International Journal of Computational Intelligence Systems, Vol. 9, No. 6 (2016) 1028-1040

Revenue Sharing Contract in a Multi-Echelon Supply Chain with Fuzzy Demand and Asymmetric Information

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Received 5 March 2015

Accepted 27 June 2016

Abstract

In this paper, we consider the revenue sharing contract between supply chain actors in a multi-echelon supply chain, where the demand of the customers and retail price are fuzzy variables. The centralized decision making system and a coordinating contract, namely, the revenue sharing contract with fuzzy demand and asymmetric information are proposed. To derive the optimal solutions, the fuzzy set theory is applied for solving these models. Finally, the optimal results of proposed models are illustrated with three numerical experiments. The effects of the fuzziness of retail price and demand, different contract parameters on the optimal strategies for supply chain actors are also analyzed.

Keywords: Revenue sharing contract, fuzzy variables, asymmetric information, multi-echelon supply chain

1. Introduction

Over the past ten years, revenue sharing (RS) contract has attracted a lot of attention from both scholars and practitioners, and has achieved much success in film studios and video rental industry. In the revenue sharing contract, the sum of the expected profits of all supply chain actors is the same as the maximum expected profit derived in the centralized decision making system.

An adequate number of papers discussed the RS contract in the two-echelon supply chains under a linear or random demand setting. Cachon and Lariviere[1] studied the strengths and limitations of the RS contract. Hou *et al.*[2] considered a RS and bargaining model in which the profit of the retail was sensitive to the lead time of the manufacturer. Giannoccaro and Pontrandolfo[3] developed an agent-based systems model to study the negotiation problem of the RS contract. Chen *et al.*[4] investigated the problem of channel coordination by using RS contract, where the demands for customers are non-linear functions of the retail price and the size of self-space. Sheu[5] proposed a RS contract to coordinate a supplier-retailer distribution channel with three types of price promotion patterns to customers. Kunter[6] investigated how to coordinate a manufacturer-retailer channel using cost and RS contact. Chen and Cheng[7] developed the price-dependent and price-independent RS contracts models in a vendor-buyer channel. They found that in price-independent RS contracts the supply chain actors can obtain higher profits than those price-dependent RS contracts. Sarathi *et al.*[8] used a mixed RS and quantity discounts contract to coordinate a two-echelon supply chain. Govindan and Popiuc[9] developed a RS contract aimed at coordinating a reverse supply chain in the personal computers industry. Recently, socially responsible of the supply chain actors was taken into

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account in the RS contract. Panda[10] used the RS contract to coordinate a socially responsible two stage supply chain with considering two cases, corporate social responsible retailer and corporate social responsibility manufacturer in a linear demand. Hsueh[11]also developed a RS contract embedding corporate social responsibility for coordinating a two-echelon supply chain with random demand. Linh and Hong[12] discussed how to set the wholesale and RS ratio in a two-period model by using the RS contract. Palsule-Desai[13] also studied the coordination problem of the supply chain in a two-period model via revenue-dependent RS contract. In addition, some studies have been done on analyzing competition problems of the supply chain actors in the RS contract. Yao *et al.* [14] used the RS contract to coordinate the supply chain with two competing retailers and one manufacturer. Pan *et al.* [15] compared RS contract to wholesale price policy with two manufacturers and one retailer, two retailers and one manufacturer under different channel power structures. Ouardighi and Kim[16] developed a differential game model with wholesale price and RS contracts between two manufacturers and one supplier. Krishnan and Winter[17] studied the coordinating role of RS contract with two competing retailers. Zhang *et al.* [18] studied the RS contract in a supply chain consisting of two competing retailers with demand disruption. Recently, Chakraborty *et al.*[19] studied the RS mechanisms with two competing manufacturers and one retailer under a linear stochastic demand. Zhang *et al.* [20] proposed a RS and cooperative investment contract for deteriorating items to coordinate a supply chain. Arani *et al.*[21] proposed a novel mixed revenue-sharing option contract for coordinating a retailer–manufacturer supply chain.

Some literature focused on the RS contract in a multi-echelon supply chain. For example, Giannoccaro and Pontrandolfo[22] studied the RS contract in a three-echelon supply chain. Rhee *et al.* [23, 24] proposed a spanning RS contract to coordinate a multi-stage supply chain with random demand. Jiang *et al.* [25] developed a spanning RS contract comprising two competing manufacturers, one distributor and one retailer with a linear demand function in a three-echelon supply chain. Feng *et al.*[26] studied the RS contract with more than one actor at some echelons in a multi-echelon supply chain. Only a few of articles addressed the RS contract in a fuzzy environment. Wang *et al.* [27] studied the RS contract with two fuzzy demand forms in a two-stage supply chain. Sang[28] developed a RS contract to coordinate a supply chain with one supplier and multiple retailers in a fuzzy demand environment.

All studies mentioned above mainly discussed the RS contract under a linear or probabilistic market demand and known retail price. However, in today's highly competitive market, shorter and shorter product life cycles make the useful statistical data less and less available. Thus, in recent years, fuzzy set theory has been adopted by more and more scholars to solve fuzzy supply chain problems. Zhou *et al.* [29] studied the price decision problem between a manufacturer and one retailer, where the demand and the manufacturing cost were considered as fuzzy variables. Sang[30] extended this work to a fuzzy supply chain with two competitive retailers and one manufacturer, where two retailers pursued the Cournot and Stackelberg games. Dang and Hong[31] developed a Cournot game in a fuzzy supply chain, where the demand and costs were treaded as triangular fuzzy numbers. Dang *et al.*[32] further studied this fuzzy Cournot game with multiple firms. Zhao *et al.*[33] considered the pricing decisions in a two-echelon supply chain, where one manufacturer sold his substitutable products to two competing retailers. Zhao *et al.*[34] studied the service and price decisions with two competing manufacturers and one retailer in a fuzzy environment. Zhao and Wang[35] also studied the pricing and service decisions with one manufacturer and two retailers in fuzzy linear demand setting. Wei and Zhao[36, 37]studied the pricing decisions problem of retail competition and reverse channel decisions in a fuzzy decision environment. Khamseh *et al.*[38] studied the pricing policies of a fuzzy two-echelon supply chain with two competing manufacturers in both cooperation and non-cooperation situations. Sang[39] studied the pricing decisions of a fuzzy two-echelon supply chain with two competing manufacturers in which the attitudes of the members to the risk were considered. Recently, some researchers developed the fuzzy newsboy model in a supply chain. Xu and Zhai[40, 41] developed a fuzzy newsboy model in a two-stage supply chain, in which demands were considered as triangular fuzzy variables. Yu and Jin[42] developed a return policy to coordinate the supply chain actors in which the demand and retail price were assumed to be fuzzy variables. Yu *et al.* [43] also studied the fuzzy newsboy model in a fuzzy price-dependent demand setting. In addiction, Chang and Yeh[44] explored a return policy with fuzzy demand in a two-stage supply chain, where unsatisfactory products were returned to the manufacturer. Zhang *et al.*[45] studied the buyback contract for coordinating a fuzzy two-stage supply chain, where demand was assumed to be a fuzzy random variable.

As far as we know, there are no studies on the RS contract in a fuzzy multi-stage supply chain. However, in real life, the rapid change of the product life cycle makes the parameters of the supply chain models more and more uncertain. These uncertainties may be the market demand, operations costs of the product, etc. In addition, a supply chain is usually made up more than two stages. Then, a natural question is how to design a coordination mechanism in a multi-stage supply chain. Therefore, in this article, we concentrate on the RS contract in a multi-stage supply chain, in which the demand, retail price and operational costs are all fuzzy. Furthermore, we discuss the impact of fuzziness for demand and retail price on the RS policy.

The contributions of this article are as follows. Firstly, we extend the works of Sang[28, 30] to a multiple echelon supply chain in a fuzzy demand environment. Secondly, we study the RS contract in an asymmetric information environment in which both the market demand and the retail price are considered as fuzzy variables. Thirdly, we analyze the impacts of the fuzziness of retail price and market demand on the optimal policies in the RS contract.

The reminder of paper is as follows. Some definitions and propositions about fuzzy set theory are introduced in Section 2. Section 3 describes the problem. In Section 4, we develop two fuzzy supply chain models with asymmetric information. In Section 5, three numerical examples are given to illustrate the solutions for proposed models. Section 6 summarizes the work and indicates future work directions.

