v_j^{**} \geq v_j^*$  for  $j=1 \dots K$ .

Accepting that the set  $\{v_1, \dots, v_K\}$  reflects the impact of a portfolio, the following premise seems obvious:

**Premise 1:** Let  $C^*$  and  $C^{**}$  be two feasible portfolios and  $V^* = \langle v_1, \dots, v_K \rangle^*$  and  $V^{**} = \langle v_1, \dots, v_K \rangle^{**}$  their respective image in the proxy objective space. Then,  $V^*$  is Pareto dominated by  $V^{**}$  only if under the available information,  $C^{**}$  is preferred to  $C^*$  from the DM point of view. Let  $R_F$  be the set of feasible portfolios (feasibility

is determined by budgetary constraints). The following proposition was given in Ref. 9:

**Proposition 1.** Under Premise 1 the best portfolio for the DM is a non-dominated solution of the problem:

$$\max_{C \in R_F} \{ \langle v_1, \dots, v_K \rangle \} \quad (2)$$

The proof is very simple. Suppose that  $C_L$  is the best portfolio being a dominated solution to Problem 2. Let  $A_p$  be the asymmetric preference relation from the DM on a subset of  $R_F$ . If  $C_L$  is dominated by  $C_j$ , then  $C_j A_p C_L$  (Premise 1). Then,  $C_L$  cannot be the best portfolio. The choosing of a set  $\{v_1, \dots, v_K\}$  depends on the available information about the portfolio impact, or how it is perceived by the DM.

The works of Fernandez et al.,<sup>13</sup> and Fernandez and Olmedo<sup>9</sup> implement particular cases of Problem 2. The approach uses a heuristic based on dividing the ranking into five categories to further construct the specific preference relation. The categories are labeled: 1) vanguard, 2) high-medium, 3) medium, 4) low-medium, and 5) rearguard. In other words, if  $P_r = \{1, 2, \dots, N\}$  is the set of projects, the above papers defined a "priority" function  $\rho: P_r \rightarrow \{1, 2, 3, 4, 5\}$  whose image is the project label (for instance,  $\rho(a)=1 \Leftrightarrow a$  is ranked in the vanguard). Through exercising such categorization, they build up several preference relations according to the ranking, as follows:

- *Absolute preference* (or  $a \gg b$ ):  $a \gg b \Leftrightarrow \rho(a) < \rho(b)$  and  $|\rho(a) - \rho(b)| \geq 2$
- *Strict preference* (or  $a > b$ ):  $a > b \Leftrightarrow (\rho(a) < \rho(b) \text{ and } |\rho(a) - \rho(b)| = 1) \text{ or } (\rho(a) = \rho(b) \text{ and } a \text{ is ranked better than } b)$
- *Weak preference* (or  $a \sim b$ ):  $a \sim b \Leftrightarrow a$  and  $b$  are ranked equal, but  $b$  needs more funds than  $a$
- *Strict outranking* ( $aSb$ ):  $S = \gg \cup > \cup \sim$ :  $S$  is an asymmetric relation expressing preference for one of the projects. Hence, a well-formed portfolio should be compatible with the information contained in  $S$ .

Using the relation  $S$ , Fernandez et al.,<sup>13</sup> and Fernandez and Olmedo<sup>9</sup> define the concept of *discrepancy* in the following manner. Let  $C$  be a project portfolio that obtains support (a subset of  $P_r$ ), with  $aSb$  and  $a \notin C$ . In such a case, the inclusion of  $b$  into  $C$  would constitute a discrepancy with respect to the information contained in  $S$ . Thus, each portfolio may be associated with a discrepancy set as follows:

$$D = \{ (a,b) \in P_r \times P_r \text{ such that } aSb, b \in C \text{ and } a \notin C \}$$

- $D$  can be partitioned in
- A set of *absolute discrepancies*  $D_a = \{ (a,b) \in P_r \times P_r \text{ such that } a \gg b, b \in C \text{ and } a \notin C \}$ ;
  - A set of *strict discrepancies*  $D_s = \{ (a,b) \in P_r \times P_r \text{ such that } a > b, b \in C \text{ and } a \notin C \}$ ;
  - A set of *weak discrepancies*  $D_w = \{ (a,b) \in P_r \times P_r \text{ such that } a \succsim b, b \in C \text{ and } a \notin C \}$ .

Let  $n_a$ ,  $n_s$ ,  $n_w$  denote the respective cardinality of the above sets. Considering also the number of supported projects in  $C$  (denoted by  $n_{pr}$ ), the *best portfolio* (under limited information) was defined in Ref. 9 as the best compromise solution of the problem:

$$\max_{C \in R_F} \left\{ \left\langle n_{pr} \right\rangle \right\}, \min_{C \in R_F} \{n_a, n_s, n_w\} \quad (3)$$

Problem 3 is an instance of Problem 2 with a particular choosing of proxy variables.

As was stated in Ref. 9, the best portfolio should be selected by the DM among the non-dominated solutions of Problem 3. It is he/she who will finally select the adequate compromise among the number of supported projects and the quality given by their rank. Compromise solutions will depend on the extent that the DM relies on the information provided by the rank ordering, and facts such as pressures exerted on him/her to either include high-ranked projects or increase the number of approved projects, as well as his/her aversion to costly projects. In our opinion, the proxy variables in Problem 3 fail in performing a good representation of the portfolio's impact and the reluctance from the DM to reject well-ranked but expensive projects.