2. Preliminaries

Definition 1. The fuzzy set $\tilde{A}=(a,b,c)$ is said to be a triangular fuzzy variable if it has a following membership function

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-a}{b-a}, & a\le x\le b,\\ \dfrac{c-x}{c-b}, & b\le x\le c,\\ 0, & x\notin(b,c).\end{cases}\qquad(1)$$

where a, b and c are real numbers with $a<b<c$. For $x\in[a,b]$, $\tilde{A}_L(x)=\dfrac{x-a}{b-a}$ is continuous and strictly increasing with respect to x. For $x\in(b,c]$, $\tilde{A}_R(x)=\dfrac{c-x}{c-b}$ is continuous and strictly decreasing with respect to x.

Definition 2. Given $\alpha\in[0,1]$, the set $\tilde{A}(\alpha)=\{x\mid\mu_{\tilde{A}}(x)\ge\alpha\}$ is said to be the α cut set of $\tilde{A}$ and is denoted by the interval $\left[\tilde{A}_L^{-1}(\alpha),\tilde{A}_R^{-1}(\alpha)\right]$, with

$$\tilde{A}_L^{-1}(\alpha)=\inf\left\{x\in R:\mu_{\tilde{A}(x)}\ge\alpha\right\},$$
$$\tilde{A}_R^{-1}(\alpha)=\sup\left\{x\in R:\mu_{\tilde{A}(x)}\ge\alpha\right\}.\qquad(2)$$

where $\tilde{A}_L^{-1}(\alpha)$ and $\tilde{A}_R^{-1}(\alpha)$ are the left and right cut sets of $\tilde{A}(\alpha)$.

Example 1. Given $\alpha\in[0,1]$, the α cut set of $\tilde{A}=(a,b,c)$ can be given as

$$\tilde{A}_L^{-1}(\alpha)=a+(b-a)\alpha\text{ , }\tilde{A}_R^{-1}(\alpha)=c+(b-c)\alpha.\qquad(3)$$

Proposition 1. Given $\alpha\in[0,1]$, let $\tilde{A}$ be a positive triangular fuzzy variable with α cut set $\left[\tilde{A}_L^{-1}(\alpha),\tilde{A}_R^{-1}(\alpha)\right]$, then

$$k\tilde{A}(\alpha)=\begin{cases}\left[k\tilde{A}_L^{-1}(\alpha),k\tilde{A}_R^{-1}(\alpha)\right], & k\in R^+,\\ \left[k\tilde{A}_R^{-1}(\alpha),k\tilde{A}_L^{-1}(\alpha)\right], & k\in R^-.\end{cases}\qquad(4)$$

Proposition 2. Given $\alpha\in[0,1]$, let $\tilde{B}$ and $\tilde{C}$ be two positive triangular fuzzy numbers with α cut set $\left[\tilde{B}_L^{-1}(\alpha),\tilde{B}_R^{-1}(\alpha)\right]$ and $\left[\tilde{C}_L^{-1}(\alpha),\tilde{C}_R^{-1}(\alpha)\right]$, respectively. Then we have

$$(1)\ \tilde{B}(\alpha)+\tilde{C}(\alpha)=\left[\tilde{B}_L^{-1}(\alpha)+\tilde{C}_L^{-1}(\alpha),\tilde{B}_R^{-1}(\alpha)+\tilde{C}_R^{-1}(\alpha)\right],$$
$$(2)\ \tilde{B}(\alpha)-\tilde{C}(\alpha)=\left[\tilde{B}_L^{-1}(\alpha)-\tilde{C}_R^{-1}(\alpha),\tilde{B}_R^{-1}(\alpha)-\tilde{C}_L^{-1}(\alpha)\right],$$
$$(3)\ \left(\tilde{A}\tilde{B}\right)_L^{-1}(\alpha)=\tilde{A}_L^{-1}(\alpha)\tilde{B}_L^{-1}(\alpha),$$
$$(4)\ \left(\tilde{A}\tilde{B}\right)_R^{-1}(\alpha)=\tilde{A}_R^{-1}(\alpha)\tilde{B}_R^{-1}(\alpha).\qquad(5)$$

Proposition 3 (Liu and Liu[46]). Let $\tilde{A}$ be a positive triangular variable with α cut set $\left[\tilde{A}_L^{-1}(\alpha),\tilde{A}_R^{-1}(\alpha)\right]$. If the expected value of $\tilde{A}$ exists, then

$$E\left[\tilde{A}\right]=\frac{1}{2}\int_0^1\left[\tilde{A}_L^{-1}(\alpha)+\tilde{A}_R^{-1}(\alpha)\right]\mathrm{d}\alpha. \tag{6}$$

Proposition 4 (Liu and Liu[46]). Let $\tilde{A}$ and $\tilde{B}$ be two independent positive triangular fuzzy variables. If their expected values exist, then for any numbers *m* and *n*

$$E\left[m\tilde{A}+n\tilde{B}\right]=mE\left[\tilde{A}\right]+nE\left[\tilde{B}\right]. \tag{7}$$

3. Problem Descriptions

Consider a linear supply chain structure with only one actor at each of the $n \geq 2$ echelons in a single period setting. Denote the supply chain actor 1 as the most downstream company and actor *n* as the most upstream company. The supply chain actor 1 sells the product to customer in a high uncertain demand setting. The uncertain demand faced by the supply chain actor 1 is supposed to be a triangular fuzzy number $\tilde{D}=(d_1,d_2,d_3)$, where $0<d_1<d_2<d_3$. d_2 is the most possible value of the demand $\tilde{D}$, this means that the demand is about d_2. d_1 and d_3 respectively denote the minimum and maximum values of the demand. The fuzzy demand has the following membership function $\mu_{\tilde{D}}(x)$

$$\mu_{\tilde{D}}(x)=\begin{cases} L(x), & x\in[d_1,d_2],\\ R(x), & x\in(d_2,d_3],\\ 0, & x\notin(d_1,d_3). \end{cases} \tag{8}$$

For convenience, the left and right spread functions of the fuzzy demand $\tilde{D}$ are denoted by $L(x)$ and $R(x)$.

In the asymmetric information environment, the supply chain actor 1 does not share his complete information of retail price with other actors. Therefore, the retail price will be estimated by the decision makers. Assuming the fuzzy retail price $\tilde{p}=(p-\Delta_1,p,p+\Delta_2)$ is considered as a positive triangular fuzzy variable, and denoted by a general membership function $\mu_{\tilde{p}}(x)$:

$$\mu_{\tilde{p}}(x)=\begin{cases} \tilde{p}_L(x), & x\in[p-\Delta_1,p],\\ \tilde{p}_R(x), & x\in(p,p+\Delta_2],\\ 0, & ix\notin(p-\Delta_1,p+\Delta_2). \end{cases} \tag{9}$$

The supply chain actor *i* faces the wholesale price w_{i+1} offered by actor $i+1$, and per unit fuzzy operational cost $\tilde{c}_i=(c_{i1},c_{i2},c_{i3})$, and can set his selling price w_i, $i=2,3,\ldots,n$. For convenience of notation it is assumed that $w_{n+1}=0$. Thus, the supply chain actor 1 faces a wholesale price w_2 and fuzzy operational cost $\tilde{c}_1$. Furthermore, actor 1 sells his product to the customers with retail price *p*, and can choose his order quantity *q*. Let $\tilde{\Pi}_1$ be the fuzzy profit for supply chain actor 1, $\tilde{\Pi}_i$ be the fuzzy profit for actor $i, i=2,3,\ldots,n$, and $\tilde{\Pi}_{SC}$ be the fuzzy profit for whole supply chain.

The assumptions related to this paper are given as follows:

Assumption 1(Risk neutral assumption). The supply chain actors are all risk neutral, and they maximize the fuzzy expected profits.

Assumption 2(Positive assumption). We assume $w_i>w_{i+1}+E[\tilde{c}_i]$, $i=2,3,\ldots,n$, and $p>\sum_{i=1}^{n}E[\tilde{c}_i]$. These ensure that the supply chain actors can obtain their positive fuzzy profits.

4. Fuzzy Supply Chain Models with Asymmetric Information

In this section, we develop the centralized decision-making system and one coordinating contract, namely, RS contract with fuzzy demand and asymmetric information, which can tell the supply chain actors how to make their decisions in a fuzzy environment.

4.1. Fuzzy centralized decision-making system with asymmetric information

In the fuzzy centralized decision-making system, all the actors in the supply chain make cooperation, which can be regarded as the supply chain possessed by an integrated decision maker. Then, we can get the fuzzy profit function for supply chain system as

$$\tilde{\Pi}_{SC}=\tilde{p}\min\left(q,\tilde{D}\right)-\sum_{i=1}^{n}\tilde{c}_i q. \tag{10}$$

The supply chain actors try to maximize their total fuzzy expected profit by setting the optimal order quantity *q*. Thus, the fuzzy model solved by the integrated decision maker is given by

$$\operatorname{Max}_q E\left[\tilde{\Pi}_{SC}\right]=E\left[\tilde{p}\min\left(q,\tilde{D}\right)-\sum_{i=1}^{n}\tilde{c}_i q\right]$$
$$\text{s.t. } d_1\leq q\leq d_3. \tag{11}$$

Since $\tilde{D}=(d_1,d_2,d_3)$ is a positive triangular fuzzy variable, then optimal order quantity *q* set by the integrated decision maker has two cases, $q\in[d_1,d_2]$ and $q\in(d_2,d_3]$.

Theorem 1. *If $q \in [d_1, d_2]$, then the optimal order quantity q^* satisfies the following equation*

$$\frac{1}{2}\int_0^{L(q^*)} \tilde{p}_L^{-1}(\alpha)\,\mathrm{d}\alpha = E[\tilde{p}] - \sum_{i=1}^n E[\tilde{c}_i]. \quad (12)$$

Proof. If $q \in [d_1, d_2]$, then the α cut set of $\min(q, \tilde{D})$ is

$$\left(\min\left(q,\tilde{D}\right)\right)(\alpha) = \begin{cases} \left[L^{-1}(\alpha), q\right], & \alpha \in [0, L(q)], \\ [q, q], & \alpha \in (L(q), 1]. \end{cases}$$

If $\alpha \in [0, L(q)]$, we can get the α cut set of $\tilde{\Pi}_{SC}(\alpha)$ as

$$\begin{aligned}\tilde{\Pi}_{SC}(\alpha) &= \left[\tilde{p}_L^{-1}(\alpha)L^{-1}(\alpha), \tilde{p}_R^{-1}(\alpha)q\right] \\ &\quad - \left[\sum_{i=1}^n \tilde{c}_{iL}^{-1}(\alpha)q, \sum_{i=1}^n c_{iR}^{-1}(\alpha)q\right] \\ &= \Big[\tilde{p}_L^{-1}(\alpha)L^{-1}(\alpha) - \sum_{i=1}^n c_{iR}^{-1}(\alpha)q, \\ &\qquad \tilde{p}_R^{-1}(\alpha)q - \sum_{i=1}^n \tilde{c}_{iL}^{-1}(\alpha)q\Big].\end{aligned}$$

If $\alpha \in (L(q), 1]$, we can get the α cut set of $\tilde{\Pi}_{SC}(\alpha)$ as

$$\begin{aligned}\tilde{\Pi}_{SC}(\alpha) &= \left[\tilde{p}_L^{-1}(\alpha)q, \tilde{p}_R^{-1}(\alpha)q\right] \\ &\quad - \left[\sum_{i=1}^n \tilde{c}_{iL}^{-1}(\alpha)q, \sum_{i=1}^n c_{iR}^{-1}(\alpha)q\right] \\ &= \Big[\tilde{p}_L^{-1}(\alpha)q - \sum_{i=1}^n c_{iR}^{-1}(\alpha)q, \\ &\qquad \tilde{p}_R^{-1}(\alpha)q - \sum_{i=1}^n \tilde{c}_{iL}^{-1}(\alpha)q\Big].\end{aligned}$$

From Eq.(6), the fuzzy expected profit $E[\tilde{\Pi}_{SC}]$ can be obtained as

$$\begin{aligned}E[\tilde{\Pi}_{SC}] &= \frac{1}{2}\int_0^{L(q)}\Big(\tilde{p}_L^{-1}(\alpha)L^{-1}(\alpha) - \sum_{i=1}^n c_{iR}^{-1}(\alpha)q \\ &\qquad + \tilde{p}_R^{-1}(\alpha)q - \sum_{i=1}^n \tilde{c}_{iL}^{-1}(\alpha)q\Big)\mathrm{d}\alpha \\ &\quad + \frac{1}{2}\int_{L(q)}^1\Big(\tilde{p}_L^{-1}(\alpha)q - \sum_{i=1}^n c_{iR}^{-1}(\alpha)q \\ &\qquad + \tilde{p}_R^{-1}(\alpha)q - \sum_{i=1}^n \tilde{c}_{iL}^{-1}(\alpha)q\Big)\mathrm{d}\alpha \\ &= -\frac{1}{2}\int_0^{L(q)} \tilde{p}_L^{-1}(\alpha)\left(q - L^{-1}(\alpha)\right)\mathrm{d}\alpha \\ &\quad + \left(E[\tilde{p}] - \sum_{i=1}^n E[\tilde{c}_i]\right)q. \quad (13)\end{aligned}$$

From Eq.(13), the first order condition of $E[\tilde{\Pi}_{SC}]$ is

$$\frac{\mathrm{d}E[\tilde{\Pi}_{SC}]}{\mathrm{d}q} = -\frac{1}{2}\int_0^{L(q)} \tilde{p}_L^{-1}(\alpha)\,\mathrm{d}\alpha + E[\tilde{p}] - \sum_{i=1}^n E[\tilde{c}_i].$$

The second order condition of $E[\tilde{\Pi}_{SC}]$ is

$$\frac{\mathrm{d}^2 E[\tilde{\Pi}_{SC}]}{\mathrm{d}q^2} = -\frac{1}{2}\tilde{p}_L^{-1}\left(L(q)\right)L'(q).$$

Note that the second order condition is negative, since $L(q)$ is increasing about q with $L'(q) > 0$ and $\tilde{p}_L^{-1}(L(q)) > 0$. Therefore, $E[\tilde{\Pi}_{SC}]$ is a concave function with respect to q.

Hence, we can get the optimal order quantity q^* by letting the first order condition be zero

$$-\frac{1}{2}\int_0^{L(q)} \tilde{p}_L^{-1}(\alpha)\,\mathrm{d}\alpha + E[\tilde{p}] - \sum_{i=1}^n E[\tilde{c}_i] = 0. \quad (14)$$

Solving Eq.(14), we can get q^* as shown in Eq.(12). Theorem 1 is proved. □

Theorem 2. *If $q \in (d_2, d_3]$, then the optimal order quantity q^* satisfies the following equation*

$$\frac{1}{2}\int_0^{R(q^*)} \tilde{p}_R^{-1}(\alpha)\,\mathrm{d}\alpha = \sum_{i=1}^n E[\tilde{c}_i]. \quad (15)$$

Proof. If $q \in (d_2, d_3]$, then the α cut set of $\min(q, \tilde{D})$ is

$$\left(\min\left(q,\tilde{D}\right)\right)(\alpha) = \begin{cases} \left[L^{-1}(\alpha), q\right], & \alpha \in [0, R(q)], \\ \left[L^{-1}(\alpha), R^{-1}(\alpha)\right], & \alpha \in (R(q), 1]. \end{cases}$$

If $\alpha \in [0, R(q)]$, we can get the α cut set of $\tilde{\Pi}_{SC}(\alpha)$ as

$$\begin{aligned}\tilde{\Pi}_{SC}(\alpha) &= \left[\tilde{p}_L^{-1}(\alpha)L^{-1}(\alpha), \tilde{p}_R^{-1}(\alpha)q\right] \\ &\quad - \left[\sum_{i=1}^n \tilde{c}_{iL}^{-1}(\alpha)q, \sum_{i=1}^n c_{iR}^{-1}(\alpha)q\right] \\ &= \Big[\tilde{p}_L^{-1}(\alpha)L^{-1}(\alpha) - \sum_{i=1}^n c_{iR}^{-1}(\alpha)q, \\ &\qquad \tilde{p}_R^{-1}(\alpha)q - \sum_{i=1}^n \tilde{c}_{iL}^{-1}(\alpha)q\Big].\end{aligned}$$