Note that the portfolio's impact is only modeled by  $n_{pr}$ ; it can be improved by considering the number of projects per category. But the main drawback of the model in (3) concerns the definitions of discrepancies. Let us consider, for instance, the portfolio represented in Figure 1. Each supported project that is ranked worse than the third and ninth positions generates one discrepancy. Hence, this portfolio is associated with 27 discrepancies, although only two well-ranked projects have been rejected. Thus, the model produces a sort of inflation of the disappointment that is actually felt by the DM. As a consequence, solving Problem 3 does not lead to finding the best portfolio.

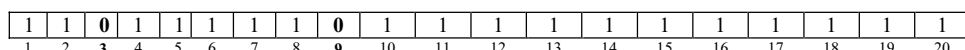


Fig.1. Representation of a portfolio composed of a set of ordered projects  
Note: “1” in the  $i$ -th position means that the project ranked in the  $i$ -th position belongs to the portfolio.

The previous situation drives the research proposed in this paper towards the development of a new optimization formulation that improves the model of the DM's preferences in PSPSOP. The following section details this new model.

### 3. An improved selection of proxy variables in Problem 2

Here, we present an improved model to reflect the portfolio's impact and the DM reluctance to reject relatively well-ranked projects.

Let us assume that the set of projects can be separated in  $M \geq 2$  ordered categories. The impact of the portfolio  $C$  can be modeled by several measures of the projects' quality. We suggest to use  $M+1$  proxy variables  $N_1, \dots, N_M$ , and  $P$ , where:  $N_i$  is the number of projects assigned to the  $i$ -th category belonging to the portfolio and  $P$  is an indicator of the average rank of the projects in the portfolio, defined as:

$$P = \frac{\sum_{a \in C} (N - \text{rank}(a) + 1)}{\text{card}(C)}, \quad (4)$$

where  $\text{rank}=1$  corresponds to the highest ranked project (the best one) and  $\text{card}(C)$  denotes the total amount of projects in the portfolio.

Comparing feasible portfolios, the vector  $\langle N_1, \dots, N_M, P \rangle$  contains the arguments favoring a particular one in terms of projects' quality. The model should be completed by introducing some measures reflecting the DM disappointment. So, we introduce a new concept of discrepancy.

The term *discrepancy* reflects the negative effect exerted on the DM's mind by the fact that one project, as compared to others, appears to have merits to belong to the portfolio and yet is not included in it. To model this concept we must introduce the reference portfolio that is created using the available budget and strictly following the priorities expressed in the ranking of projects. Let  $C_{ref}$  be a reference portfolio and  $a$  and  $b$  two  $P_r$  projects. It holds that rank ( $a$ ) is better than rank ( $b$ )  $\Rightarrow (b \in C_{ref} \Rightarrow a \in C_{ref})$ .

A distinction should be made between three types of discrepancies:

- *Weak discrepancy*: There is a weak discrepancy in the portfolio  $C$  generated by the project  $a$  if the following conditions are met: i)  $a \in C_{ref}$ ; ii)  $a \notin C$ ; iii) the budget necessary for the project  $a$  is much higher than the average budget of the same-category projects belonging to  $C$ .
- *Strong discrepancy*: There is a strong discrepancy in the portfolio  $C$  generated by the project  $a$  if the following conditions are met: i)  $a \in C_{ref}$ ; ii)  $a \notin C$ ; iii) the budget necessary for the project  $a$  is significantly higher than the average budget of the same-category projects belonging to  $C$ , but not much higher.
- *Unacceptable discrepancy*: There is an unacceptable discrepancy in the portfolio  $C$  generated by the project  $a$  if any of the following conditions are met. First: i)  $a \in C_{ref}$ ; ii)  $a \notin C$ ; iii) the budget necessary for the project  $a$  is not significantly higher than the average budget of the same-category projects belonging to  $C$ . Second: There is  $b \in C$  such that: i) rank ( $a$ ) better than rank ( $b$ ); ii)  $a \notin C$ ; iii) the budget for  $a$  is not significantly higher than the budget for  $b$ .

Weak and strong discrepancies are separated taking into account the category of the discrepant project. Let us denote by  $n_{sdi}$  and  $n_{wdi}$  the number of strong and weak discrepancies in the  $i$ -th category. Let us also denote by  $UND$  the set of portfolios that contain unacceptable discrepancies and by  $R_F$  the region defined as  $R_F - UND$ . So, we propose to solve the vector optimization Problem 5.

$$\text{Optimize}_{C \in R_F} \{ \langle N_1, n_{wd1}, n_{sd1}, \dots, N_M, n_{wdM}, n_{sdM}, P \rangle \}, \quad (5)$$

where  $\langle N_1, \dots, N_M, P \rangle$  contains the objectives to be maximized; the remaining objectives should be minimized.

Compared with (3), the formulation in (5) is a clearly improved preference model. Its main disadvantage is a greater number of objective functions, which increases linearly with the number of categories. We suggest using 2-3 categories, which leads to 7-10 objective functions in Problem 3.