If $\alpha \in (R(q), 1]$, we can get the α cut set of $\tilde{\Pi}_{SC}(\alpha)$ as

$$\tilde{\Pi}_{SC}(\alpha) = \left[\tilde{p}_L^{-1}(\alpha)L^{-1}(\alpha), \tilde{p}_R^{-1}(\alpha)R^{-1}(\alpha)\right]$$

$$-\left[\sum_{i=1}^{n}\tilde{c}_{iL}^{-1}(\alpha)q,\sum_{i=1}^{n}\tilde{c}_{iR}^{-1}(\alpha)q\right]$$
$$=\left[\tilde{p}_L^{-1}(\alpha)L^{-1}(\alpha)-\sum_{i=1}^{n}c_{iR}^{-1}(\alpha)q,\right.$$
$$\left.\tilde{p}_R^{-1}(\alpha)R^{-1}(\alpha)-\sum_{i=1}^{n}\tilde{c}_{iL}^{-1}(\alpha)q\right].$$

From Eq.(6), the fuzzy expected profit $E\left[\tilde{\Pi}_{SC}\right]$ can be obtained as

$$E\left[\tilde{\Pi}_{SC}\right]=\frac{1}{2}\int_0^{R(q)}\Big(\tilde{p}_L^{-1}(\alpha)L^{-1}(\alpha)-\sum_{i=1}^{n}\tilde{c}_{iL}^{-1}(\alpha)q$$
$$+\tilde{p}_R^{-1}(\alpha)q-\sum_{i=1}^{n}\tilde{c}_{iL}^{-1}(\alpha)q\Big)\mathrm{d}\alpha$$
$$+\frac{1}{2}\int_{R(q)}^{1}\Big(\tilde{p}_L^{-1}(\alpha)L^{-1}(\alpha)-\sum_{i=1}^{n}\tilde{c}_{iR}^{-1}(\alpha)q$$
$$+\tilde{p}_R^{-1}(\alpha)R^{-1}(\alpha)-\sum_{i=1}^{n}\tilde{c}_{iL}^{-1}(\alpha)q\Big)\mathrm{d}\alpha$$
$$=\frac{1}{2}\int_0^{R(q)}\tilde{p}_R^{-1}(\alpha)\left(q-R^{-1}(\alpha)\right)\mathrm{d}\alpha$$
$$+E\left[\tilde{p}\tilde{D}\right]-\sum_{i=1}^{n}E\left[\tilde{c}_i\right]q\,. \quad (16)$$

From Eq.(16), the first order condition of $E\left[\tilde{\Pi}_{SC}\right]$ is

$$\frac{\mathrm{d}E\left[\tilde{\Pi}_{SC}\right]}{\mathrm{d}q}=\frac{1}{2}\int_0^{R(q)}\tilde{p}_R^{-1}(\alpha)\mathrm{d}\alpha-\sum_{i=1}^{n}E\left[\tilde{c}_i\right].$$

The second order condition of $E\left[\tilde{\Pi}_{SC}\right]$ is

$$\frac{\mathrm{d}^2E\left[\tilde{\Pi}_{SC}\right]}{\mathrm{d}q^2}=\frac{1}{2}\tilde{p}_R^{-1}\left(R(q)\right)R'(q).$$

Note that the second order condition is negative, since $R(q)$ is decreasing about q with $R'(q)<0$ and $\tilde{p}_R^{-1}\left(R(q)\right)>0$. Therefore, $E\left[\tilde{\Pi}_{SC}\right]$ is a concave function with respect to q.

Hence, we can get the optimal order quantity q^* by letting the first order condition be zero

$$\frac{1}{2}\int_0^{R(q)}\tilde{p}_R^{-1}(\alpha)\mathrm{d}\alpha-\sum_{i=1}^{n}E\left[\tilde{c}_i\right]=0\,. \quad (17)$$

Solving Eq.(17), we can get q^* as shown in Eq.(15). Theorem 2 is proved. □

Theorem 3. *If $\tilde{p}=\left(p-\Delta_1,p,p+\Delta_2\right)$, then the optimal order quantity q^* can be expressed as*

(1) if $p\in\left(\sum_{i=1}^{n}E\left[\tilde{c}_i\right],2\sum_{i=1}^{n}E\left[\tilde{c}_i\right]-0.5\Delta_2\right]$, then

$$q^*=L^{-1}\left(\frac{\sqrt{0.25\left(p-\Delta_1\right)^2+\left(p+0.25\Delta_2-0.25\Delta_1-\sum_{i=1}^{n}E\left[\tilde{c}_i\right]\right)\Delta_1}-0.5\left(p-\Delta_1\right)}{0.5\Delta_1}\right).$$

(2) if $p\in\left(2\sum_{i=1}^{n}E\left[\tilde{c}_i\right]-0.5\Delta_2,+\infty\right)$, then

$$q^*=R^{-1}\left(\frac{0.5\left(p+\Delta_2\right)-\sqrt{0.25\left(p+\Delta_2\right)^2-\sum_{i=1}^{n}E\left[\tilde{c}_i\right]\Delta_2}}{0.5\Delta_2}\right).$$

Proof. If $\tilde{p}=\left(p-\Delta_1,p,p+\Delta_2\right)$, then left and right cut set boundary of $\tilde{p}$ with $\tilde{p}_L^{-1}(\alpha)$ and $\tilde{p}_R^{-1}(\alpha)$, respectively, are

$$\tilde{p}_L^{-1}(\alpha)=p-(1-\alpha)\Delta_1\text{, and }\tilde{p}_R^{-1}(\alpha)=p+(1-\alpha)\Delta_2\,.$$

Case 1. $q\in\left[d_1,d_2\right]$

Substituting $\tilde{p}_L^{-1}(\alpha)$ and $\tilde{p}_R^{-1}(\alpha)$ into Eq. (12), we have

$$0.25\Delta_1L^2\left(q^*\right)+0.5\left(p-\Delta_1\right)L\left(q^*\right)-p$$
$$-0.25\Delta_2+0.25\Delta_1+\sum_{i=1}^{n}E\left[\tilde{c}_i\right]=0 \quad (18)$$

Solving Eq.(18) leads to

$$L\left(q^*\right)=\frac{\sqrt{0.25\left(p-\Delta_1\right)^2+\left(p+0.25\Delta_2-0.25\Delta_1-\sum_{i=1}^{n}E\left[\tilde{c}_i\right]\right)\Delta_1}-0.5\left(p-\Delta_1\right)}{0.5\Delta_1}. \quad (19)$$

Since $0\le L\left(q^*\right)\le1$, thus we get $p\le2\sum_{i=1}^{n}E\left[\tilde{c}_i\right]-0.5\Delta_2$.

Case 2. $q\in\left(d_2,d_3\right]$

Substituting $\tilde{p}_L^{-1}(\alpha)$ and $\tilde{p}_R^{-1}(\alpha)$ into Eq. (15), we have

$$0.25\Delta_2R^2\left(q^*\right)-0.5\left(p+\Delta_2\right)R\left(q^*\right)+\sum_{i=1}^{n}E\left[\tilde{c}_i\right]=0\,. \quad (20)$$

Solving Eq.(20) leads to

$$R\left(q^*\right)=\frac{0.5\left(p+\Delta_2\right)-\sqrt{0.25\left(p+\Delta_2\right)^2-\sum_{i=1}^{n}E\left[\tilde{c}_i\right]\Delta_2}}{0.5\Delta_2}. \quad (21)$$

Since $0\le R\left(q^*\right)\le1$, thus we get $p>2\sum_{i=1}^{n}E\left[\tilde{c}_i\right]-0.5\Delta_2$.

Note that if $p=2\sum_{i=1}^{n}E\left[\tilde{c}_i\right]-0.5\Delta_2$, and then we have the following equation

$$\frac{\sqrt{0.25\left(p-\Delta_1\right)^2+\left(p+0.25\Delta_2-0.25\Delta_1-\sum_{i=1}^{n}E\left[\tilde{c}_i\right]\right)\Delta_1}-0.5\left(p-\Delta_1\right)}{0.5\Delta_1}$$
$$=\frac{0.5\left(p+\Delta_2\right)-\sqrt{0.25\left(p+\Delta_2\right)^2-\sum_{i=1}^{n}E\left[\tilde{c}_i\right]\Delta_2}}{0.5\Delta_2}.$$

That is $L\left(q^*\right)=R\left(q^*\right)$.