In case of three categories (*priority*, *satisfactory*, *acceptable*), Problem 5 is transformed to Problem 6.

$$\text{Optimize}_{C \in R_F} \{ \langle N_1, n_{wd1}, n_{sd1}, N_2, n_{wd2}, n_{sd2}, N_3, n_{wd3}, n_{sd3}, P \rangle \}. \quad (6)$$

In Problem 6 the objectives  $N_1, N_2, N_3, P$  should be maximized looking for a portfolio with a greater

cardinality and average rank. On the other hand, the objectives  $n_{wd1}, n_{sd1}, n_{wd2}, n_{sd2}, n_{wd3}, n_{sd3}$  should be minimized in favor of a portfolio with the minimum number of discrepancies.

#### 4. An ant colony optimization algorithm

Dorigo's Ant Colony System (cf. Ref. 14) has been used to solve a large variety of multi-objective optimization problems (e.g. Ref. 15). Cruz et al.<sup>16</sup> reported good results in problems with 9 and 16 objective functions working with an ant-colony based optimization method. Inspired in this works, we propose an ant colony algorithm, called *Ant Colony Algorithm for Solving the Problem with the Ranking Information* (ACO-SPRI), to solve Problem 6. The main changes of our ACO-SPRI are in: a) the pheromone representation, b) the selection rule, and c) the local search function. The remainder of this section details each of these components.

##### 4.1. Pheromone representation and updating

The *pheromone representation* is a bi-dimensional measurement that records learning of all ants to find the best solutions. Pheromone is usually represented by the letter  $\tau$  and is modeled in ACO-SPRI as a two dimensional array of size  $N \times N$ , where  $N$  is the total number of applicant project proposals. The pheromone between two projects  $i$  and  $j$  is represented as  $\tau_{ij}$ , and indicates how good it is that both projects receive financial support.

The pheromone matrix acts as a reinforcement learning structure reflecting the knowledge gained by the ants that formed high-quality portfolios.

In every iteration of the ant colony algorithm, each ant constructs a new solution (a feasible portfolio). This new set of solutions, denoted by  $SC$ , is subject of a local search to produce a set  $SC'$  of modified solutions. The set  $SC'$  is subdivided into domination fronts using the *Fast-Non-Dominated-Sorting* procedure,<sup>17</sup> which are used to update the pheromone matrix. The fronts are obtained by considering the objectives of Problem 6. The set composed by these fronts is denoted by  $F = \{F_0, F_1, F_2, \dots, F_k, F_{k+1}, \dots\}$ , where  $F_0$  contains the non-dominated solutions,  $F_1$  contains the portfolios that are dominated by only one solution,  $F_2$  those dominated by two solutions, and so forth. The set  $F$  is used in the pheromone intensification in order to increase the selective pressure towards the best compromise. For

each solution  $C$ , each pair of projects  $(i, j)$  lays pheromone trail according to Eq. (7) and Eq. (8).

$$\tau_{i,j} = \Delta\tau_{i,j} + \tau_{i,j} \quad (7)$$

$$\Delta\tau_{i,j} = \frac{|F| - k + 1}{|F|} (1 - \tau_{i,j}) \quad (8)$$

where  $k$  indicates the front containing the portfolio  $C \in F_k$  in the solution set constructed by the ants (and improved by local search). To prevent premature convergence, the colony includes a strategy known as evaporation that reduces pheromone trails for fixed periods of time; this strategy is modeled in Eq. (9):

$$\tau_{i,j} = (1 - \rho)(\tau_{i,j}), \quad (9)$$

where  $\rho$  is a parameter known as evaporation rate.

#### 4.2. Selection rule

The *selection rule* consists in that each ant builds its portfolio by selecting the projects one by one, taking into account the past best result of the search (global knowledge) and information related with the problem under study (local knowledge). The *global knowledge* is represented in the pheromone matrix; this takes into account the experience of previous generations of ants. The global knowledge for project  $i$  to be included in a portfolio  $C$  is denoted by  $\overline{\tau}_{C,i}$  and defined by Eq. (10):

$$\overline{\tau}(C, i) = \frac{\sum_{j=1}^N \varphi(C_j) \tau_{i,j}}{\sum_{j=1}^N \varphi(C_j)}, \quad (10)$$

where  $N$  is the total number of applicant projects,  $\tau_{i,j}$  is the pheromone for projects  $i$  and  $j$  and  $\varphi(C_j)$  is the binary value indicating whether the  $j$ -th project is included in the portfolio  $C$ . The numerator in Equation (10) is the total sum of pheromone between  $i$  and each project in portfolio  $C$ . The *local knowledge* ( $\eta_i$ ) considers the preference of the DM ( $r_i$ ) and how many resources the project consumes ( $c_i$ ); it is calculated according to Eq. (11).

$$\eta_i = \frac{1}{\frac{r_i}{c_i}} \quad (11)$$

We defined a *candidate list* to incorporate a new project into a partially constructed portfolio  $C_p$ . Among all the project proposals, only those that are not part of  $C_p$ , and whose inclusion favors the fulfillment of budgetary constraints, are included in the candidate list  $N' \subset P_r$ . The choice of which project  $j \in N'$  will be

added is made by using the selection rule defined by Eq. (12):

$$j = \begin{cases} \arg \max_{i \in N'} \{\Omega(C, i)\} & \text{if } p \leq \alpha_1, \\ S_{i \in N'} \{\Omega(C, i)\} & \text{if } \alpha_1 < p \leq \alpha_2, \\ S_{i \in N'} & \text{otherwise,} \end{cases} \quad (12)$$

where  $\alpha_1$  is the intensification threshold,  $\alpha_2$  is the diversification threshold,  $p$  is a pseudorandom number between zero and one,  $\Omega(C, i)$  is the intensification effect,  $S_{i \in N'}$  is a middle state effect between intensification and diversification, and  $S_{i \in N'}$  is the diversification effect. The value of  $\Omega(C, i)$ , which is the intensification criteria, is calculated according to Eq. (13), where  $w$  represents the weight given to the preference between the two types of knowledge  $\eta_i$  and  $\overline{\tau}_{C,i}$ , and  $j$  is the considered project to select.

$$\Omega(C, i) = (w)(\eta_i) + (1 - w)(\overline{\tau}_{C,i}) \quad (13)$$

The selection scheme  $S_{i \in N'}$  is based on the known roulette wheel technique,<sup>16</sup> with the probability of each project calculated as in Eq. (14). Note that  $\alpha_2 - \alpha_1$  is the probability of triggering a middle state between intensification and diversification.

$$S_i = \frac{\Omega(C, i)}{\sum_{i \in N'} \Omega(C, i)} \quad (14)$$

The diversification effect  $S_{i \in N'}$  is also a selection scheme that is exerted by choosing a project uniformly at random. Once that the ants have constructed the portfolios a local search process is used with the purpose of achieving better resources. This process is described in detail in the next section.

#### 4.3. Local Search

The intensification algorithm is performed by a greedy variable-neighborhood local search that is carried out on the solutions constructed by the ants. The importance of such a search in a similar framework was analyzed by Cruz et al.<sup>16</sup> This search explores regions near the best known solutions by a simple scheme consisting of selecting  $v$  projects at random, and generating all possible combinations of them for each solution. Small values for  $v$  provoke behavior that is too greedy, whereas large values produce intolerable computation times. In our experiments, we obtained a good balance between these by using  $v = \lfloor \ln N \rfloor$ . The algorithmic outline of the local search is illustrated in Algorithm 1. As observed, the search starts by choosing  $v$  projects at random (Line 3), and generating the binary

power set  $S$  of cardinality  $v$  (Line 4). Each portfolio  $C$  built by an ant is modified in  $C_{new}$  according to each binary combination of actions  $s \in S$  on chosen projects (e.g.  $SP = \{1,3\}$ ,  $s = \{0,1\}$ ,  $C = \{1,2,6,7,10,16\}$ ,  $C_{new} = \{2,3,6,7,10,16\}$ ) (Lines 6 to 8). Whenever a new portfolio  $C_{new}$  is built using a portfolio  $C$  and a combination  $s$ , it is repaired in accordance with its rank and its budget value (Lines 9 to 14). Observe that the repairing procedure has two main goals: 1) bringing unfeasible portfolios to the feasible region, and 2) improving clearly suboptimal portfolios. If the budget of a new portfolio  $C_{new}$  is greater than the available budget  $B$  then it is unfeasible; in this situation the repairing process removes the projects with the lowest ranks, until the budget is not surpassed (Line 11); no project chosen by the current combination  $s$  can be removed from  $C_{new}$ . On the other hand, whenever the new portfolio  $C_{new}$  is partially constructed, i.e. its budget allows the incorporation of more projects, the repairing process adds projects to it, according to their ranking, but keeping the projects chosen by  $s$  (Line 13). This process repeats until no project can be added to the portfolio. Each repaired portfolio  $C_{new}$  is included into a new set of solutions  $SC'$ , containing  $SC$  and its modified set of solutions. Finally, the local search algorithm (Line 18) returns  $SC'$ .

#### 4.4. ACO-SPRI algorithm description

Algorithm 2 details ACO-SPRI. This algorithm constructs the set  $SC^+$  formed by the non-dominated portfolios taken from the front  $F_0$  derived from Problem 6 at each iteration of the algorithm. Problem 6 is used again on the final set  $SC^+$  to extract the overall non-dominated set of solutions, denoted by  $F^*$ . Finally, given that  $F_0^*$  is the best compromise for Problem 6, it is returned by ACO-SPRI as the final solution. The ACO-SPRI algorithm initializes the number of iterations, the reference portfolio  $C_{ref}$ , the pheromone matrix  $\tau$ , and the set  $SC^+$  (Lines 1 to 4). Each iteration of the algorithm is depicted between Lines 5 and 24. During this process, the ants construct a set of feasible solutions  $SC$  according to the available budget (Lines 8 to 17). After that, the set of solution  $SC$  is improved using the local search algorithm proposed in this paper, producing the new set of solutions  $SC'$  (Line 18). The next step is to perform the non-dominated sorting based on Problem 6 (Line 19), which will produce the set  $F$  that in turn will be used to update the pheromone matrix (Lines 20 and 21). Once the iteration process of ACO-SPRI is ended, the best compromise  $F_0^*$  is identified in the set  $F^*$  (Lines 25 and 26). The following section presents some results derived from the experiments performed to solve instances of PSPSOP using our approach.