Theorem 3 is proved. □

Theorem 4. *If* $\Delta_1 \to 0$ *and* $\Delta_2 \to 0$ *, then the retail price* $\tilde{p}$ *reduces to the crisp real number, the results in Theorem 3 are reduced to*

$$q^*=\begin{cases} L^{-1}\left(\dfrac{2\left(p-\sum_{i=1}^n E[\tilde{c}_i]\right)}{p}\right), & p\in\left(\sum_{i=1}^n E[\tilde{c}_i],2\sum_{i=1}^n E[\tilde{c}_i]\right], \\ R^{-1}\left(\dfrac{2\sum_{i=1}^n E[\tilde{c}_i]}{p}\right), & p\in\left(2\sum_{i=1}^n E[\tilde{c}_i],+\infty\right). \end{cases}$$

There are just the solutions with symmetric information.

Proof. Case 1. $q\in[d_1,d_2]$

If $q\in[d_1,d_2]$, that is $p\in\left(\sum_{i=1}^n E[\tilde{c}_i],2\sum_{i=1}^n E[\tilde{c}_i]\right]$, then let $\Delta_1\to 0$ and $\Delta_2\to 0$, we can get

$$q^*=\lim_{\Delta_1\to 0,\Delta_2\to 0} L^{-1}\left(\frac{\sqrt{0.25(p-\Delta_1)^2+\left(p+0.25\Delta_2-0.25\Delta_1-\sum_{i=1}^n E[\tilde{c}_i]\right)\Delta_1}-0.5(p-\Delta_1)}{0.5\Delta_1}\right)$$

$$=\lim_{\Delta_1\to 0,\Delta_2\to 0} L^{-1}\left(\frac{\dfrac{0.5p+0.25\Delta_2-\sum_{i=1}^n E[\tilde{c}_i]}{\sqrt{0.25(p-\Delta_1)^2+\left(p+0.25\Delta_2-0.25\Delta_1-\sum_{i=1}^n E[\tilde{c}_i]\right)\Delta_1}}+0.5}{0.5}\right)$$

$$=L^{-1}\left(\frac{2\left(p-\sum_{i=1}^n E[\tilde{c}_i]\right)}{p}\right).$$

Case 2. $q\in(d_2,d_3]$

If $q\in(d_2,d_3]$, that is $p\in\left(2\sum_{i=1}^n E[\tilde{c}_i],+\infty\right)$, then let $\Delta_1\to 0$ and $\Delta_2\to 0$, we can get

$$q^*=\lim_{\Delta_2\to 0} R^{-1}\left(\frac{0.5(p+\Delta_2)-\sqrt{0.25(p+\Delta_2)^2-\sum_{i=1}^n E[\tilde{c}_i]\Delta_2}}{0.5\Delta_2}\right)$$

$$=\lim_{\Delta_2\to 0} R^{-1}\left(\frac{0.5-\dfrac{0.5(p+\Delta_2)-\sum_{i=1}^n E[\tilde{c}_i]}{2\sqrt{0.25(p+\Delta_2)^2-\sum_{i=1}^n E[\tilde{c}_i]\Delta_2}}}{0.5}\right)$$

$$=R^{-1}\left(\frac{2\sum_{i=1}^n E[\tilde{c}_i]}{p}\right).$$

Theorem 4 is proved. □

From Eqs.(13), (16) and (18), we can easily derive the optimal fuzzy expected profits for supply chain system as follows

Case 1. $p\in\left(\sum_{i=1}^n E[\tilde{c}_i],2\sum_{i=1}^n E[\tilde{c}_i]-0.5\Delta_2\right]$

$$E\left[\tilde{\Pi}_{SC}\right]^*=\frac{1}{2}\int_0^{L(q^*)}\tilde{p}_L^{-1}(\alpha)L^{-1}(\alpha)\mathrm{d}\alpha. \tag{22}$$

where

$$q^*=L^{-1}\left(\frac{\sqrt{0.25(p-\Delta_1)^2+\left(p+0.25\Delta_2-0.25\Delta_1-\sum_{i=1}^n E[\tilde{c}_i]\right)\Delta_1}-0.5(p-\Delta_1)}{0.5\Delta_1}\right).$$

Case 2. $p\in\left(2\sum_{i=1}^n E[\tilde{c}_i]-0.5\Delta_2,+\infty\right)$

$$E\left[\tilde{\Pi}_{SC}\right]^*=E\left[\tilde{p}\tilde{D}\right]-\frac{1}{2}\int_0^{R(q^*)}\tilde{p}_R^{-1}(\alpha)R^{-1}(\alpha)\mathrm{d}\alpha. \tag{23}$$

where

$$q^*=R^{-1}\left(\frac{0.5(p+\Delta_2)-\sqrt{0.25(p+\Delta_2)^2-\sum_{i=1}^n E[\tilde{c}_i]\Delta_2}}{0.5\Delta_2}\right).$$

4.2. Fuzzy revenue sharing contract with asymmetric information

In the RS contract, the supply chain actor 1 shares his fuzzy profit with other actor i, $i=2,3,\ldots,n$, and the portion is denoted by Φ_i, with $\Phi_i\in(0,1)$. Then the portion kept by the actor 1 is $1-\sum_{i=2}^n\Phi_i$, with $\sum_{i=2}^n\Phi_i\in(0,1)$.

Thus, the fuzzy profit function for actor 1 can be expressed as follows

$$\tilde{\Pi}_1=\left(1-\sum_{i=2}^n\Phi_i\right)\tilde{p}\min\left(q,\tilde{D}\right)-\left(w_2+\tilde{c}_1\right)q. \tag{24}$$

The supply chain actor 1 wants to get the order quantity q which maximizes his expected profit $E\left[\tilde{\Pi}_1\right]$. Thus, the optimal objection function of actor 1 is

$$\mathrm{Max}_q\ E\left[\tilde{\Pi}_1\right]=E\left[\left(1-\sum_{i=2}^n\Phi_i\right)\tilde{p}\min\left(q,\tilde{D}\right)-\left(w_2+\tilde{c}_1\right)q\right]$$
$$\text{s.t. } d_1\le q\le d_3. \tag{25}$$

In the RS contract, we can also get the profit function for supply chain actor i, $i=2,3,\ldots,n$, as

$$\tilde{\Pi}_i = \Phi_i \tilde{p} \min\left(q, \tilde{D}\right) + \left(w_i - w_{i+1} - \tilde{c}_i\right) q . \qquad (26)$$

The supply chain actor i, $i = 2,3,\ldots,n$, also wants to maximize the expected profit $E\left[\tilde{\Pi}_i\right]$.thus, the optimal objection function of actor i is

$$\text{Max}_q\ E\left[\tilde{\Pi}_i\right] = E\left[\Phi_i \tilde{p} \min\left(q, \tilde{D}\right) + \left(w_i - w_{i+1} - \tilde{c}_i\right) q\right]$$
$$\text{s.t. } d_1 \le q \le d_3 . \qquad (27)$$

Theorem 5. *The optimal wholesale price* w_i^* $(i = 2,3,\ldots,n)$ *that the actor* i *charges the actor* $i-1$, *in the RS contract satisfies the following equations*

$$w_2^* = \left(1 - \sum_{i=2}^{n} \Phi_i\right) \sum_{i=1}^{n} E\left[\tilde{c}_i\right] - E\left[\tilde{c}_1\right],$$
$$w_{i+1}^* = w_i^* - E\left[\tilde{c}_i\right] + \Phi_i \sum_{i=1}^{n} E\left[\tilde{c}_i\right] \text{ and } w_{n+1} = 0 . \quad (28)$$

Proof. Case 1. $q \in \left[d_1, d_2\right]$

Similar to the solution process in Eq.(13), the fuzzy expected profit for supply chain actor 1 can be easily obtained as follows

$$E\left[\tilde{\Pi}_1\right] = -\frac{1}{2}\left(1 - \sum_{i=2}^{n} \Phi_i\right) \int_0^{L(q)} \tilde{p}_L^{-1}(\alpha)\left(q - L^{-1}(\alpha)\right) \mathrm{d}\alpha$$
$$+\left(\left(1 - \sum_{i=2}^{n} \Phi_i\right) E\left[\tilde{p}\right] - w_2 - E\left[\tilde{c}_1\right]\right) q . \quad (29)$$