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##### Algorithm 1. Local Search

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**Input:**  $SC, Pr, B$

$SC$ : Solution set built by ants

$Pr$ : Set of projects

$B$ : Total budget

**Output:** The new set of portfolios  $SC'$

1:  $N = |Pr|$

2:  $v = \lceil \ln N \rceil$

3:  $SP \leftarrow \text{select\_projects}(v, Pr)$

4:  $S \leftarrow \text{generate\_power\_set}(v)$

5:  $SC' \leftarrow SC$

6: **for each**  $C \in SC$  **do**

7:   **for each**  $s \in S$  **do**

8:      $C_{new} \leftarrow \text{modify\_portfolio}(s, C)$

9:      $BC_{new} \leftarrow \text{evaluate\_budget}(C_{new})$

10:    **if**  $BC_{new} > B$  **then**

11:      $C_{new} \leftarrow \text{remove\_projects}(C_{new}, B)$

12:    **else**

13:      $C_{new} \leftarrow \text{append\_projects}(C_{new}, B)$

14:    **end if**

15:    $SC' \leftarrow SC' \cup C_{new}$

16: **end for**

17: **end for**

18: **return**  $SC'$

---

// randomly taken

// Generate  $v$  binary combinations in  $S$

//  $C$  is modified according to  $s \in S$

//Repair according to the budget value

//using the rank

//using the rank and the available budget

## 5. Computer Experiments

We consider one experiment to analyze the quality of solution and robustness of the algorithm ACO-SPRI with differently sized instances of PSPSOP. The experiments shown below prove that the ACO-SPRI algorithm is able to improve previous approaches.

studied in this work, and it was characterized by four attributes: identifier ( $Id$ ), total budget ( $B$ ), cost of project ( $c$ ) and project ranking ( $r$ ). This instance was taken from Ref. 9, and its structures are presented in Table 1.

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### Algorithm 2: ACO-SPRI

---

**Input:**  $P_r, B, tot\_iter, n_a$   
 $P_r$ : Set of ranked projects  
 $B$ : Total budget  
 $tot\_iter$ : maximum number of iterations  
 $n_a$ : maximum number of ants

**Output:**  $F_0^*$   
 $F_0^*$ : non-dominated solutions to Problem 6 // Section 3

- 1: Initialize:  $Iter \leftarrow 0$
- 2:  $C_{ref} \leftarrow \text{build\_initial\_portfolio}(P_r, B)$  // Section 3
- 3: initialize\\_pheromone\\_matrix() // a tuned parameter in Section 5.1
- 4:  $SC^+ \leftarrow \emptyset$
- 5: **Repeat**
- 6:    $SC \leftarrow \emptyset$
- 7:   **for each**  $ant$  in the colony **do**
- 8:      $C_p \leftarrow \emptyset$
- 9:     **Repeat**
- 10:        $N' \leftarrow \text{init\_candidate\_list}(P_r, C_p)$  // Section 4.2
- 11:        $j \leftarrow \text{rule\_selection}(N')$  // Equation 12
- 12:        $C_p \leftarrow C_p \cup j$
- 13:        $BC_p \leftarrow \text{evaluate\_budget}(C_p)$  // Section 4.1
- 14:       **until**  $P_r - C_p = \emptyset$  **or**  $BC_p > B$
- 15:       **if**  $BC_p > B$  **then**  $C_p \leftarrow C_p - j$
- 16:        $SC \leftarrow SC \cup C_p$
- 17:     **end for each ant colony**
- 18:    $SC' \leftarrow \text{local\_search}(SC, P_r, B)$  // Section 4.3
- 19:    $F \leftarrow \text{fast-non-dominating-sorting}(SC', C_{ref}, n_a)$  // Section 4.1
- 20:   lay\\_pheromon ( $F_0$ ),  $F_0 \in F$  // Equation (7)
- 21:   propagate\\_evaporation ( $F$ ) // Equation (9)
- 22:    $SC^+ \leftarrow SC^+ \cup F_0, F_0 \in F$
- 23:    $iter = iter + 1$
- 24: **until** ( $iter = tot\_iter$ )
- 25:  $F^* \leftarrow \text{fast-non-dominating-sorting}(SC^+, C_{ref})$
- 26: **return**  $F_0^* \in F^*$

---

### 5.1. Description of experiments

With the aim of giving evidence that our approach acceptably approximates the Pareto frontier we solved the same PSPSOP instances reported in the literature. For these instances, an approximation to the Pareto frontier is obtained by ACO-SPRI with the configuration given below.

*A. Test Environment.* The algorithm ACO-SPRI was implemented in Java Programming Language. In order to compute the results, the program was run in a computer with the following characteristics: Intel Core i7 2.8 GHz CPU, 4 GB of RAM, and Mac OS X Lion 10.7.5 (11G63b) as operating system. One instance was

Note that the amounts for budget and cost are expressed in billions of dollars. It is also important to remark that projects with equal ranking  $r$  should have the same priority to be selected, so they must be assigned to the same category (as will be seen below). The solutions of the instance were obtained from 30 independent executions of ACO-SPRI.

*B. Algorithm Configuration.* In order to set the ACO-SPRI algorithm parameters described in Section 4, we experimentally made small changes to the configuration given by Cruz et al.<sup>16</sup>, paper that is a main reference on Ant Colony-based optimization of project portfolios. The tuning process involved exploring 256



combinations of the values of  $\alpha_1$ ,  $\alpha_2$ ,  $w$  and  $\rho$ , which were selected as the most significant parameters by using a correlation analysis.

According to the results of the fine tuning process, the best configuration of values was:  $tot\_iter = 200$  iterations,  $n_a = 300$  ants,  $w = 0.63$ ,  $\eta = 0.1$ ,  $\rho = 0.9$ ,  $\alpha_1 = 0.65$  and  $\alpha_2 = 0.75$ .

These parameter values were used to obtain the experimental results reported in this section. Initially, a heuristic based on dividing symmetrically the set of projects into categories was applied considering that projects with equal ranking  $r$  must remain in the same category. For the experimentation, three categories were considered: 1) priority, 2) satisfactory and 3) acceptable (e.g. in Table 1,  $I_{100}$ , Projects 1-33 are priority, 34-66 are satisfactory and 67-100 are acceptable projects).

As part of the initialization, ACO-SPRI creates a reference portfolio  $C_{ref}$  in order to determine discrepancies derived from the exclusion of a project, when it appears to have merits to belong to the portfolio. Here the reference portfolio is created using a traditional heuristic that considers giving support by following the rank order until the available resources run out. If all the participating projects from Table 1 were funded, the amounts of 5.522 billion would be required; this would take us to support a reduced set of projects. Besides, if the DM accepts some discrepancies, the reference portfolio would be clearly outranked by many other solutions (e.g. in Table 3, the first row shows the reference portfolio  $C_{ref}$ , which is outranked by all others).

**C. Evaluation.** In multi-objective evolutionary optimization, the algorithm performance is measured in terms of the non-dominated solutions of the final generation; this set is called approximate Pareto front. In order to determine if the non-dominated solutions obtained by the algorithm ACO-SPRI have better quality than the solutions reported in the literature, the latter were evaluated with the objective functions of Problem 6.

## 5.2. Analysis of results

We present the results of an experiment designed to verify the validity and advantages of our approach for solving the case of study. The analysis proves that our approach has good potential for solving real resource-allocation problems. To accomplish this, two types of analysis are presented. The computational time was excluded from analysis because it was not reported in the considered literature.

- Quality Analysis.** To determine if our method provides the best quality solutions, the non-dominated solutions of ACO-SPRI were compared against the solutions selected from the literature, considering various realistic attitudes of DMs.
- Robustness Analysis.** We perform 30 independent runs of ACO-SPRI per instance. Here we consider that the algorithm has a reasonable robustness if it provides solutions that exhibit a specified level of quality during twenty or more runs.

Table 1. The test instance ( $N=100$ )

$I_{100}$			$I_{100}$			$I_{100}$			$I_{100}$		
<i>id</i>	<i>c</i>	<i>r</i>	<i>id</i>	<i>c</i>	<i>r</i>	<i>id</i>	<i>c</i>	<i>r</i>	<i>id</i>	<i>c</i>	<i>r</i>
<b>Category Priority</b>			<b>Category Priority</b>			<b>Category Satisfactory</b>			<b>Category Acceptable</b>		
1	84	1	26	31.3	26	51	27.5	51	76	46.5	76
2	125	2	27	26.5	27	52	41.3	52	77	44	77
3	130	3	28	36.3	28	53	29.5	53	78	25.8	78
4	148	4	29	50	29	54	25.3	54	79	38.3	79
5	126	5	30	34.8	30	55	40	55	80	40.7	80
6	137	6	31	48.3	31	56	30.8	56	81	42.8	81
7	96	7	32	46	32	57	39	57	82	43	82
8	84.8	8	33	36.8	33	58	44.5	58	83	32.3	83
9	93	9	<b>Category Satisfactory</b>			59	47.5	59	84	37.8	84
10	122	10	34	34	34	60	36	60	85	44.8	85
11	103	11	35	26	35	61	28.5	61	86	27	86
12	142	12	36	31.8	36	62	29	62	87	39.5	87
13	106	13	37	29.8	37	63	30.3	63	88	30	88
14	98.3	14	38	37.3	38	<b>Category Acceptable</b>			89	37.5	89
15	101	15	39	26.8	39	64	49.5	64	90	49	90
16	83.3	16	40	43.8	40	65	33	65	91	41.8	91
17	110	17	41	27.3	41	66	38.5	66	92	39.3	92
18	107	18	42	47	42	67	33.5	67	93	34.5	93
19	135	19	43	41	43	68	48.5	68	94	49.8	94
20	97.5	20	44	30.5	44	69	35	69	95	48	95
21	128	21	45	45.3	45	70	28.8	70	96	29.3	96
22	114	22	46	26.3	46	71	25.5	71	97	47.8	97
23	107	23	47	25.5	47	72	40.3	72	98	42.3	98
24	94.5	24	48	44.3	48	73	38.8	73	99	46.3	99
25	43.5	25	49	48.8	49	74	46.8	74	100	39.8	100
			50	33.3	50	75	37	75	B=2500		

Results derived from  $I_{100}$ , an instance with 100 projects

The average computer time of ACO-SPRI was 91 minutes and 51 seconds. Table 2 and 3 show a set of non-dominated solutions in the objective and project space. No solution reported in the literature was included in these tables because after being evaluated by the functions of the Problem 6 objective, all of them presented unacceptable discrepancies.