From Eq.(31), the first order condition of $E\left[\tilde{\Pi}_1\right]$ is

$$\frac{\mathrm{d} E\left[\tilde{\Pi}_1\right]}{\mathrm{d} q} = -\frac{1}{2}\left(1 - \sum_{i=2}^{n} \Phi_i\right) \int_0^{L(q)} \tilde{p}_L^{-1}(\alpha) \mathrm{d}\alpha$$
$$+\left(1 - \sum_{i=2}^{n} \Phi_i\right) E\left[\tilde{p}\right] - w_2 - E\left[\tilde{c}_1\right].$$

The second order condition of $E\left[\tilde{\Pi}_1\right]$ is

$$\frac{\mathrm{d}^2 E\left[\tilde{\Pi}_1\right]}{\mathrm{d} q^2} = -\frac{1}{2}\left(1 - \sum_{i=2}^{n} \Phi_i\right) \tilde{p}_L^{-1}\left(L(q)\right) L^{'}(q).$$

Note that the second order condition is negative, since $L(q)$ is increasing about q with $L^{'}(q) > 0$, $1 - \sum_{i=2}^{n} \Phi_i > 0$ and $\tilde{p}_L^{-1}\left(L(q)\right) > 0$. Therefore, $E\left[\tilde{\Pi}_1\right]$ is a concave function with respect to q.

Hence, we can get the optimal order quantity q^{**} by letting the first order condition be zero

$$-\frac{1}{2}\left(1 - \sum_{i=2}^{n} \Phi_i\right) \int_0^{L(q^{**})} \tilde{p}_L^{-1}(\alpha) \mathrm{d}\alpha$$
$$+\left(1 - \sum_{i=2}^{n} \Phi_i\right) E\left[\tilde{p}\right] - w_2 - E\left[\tilde{c}_1\right] = 0 . \qquad (30)$$

That is

$$\frac{1}{2}\int_0^{L(q^{**})} \tilde{p}_L^{-1}(\alpha) \mathrm{d}\alpha = E\left[\tilde{p}\right] - \frac{w_2 + E\left[\tilde{c}_1\right]}{1 - \sum_{i=2}^{n} \Phi_i} . \qquad (31)$$

For coordinating this supply chain, $q^{**} = q^*$ must be hold. This means that the optimal order chosen by supply chain actor 1 in fuzzy RS contract is the same as in fuzzy centralized decision-making system.

Comparing Eq.(31) with Eq.(12), we have

$$w_2^* = \left(1 - \sum_{i=2}^{n} \Phi_i\right) \sum_{i=1}^{n} E\left[\tilde{c}_i\right] - E\left[\tilde{c}_1\right]$$

Similar to the solution process in Eq.(13), we can also easily obtain the fuzzy expected profit for supply chain actor i as

$$E\left[\tilde{\Pi}_i\right] = -\frac{1}{2}\Phi_i \int_0^{L(q)} \tilde{p}_L^{-1}(\alpha)\left(q - L^{-1}(\alpha)\right) \mathrm{d}\alpha$$
$$+\left(\Phi_i E\left[\tilde{p}\right] + w_i - w_{i+1} - E\left[\tilde{c}_i\right]\right) q \qquad (32)$$

From Eq.(32), the first order condition of $E\left[\tilde{\Pi}_i\right]$ is

$$\frac{\mathrm{d} E\left[\tilde{\Pi}_i\right]}{\mathrm{d} q} = -\frac{1}{2}\Phi_i \int_0^{L(q)} \tilde{p}_L^{-1}(\alpha) \mathrm{d}\alpha + \Phi_i E\left[\tilde{p}\right] + w_i - w_{i+1} - E\left[\tilde{c}_i\right]$$

The second order condition of $E\left[\tilde{\Pi}_i\right]$ is

$$\frac{\mathrm{d}^2 E\left[\tilde{\Pi}_i\right]}{\mathrm{d} q^2} = -\frac{1}{2}\Phi_i \tilde{p}_L^{-1}\left(L(q)\right) L^{'}(q)$$

Note that the second order condition is negative, since $L(q)$ is increasing about q with $L^{'}(q) > 0$, $\Phi_i > 0$ and $\tilde{p}_L^{-1}\left(L(q)\right) > 0$. Therefore, $E\left[\tilde{\Pi}_i\right]$ is a concave function with respect to q.

Hence, we can get the optimal order quantity q^{**} by letting the first order condition be zero

$$-\frac{1}{2}\Phi_i \int_0^{L(q^{**})} \tilde{p}_L^{-1}(\alpha) \mathrm{d}\alpha + \Phi_i E\left[\tilde{p}\right] + w_i - w_{i+1} - E\left[\tilde{c}_i\right] = 0 \qquad (33)$$

That is

$$\frac{1}{2}\int_0^{L(q^{**})} \tilde{p}_L^{-1}(\alpha) \mathrm{d}\alpha = E\left[\tilde{p}\right] + \frac{w_i - w_{i+1} - E\left[\tilde{c}_i\right]}{\Phi_i} . \qquad (34)$$

For coordinating this supply chain system, $q^{**} = q^*$ must be hold.

Comparing Eq.(34) with Eq.(12), we have

$$w_{i+1}{}^{*} = w_i^{*} - E[\tilde{c}_i] + \Phi_i \sum_{i=1}^{n} E[\tilde{c}_i].$$

Case 2. $q \in (d_2, d_3]$

Similar to the solution process in Eq.(16), the fuzzy expected profit for supply chain actor 1 $E[\tilde{\Pi}_1]$ is

$$E[\tilde{\Pi}_1] = \frac{1}{2}\left(1-\sum_{i=2}^{n}\Phi_i\right)\int_0^{R(q)} \tilde{p}_R^{-1}(\alpha)\left(q - R^{-1}(\alpha)\right)\mathrm{d}\alpha + \left(1-\sum_{i=2}^{n}\Phi_i\right)E[\tilde{p}\tilde{D}] - \left(w_2 + E[\tilde{c}_1]\right)q. \quad (35)$$

From Eq.(35), the first order condition of $E[\tilde{\Pi}_1]$ is

$$\frac{\mathrm{d}E[\tilde{\Pi}_1]}{\mathrm{d}q} = \frac{1}{2}\left(1-\sum_{i=2}^{n}\Phi_i\right)\int_0^{R(q)} \tilde{p}_R^{-1}(\alpha)\mathrm{d}\alpha - w_2 - E[\tilde{c}_1].$$

The second order condition of $E[\tilde{\Pi}_1]$ is

$$\frac{\mathrm{d}^2 E[\tilde{\Pi}_1]}{\mathrm{d}q^2} = \frac{1}{2}\left(1-\sum_{i=2}^{n}\Phi_i\right)\tilde{p}_R^{-1}\left(R(q)\right)R'(q).$$

Note that the second order condition is negative, since $R(q)$ is decreasing about q with $R'(q) < 0$, $1-\sum_{i=2}^{n}\Phi_i > 0$ and $\tilde{p}_R^{-1}(R(q)) > 0$. Therefore, $E[\tilde{\Pi}_1]$ is a concave function with respect to q.

Hence, we can get the optimal order quantity q^{**} by letting the first order condition be zero

$$\frac{1}{2}\left(1-\sum_{i=2}^{n}\Phi_i\right)\int_0^{R(q^{**})} \tilde{p}_R^{-1}(\alpha)\mathrm{d}\alpha - w_2 - E[\tilde{c}_1] = 0. \quad (36)$$

That is

$$\frac{1}{2}\int_0^{R(q^{**})} \tilde{p}_R^{-1}(\alpha)\mathrm{d}\alpha = \frac{w_2 + E[\tilde{c}_1]}{1-\sum_{i=2}^{n}\Phi_i}. \quad (37)$$

For coordinating this supply chain, $q^{**} = q^{*}$ must be hold.

Comparing Eq.(37) with Eq.(15), we have

$$w_2^{*} = \left(1-\sum_{i=2}^{n}\Phi_i\right)\sum_{i=1}^{n} E[\tilde{c}_i] - E[\tilde{c}_1]$$

Similar to the solution process in Eq.(16), the fuzzy expected profit for supply chain actor i $E[\tilde{\Pi}_i]$ is

$$E[\tilde{\Pi}_i] = \frac{1}{2}\Phi_i\int_0^{R(q)} \tilde{p}_R^{-1}(\alpha)\left(q - R^{-1}(\alpha)\right)\mathrm{d}\alpha + \Phi_i E[\tilde{p}\tilde{D}] + \left(w_i - w_{i+1} - E[\tilde{c}_i]\right)q. \quad (38)$$

From Eq.(38), the first order condition of $E[\tilde{\Pi}_i]$ is

$$\frac{\mathrm{d}E[\tilde{\Pi}_i]}{\mathrm{d}q} = \frac{1}{2}\Phi_i\int_0^{R(q)} \tilde{p}_R^{-1}(\alpha)\mathrm{d}\alpha + w_i - w_{i+1} - E[\tilde{c}_i].$$

The second order condition of $E[\tilde{\Pi}_i]$ is

$$\frac{\mathrm{d}^2 E[\tilde{\Pi}_i]}{\mathrm{d}q^2} = \frac{1}{2}\Phi_i \tilde{p}_R^{-1}\left(R(q)\right)R'(q).$$

Note that the second order condition is negative, since $R(q)$ is decreasing about q with $R'(q) < 0$, $\Phi_i > 0$ and $\tilde{p}_R^{-1}(R(q)) > 0$. Therefore, $E[\tilde{\Pi}_i]$ is a concave function with respect to q.

Hence, we can get the optimal order quantity q^{**} by letting the first order condition be zero

$$\frac{1}{2}\Phi_i\int_0^{R(q)} \tilde{p}_R^{-1}(\alpha)\mathrm{d}\alpha + w_i - w_{i+1} - E[\tilde{c}_i] = 0. \quad (39)$$

That is

$$\frac{1}{2}\int_0^{R(q^{**})} \tilde{p}_R^{-1}(\alpha)\mathrm{d}\alpha = \frac{w_{i+1} - w_i + E[\tilde{c}_i]}{\Phi_i}. \quad (40)$$

For coordinating this supply chain system, $q^{**} = q^{*}$ must hold.

Comparing Eq.(40) with Eq.(15), we have

$$w_{i+1}{}^{*} = w_i^{*} - E[\tilde{c}_i] + \Phi_i \sum_{i=1}^{n} E[\tilde{c}_i].$$

Theorem 5 is proved. □

Theorem 6. *In fuzzy RS contract, the supply chain actor 1 and actor i, $i = 2,3,\ldots,n$, obtain their optimal fuzzy profits at w_i^{*} as follows*

$$E[\tilde{\Pi}_1]^{*} = \left(1-\sum_{i=2}^{n}\Phi_i\right)E[\tilde{\Pi}_{SC}]^{*},$$

$$E[\tilde{\Pi}_i]^{*} = \Phi_i E[\tilde{\Pi}_{SC}]^{*}.$$

Proof. Case 1. $q \in [d_1, d_2]$

Substituting w_2^{*} in Eq.(28) and $q^{**} = q^{*}$ into Eq.(29), we have the optimal fuzzy profit for actor 1 in fuzzy RS contract as

$$E[\tilde{\Pi}_1]^{*} = \frac{1}{2}\left(1-\sum_{i=2}^{n}\Phi_i\right)\int_0^{L(q^{*})} \tilde{p}_L^{-1}(\alpha)L^{-1}(\alpha)\mathrm{d}\alpha = \left(1-\sum_{i=2}^{n}\Phi_i\right)E[\tilde{\Pi}_{SC}]^{*}.$$

Substituting $w_{i+1}{}^{*}$ in Eq.(28) and $q^{**} = q^{*}$ into Eq.(32), we have the optimal fuzzy profit for actor i as follows

$$E\left[\tilde{\Pi}_i\right]^* = \frac{1}{2}\Phi_i\left(1-\sum_{i=2}^{n}\Phi_i\right)\int_0^{L(q^*)}\tilde{p}_L^{-1}(\alpha)L^{-1}(\alpha)\,\mathrm{d}\alpha$$
$$= \Phi_i E\left[\tilde{\Pi}_{SC}\right]^*.$$

Case 2. $q \in (d_2, d_3]$

Substituting w_2^* in Eq.(28) and $q^{**} = q^*$ into Eq.(35), we have the optimal fuzzy profit for actor 1 in fuzzy RS contract as

$$E\left[\tilde{\Pi}_1\right]^* = \left(1-\sum_{i=2}^{n}\Phi_i\right)E\left[\tilde{p}\tilde{D}\right]$$
$$-\frac{1}{2}\left(1-\sum_{i=2}^{n}\Phi_i\right)\int_0^{R(q^*)}\tilde{p}_R^{-1}(\alpha)R^{-1}(\alpha)\,\mathrm{d}\alpha$$
$$= \left(1-\sum_{i=2}^{n}\Phi_i\right)E\left[\tilde{\Pi}_{SC}\right]^*.$$

Substituting w_{i+1}^* in Eq.(30) and $q^{**} = q^*$ into Eq.(40), we have the optimal fuzzy profit for actor i as follows

$$E\left[\tilde{\Pi}_i\right]^* = \Phi_i E\left[\tilde{p}\tilde{D}\right] - \frac{1}{2}\Phi_i\int_0^{R(q^*)}\tilde{p}_R^{-1}(\alpha)R^{-1}(\alpha)\,\mathrm{d}\alpha$$
$$= \Phi_i E\left[\tilde{\Pi}_{SC}\right]^*.$$

Theorem 6 is proved. □

5. Numerical Examples

For further elucidating above proposed models, we provide numerical examples in this section. Taking a three-echelon supply chain as an example, Let actor 1, 2 and 3 denotes the retailer, the distributor and the manufacturer, respectively. We discuss the impacts of fuzziness of retail price $\tilde{p}$ and demand $\tilde{D}$, and the values of contract parameters on optimal results in the RS contract.

5.1. Discussion 1

In this subsection, we discuss the impact of fuzziness of retail price $\tilde{p}$ on the optimal results in the RS contract.

The fuzzy demand estimated by the decision maker's experience is supposed to be nearly 300, but not less than 200 and not greater than 400, that is $\tilde{D} = (200, 300, 400)$. Similarly, the operational cost of the retailer is about \$ 2, but not less than 1 and not greater than \$ 3, that is $\tilde{c}_1 = (1, 2, 3)$. The operational cost of the distributor is about \$ 3, but not less than 2 and not greater than \$ 4, that is $\tilde{c}_2 = (2, 3, 4)$. The operational cost of the distributor is about \$ 15, but not less than 13 and not greater than \$ 17, that is $\tilde{c}_3 = (13, 15, 17)$.

From Theorem 5, we can obtain the optimal wholesale prices

$$w_2^* = 8.00 \text{ and } w_3^* = 9.00.$$

Since the optimal order quantity q^* has two cases, then the other optimal policies and expected profit for supply chain actors in the RS contract can be listed in Tables 1 and 2.

Table 1. The RS contract policies when $20 < p < 40 - 0.5\Delta_2$

$\tilde{p}$	q^*	$E[\tilde{\Pi}_1]^*$	$E[\tilde{\Pi}_2]^*$	$E[\tilde{\Pi}_3]^*$
(30, 30, 30)	266.67	1166.67	700.00	466.67
(29, 30, 31)	268.16	1171.07	702.64	468.43
(28, 30, 32)	269.69	1175.65	705.39	470.26
(27, 30, 33)	271.25	1180.40	708.24	472.16
(26, 30, 34)	272.84	1185.33	711.19	474.13
(25, 30, 35)	274.46	1190.44	714.26	476.18

Table 2. The RS contract policies when $p > 40 - 0.5\Delta_2$

$\tilde{p}$	q^*	$E[\tilde{\Pi}_1]^*$	$E[\tilde{\Pi}_2]^*$	$E[\tilde{\Pi}_3]^*$
(50, 50, 50)	320.00	3900.00	2340.00	1560.00
(49, 50, 51)	320.96	3910.86	2351.45	1567.64
(48, 50, 52)	321.90	3921.83	2362.62	1575.08
(47, 50, 53)	322.84	3932.91	2373.53	1582.35
(46, 50, 54)	323.77	3944.11	2384.18	1589.45
(45, 50, 55)	324.70	3955.41	2394.60	1596.40

From Tables 1 and 2, we can get the results as follows

(1) When $20 < p < 40 - 0.5\Delta_2$, q^* lies in the left side of the most possible value of parameter $\tilde{D}$, when $p > 40 - 0.5\Delta_2$, the optimal order quantity q^* lies in the right side of the most possible value of parameter $\tilde{D}$.