The following facts can be concluded from Table 2:

- As was mentioned before, the reference portfolio is denoted as  $C_{ref}$ . If the DM accepts some discrepancies, the reference portfolio would be clearly outranked by many other solutions, in both the cardinality of the supported projects and their distribution per categories.
- With the exception of the reference portfolio, solutions with less significant discrepancies are  $ACO-SPRI_6$  and  $ACO-SPRI_4$ . The first one differs relatively little from the reference portfolio, but

broadly outperforms it in the total amount of supported projects, and the number of projects belonging to the priority category.  $ACO-SPRI_6$  would probably be the choice of a DM who shows aversion to detach him/herself from the reference portfolio.

- iii. A DM specially inclined towards supporting the priority category projects would probably choose  $ACO-SPRI_4$ .
- iv. A DM, whose preferences are mainly directed towards the total amount of supported projects and prepared to manage high levels of discrepancies, will be interested in the solutions  $ACO-SPRI_1$ ,  $ACO-SPRI_2$ ,  $ACO-SPRI_3$ ,  $ACO-SPRI_9$  and  $ACO-SPRI_{10}$ .
- v. A compromise solution between the attitudes described above is the solution  $ACO-SPRI_{11}$ , which has a manageable level of discrepancies and satisfactory levels of the total amount of projects and priority category projects.

Table 2. Results in the objective space

Portfolio Solution C	Values of objective functions										Card (C)
	$N_1$	$n_{wd1}$	$n_{sd1}$	$N_2$	$n_{wd2}$	$n_{sd2}$	$N_3$	$n_{wd3}$	$n_{sd3}$	P	
$C_{ref}$	22	0	0	0	0	0	0	0	0	89.5	22
$ACO-SPRI_1$	20	3	3	8	0	0	11	0	0	57.79	39
$ACO-SPRI_2$	20	3	3	7	0	0	12	0	0	56.92	39
$ACO-SPRI_3$	21	3	2	5	0	0	12	0	0	58.18	38
<b><math>ACO-SPRI_4</math></b>	<b>27</b>	<b>4</b>	<b>0</b>	<b>6</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>81.03</b>	<b>33</b>
$ACO-SPRI_5$	25	2	1	5	0	0	0	0	0	83	30
<b><math>ACO-SPRI_6</math></b>	<b>26</b>	<b>2</b>	<b>1</b>	<b>4</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>83.3</b>	<b>30</b>
$ACO-SPRI_7$	26	2	1	1	0	0	3	0	0	79.43	30
$ACO-SPRI_8$	25	2	1	2	0	0	3	0	0	79.46	30
$ACO-SPRI_9$	14	6	3	17	0	0	17	0	0	49.85	48
$ACO-SPRI_{10}$	18	12	1	15	0	0	16	0	0	50.65	49
<b><math>ACO-SPRI_{11}</math></b>	<b>25</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>85.69</b>	<b>26</b>

The best solutions ( $ACO-SPRI_4$ ,  $ACO-SPRI_6$  and  $ACO-SPRI_{11}$ ) according to the DM's attitudes taken into consideration were consistently present in 26 out of 30 runs.

Tables 4 and 5 show the characteristics of the solutions ( $Sol_1$  and  $Sol_2$ ) reported in Ref. 9. These solutions are clearly outperformed by those suggested in our proposal in both the portfolio cardinality and the support to the priority category.

It should also be noted that the solutions in Tables 4-5 contain an unacceptable discrepancy: the unsupported project 20 belongs to the reference portfolio and its budgetary requirement (97.5 millions) is slightly lower than the average cost of the supported projects of its category.

Table 6 shows two of our best solutions when evaluated in terms of the objectives of the model proposed in Ref.9. Please note that even though  $ACO-SPRI_4$  contains 18 of 22 projects belonging to the reference portfolio (with only 4 weak discrepancies according to the model proposed herein), it would be assigned a large number of strict discrepancies if it were evaluated according to the model proposed in Ref.9 and described in Section 2. This effect that we have called inflation of discrepancies will avoid the DM accept these solutions for considering as the best compromise.

Table 4. Solutions suggested in Ref. 9

Solution	$N_1$	$n_{wd1}$	$n_{sd1}$	$N_2$	$n_{wd2}$	$n_{sd2}$	$N_3$	$n_{wd3}$	$n_{sd3}$	P	Card	ACSP
$Sol_1$	24	0	0	0	0	0	0	0	0	87.62	24	102.19
$Sol_2$	25	0	0	0	0	0	0	0	0	86.96	25	94.57

Table 6. Solutions evaluated by the model in Ref. 9

Solution	$n_{pr}$	$n_a$	$n_s$	$n_w$	Card (C)
$ACO-SPRI_6$	30	0	57	0	30
$ACO-SPRI_4$	33	0	98	0	33

Table 3. Results in the decision space

Portfolio Solution C	Selection of Projects																																		
	Priority Category																																		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33		
$C_{ref}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
$ACO-SPRI_1$	1	1	1	1	1	1	0	1	1	0	1	0	0	1	1	1	0	0	0	1	1	0	0	1	1	0	0	1	1	0	1	1	0	1	1
$ACO-SPRI_2$	1	1	1	1	1	1	0	1	1	0	1	0	0	1	1	1	0	0	0	1	1	0	0	0	1	1	0	1	1	0	1	1	0	1	1
$ACO-SPRI_3$	1	1	1	1	1	1	0	1	1	0	1	0	0	1	1	1	0	1	0	1	1	0	0	1	1	0	0	1	1	0	1	1	0	1	1
$ACO-SPRI_4$	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
$ACO-SPRI_5$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0	1	1	1	1	1	0	1	0	0	1	1	
$ACO-SPRI_6$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0	1	1	1	1	1	1	1	0	0	1	1	
$ACO-SPRI_7$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0	1	1	0	1	1	1	1	1	1	1	0	
$ACO-SPRI_8$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0	1	1	0	1	1	1	1	1	0	1	1	
$ACO-SPRI_9$	1	1	1	1	1	0	0	0	1	0	0	0	0	0	1	0	1	0	0	0	0	0	0	1	1	1	0	0	1	1	1	1	1	0	
$ACO-SPRI_{10}$	1	1	0	0	0	0	0	1	0	0	1	0	0	0	1	1	0	1	0	1	1	0	1	1	0	1	1	1	1	1	1	1	1	0	
$ACO-SPRI_{11}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	0	1	0	0	1	0	1	0	0	1	0	1	

Table 3. (Continued)

Portfolio Solution C	Selection of Projects																																													
	Satisfactory Category																																													
	3	3	3	3	3	3	4	4	4	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	6	6	6	6	6	6	6	6					
$C_{ref}$	0	4	5	6	7	8	9	0	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0	0	0	0	0	0	0	
ACO-SPRI <sub>1</sub>	0	0	0	0	0	0	0	1	1	0	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	1	0	1	0	1	0	0	0	0	0	0		
ACO-SPRI <sub>2</sub>	0	0	0	0	0	0	0	1	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	1	0	1	0	1	0	0	0	0	0		
ACO-SPRI <sub>3</sub>	0	0	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0		
ACO-SPRI <sub>4</sub>	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
ACO-SPRI <sub>5</sub>	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
ACO-SPRI <sub>6</sub>	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
ACO-SPRI <sub>7</sub>	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
ACO-SPRI <sub>8</sub>	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
ACO-SPRI <sub>9</sub>	0	0	1	1	1	1	0	1	0	1	1	0	1	0	0	1	1	1	0	1	1	1	0	1	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
ACO-SPRI <sub>10</sub>	0	1	1	1	0	1	1	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	
ACO-SPRI <sub>11</sub>	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	

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Table 5. Additional information about solutions from Ref.9

<i>Solution</i>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
<i>Sol</i> <sub>1</sub>	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0	1	1	1	1	0	0	0	0
<i>Sol</i> <sub>2</sub>	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0	1	1	1	1	1	0	0	0

<i>Solution</i>	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66
<i>Sol</i> <sub>1</sub>	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
<i>Sol</i> <sub>2</sub>	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

<i>Solution</i>	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
<i>Sol</i> <sub>1</sub>	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
<i>Sol</i> <sub>2</sub>	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	

## 6. Some conclusions

Multi-criteria analysis is a powerful tool for handling conflicting preferences in decision-making problems concerning projects, but only project rankings can be derived from these applications. In project portfolio problems, when as a result of some previous evaluation the only available decision maker preference information is a project rank ordering, the use of formal methods to optimize the portfolio is critically limited. The usual way of allocating resources according to the priority derived from the rank ordering is questionable when at least one of the following conditions is held:

- The DM is willing to support as many projects as possible; so he/she accepts trade-offs between the impact of particular projects and the number of supported projects;
- The DM is averse to exert excessive effort for modeling his preferences;
- The DM's preferences are ambiguous or contradictory as happens when the decision-maker is a non-homogenous group;
- The DM does not trust plenary regarding the quality information conveyed in the ranking;
- The DM is averse to projects whose costs exceed the average considerably.

This proposal uses a project ranking as input information, so it can be considered as complementary to multi-criteria analysis in the framework of resource allocation among competing projects. Compared with previous approaches, the main features of the new method are:

- a) An improved multi-criteria description of the portfolio quality, measured through proxy variables, which takes into account the number and level of priority of the supported projects, discrepancies with respect to the portfolio derived from the rank ordering, and an indicator of the average rank of the supported projects;
- b) An ant colony multi-objective metaheuristic to solve the optimization Problem 6 and find the best compromise.

In several examples of real size, our proposal performed very well in the sense of the quality of solutions, with reasonable robustness. We may point out that:

- i) The reference solution (that strictly follows the rank ordering information) is clearly outperformed by other solutions that increase the number of projects in the portfolio and the number of priority projects;
- ii) Though admitting some discrepancies, the solutions with portfolio cardinality much higher than reference may be acceptable for the DM. A DM exhibiting certain aversion to expensive projects would find revenues in our method, particularly with respect to what we call reference solutions;
- iii) Considerable increases in portfolio cardinality can be found without introducing strong discrepancies with respect to the reference portfolio;
- iv) Although more complex, the proposed multi-criteria description of portfolio quality improves the modeling of decision maker preferences when compared with previous approaches. The new model eliminates the “inflation of discrepancies” effect, which appears as one of the serious drawbacks of other proposals. Besides, this model allows centering the DM attention on project’s different levels of priority.

As a consequence, our proposal outperforms previous approaches concerning the quality of solutions. Although the experimental evidence is still limited, it seems that the advantage of our results increases with the complexity of the problem.

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