(2)The change of fuzziness of retail price $\tilde{p}$ will not affect wholesale prices w_2^* and w_3^*. In addition, the optimal wholesale prices w_2^* and w_3^* in case 1 showed in Table 1 are the same in case 2 showed in Table 2. This is because the fuzzy retail price $\tilde{p}$ will not affect optimal wholesale prices, and the wholesale prices are impacted only by the operational costs $\tilde{c}_1$, $\tilde{c}_2$ and $\tilde{c}_3$.

(3) q^* and the expected profits for actors will increase slightly, as the fuzziness of retail price $\tilde{p}$ increases. This is because an increase in fuzziness of retail price results in an increase in order quantity. This results in the

increase of the fuzzy expected profit for the supply chain members. Therefore, in two cases, actors should seek as high fuzziness of retail price $\tilde{p}$ as possible.

(4) If $\Delta_1 = \Delta_2 = 0$, then the results in this paper can reduce to the solutions in the symmetric information environment as showed in the second rows in Tables 1 and 2. Comparing these solutions, we can find that q^* and expected profits for actors in the asymmetric information environment are higher that those in the symmetric information environment. It indicates that all the supply chain actors can benefit from the asymmetric information in a fuzzy environment.

5.2. Discussion 2

In this subsection, we discuss the impact of the fuzziness of parameter $\tilde{D}$ on the RS contract policies. The values of the costs for supply chain actors are considered as before.

From Theorem 5, we can get the optimal wholesale prices in this discussion as $w_2^* = 8.00$ and $w_3^* = 9.00$.

The other optimal solutions derived are shown in Tables 3and 4.

Table 3. The RS contract policies when $\tilde{p} = (28,30,32)$

$\tilde{D}$	q^*	$E[\tilde{\Pi}_1]^*$	$E[\tilde{\Pi}_2]^*$	$E[\tilde{\Pi}_3]^*$
(200, 300, 400)	269.69	1175.65	705.39	470.26
(210, 300, 390)	272.72	1208.08	724.85	483.23
(220, 300, 380)	275.76	1240.52	744.31	496.21
(230, 300, 370)	278.79	1272.95	763.77	509.18
(240, 300, 360)	281.82	1305.39	783.23	522.15
(250, 300, 350)	284.85	1337.82	802.69	535.13

Table 4. The RS contract policies when $\tilde{p} = (48,50,52)$

$\tilde{D}$	q^*	$E[\tilde{\Pi}_1]^*$	$E[\tilde{\Pi}_2]^*$	$E[\tilde{\Pi}_3]^*$
(200, 300, 400)	321.90	3921.83	2353.10	1568.73
(210, 300, 390)	319.71	3979.65	2387.79	1591.86
(220, 300, 380)	317.52	4037.46	2422.48	1614.99
(230, 300, 370)	315.33	4095.28	2457.17	1638.11
(240, 300, 360)	313.14	4153.10	2491.86	1661.24
(250, 300, 350)	310.95	4210.91	2526.55	1684.37

(5) q^* will rise as the fuzziness of demand $\tilde{D}$ falls when $20 < p < 40 - 0.5\Delta_2$. While, q^* will drop as the fuzziness of demand $\tilde{D}$ falls when $p > 40 - 0.5\Delta_2$. In two cases, the change of fuzziness of demand $\tilde{D}$ will not affect optimal wholesale prices w_2^* and w_3^*.

(6) q^* and the expected profits for actors will all increase slightly, when the fuzziness of demand $\tilde{D}$ decreases. That is to say, the manufacturer, distributor and retailer all gain more expected profit when the fuzziness of demand is lower. This is intuitive because the lower the fuzziness of demand, the more efficient of the supply chain system. Therefore, actors should seek as low fuzziness of demand $\tilde{D}$ as possible.

5.3. Discussion 3

In this subsection, we discuss the impact of parameters Φ_2 and Φ_3 on the RS contract policies. The values of the costs for supply chain actors are considered as before.

When the fuzzy retail price $\tilde{p} = (28,30,32)$, From Theorem3, we can get the optimal order quantity as

$$q^* = 269.69.$$

When the fuzzy retail price $\tilde{p} = (48,50,52)$, From Theorem3, we can get the optimal order quantity as

$$q^* = 321.90.$$

The other optimal solutions derived are shown in Tables 5and 6.

Table 5. The RS contract policies when $\tilde{p} = (28,30,32)$

(Φ_2, Φ_3)	w_2^*	w_3^*	$E[\tilde{\Pi}_1]^*$	$E[\tilde{\Pi}_2]^*$	$E[\tilde{\Pi}_3]^*$
(0.30, 0.20)	8.00	9.00	1175.65	705.39	470.26
(0.30, 0.25)	9.00	9.00	1058.08	705.39	587.82
(0.30, 0.30)	10.00	9.00	940.52	705.39	705.39
(0.35, 0.20)	9.00	8.00	1058.08	822.95	470.26
(0.40, 0.20)	10.00	7.00	940.52	940.52	470.26
(0.45, 0.20)	11.00	6.00	822.95	1058.08	470.26

Table 6. The RS contract policies when $\tilde{p} = (48,50,52)$

(Φ_2, Φ_3)	w_2^*	w_3^*	$E[\tilde{\Pi}_1]^*$	$E[\tilde{\Pi}_2]^*$	$E[\tilde{\Pi}_3]^*$
(0.30, 0.20)	8.00	9.00	3921.83	2353.10	1568.73
(0.30, 0.25)	9.00	9.00	3529.65	2353.10	1960.91
(0.30, 0.30)	10.00	9.00	3137.46	2353.10	2353.10
(0.35, 0.20)	9.00	8.00	3529.65	2745.28	1568.73
(0.40, 0.20)	10.00	7.00	3137.46	3137.46	1568.73
(0.45, 0.20)	11.00	6.00	2745.28	3529.65	1568.73

(7)The change of the parameters Φ_2 and Φ_3 will not affect the optimal order quantity q^*. With the increasing of Φ_2, w_2^* will increase, but w_3^* will decrease, when the parameter Φ_3 is fixed. With the increasing of Φ_3,

w_2^* will increase, and w_3^* will not vary, when the parameter Φ_2 is fixed.

(8)The optimal expected profit for actor 2 increases as the parameter Φ_2 increase. As the parameter Φ_3 drops, the expected profit for actor 3 falls. The expected profit for actor 1 decreases when the value of sum of Φ_2 and Φ_3 increases. In addition, if $\Phi_2 = \Phi_3$, then the expected profit for actor 2 is the same as that for actor 3.Therefore, the RS contract is an effective tool in coordinating the supply chain, as we can set the reasonable values of parameters Φ_2 and Φ_3 by negotiating between supply chain actors without sacrificing the expected maximum profit for supply chain system.

6. Conclusions

This paper deals with the coordination strategy in a multi-stage supply chain, where actors adopt the RS contract to coordinate the supply chain. For examining the performance of supply chain models with fuzzy demand and asymmetric information, we use the fuzzy set theory to solve these problems. We find that the optimal wholesale prices do not vary as the fuzziness of the retail price and demand decrease, the supply chain members should seek as low fuzziness of demand as possible, and all the supply chain actors can benefit from the asymmetric information in a fuzzy environment.

Based on the discussions above, the following findings can be obtained. Firstly, the optimal order quantity and the expected profits for actors in the asymmetric information environment are higher that those in the symmetric information environment. Secondly, the optimal order quantity and the expected profits for actors will all increase slightly, when the fuzziness of market demand decreases. Thirdly, the RS contract is an effective tool in coordinating the multiple echelon supply chain, as we can set the reasonable values of parameters in the contract by negotiating between supply chain actors without sacrificing the expected maximum profit for supply chain system.

One limitation of this article is that we only consider one actor in each echelon supply chain. Another limitation is that supply chain actors are all risk neutral. It is interesting to extend our modes to conditions with multiple competing supply chain actors. Still, we will discuss the problem how to design the contract policies when the actors are risk averse in a fuzzy environment.

Acknowledgements

This work was supported by the Shandong Provincial Natural Science Foundation, China (No. ZR2015GQ 001), and the Project of Shandong Provincial Higher Educational Humanity and Social Science Research Program (No. J15WB04).

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