

# A Novel Procedure to Model and Forecast Mobile Communication Traffic by ARIMA/GARCH Combination Models

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**Abstract**—Mobile traffic modeling and forecasting are the key techniques in terms of network optimization and management because better network management can be achieved through improving the forecasting accuracy. While mobile traffic has been studied extensively and proved to be effectively modeled with ARIMA models, the volatility effect in mobile traffic series that results in forecasting errors was seldom mentioned. In this study, a multiplicative seasonal ARIMA/GARCH building procedure is proposed to show that volatility effect appearing in mobile traffic series can be processed by GARCH models. Our proposed procedure combines several evaluating parameters such as Akaike Information Criterion (AIC), Schwarz Criterion (SIC), forecast performance evaluation information and residual correlogram to find out the most suitable model, based on which descriptive statistics are used to get the final choice. This work indicates that the mobile traffic series can be better modeled and forecasted by applying GARCH models based on a multiplicative seasonal ARIMA.

**Keywords**—ARIMA; GARCH; GARCH-M; traffic forecasting, traffic modeling

## I. INTRODUCTION

Mobile technologies have a long development history of several decades, since the first mobile phone which can only make and receive phone calls, until now that a mobile phone can fulfil not only basic functions like calls, messages, webs, locations, emails services, but also advanced functions such as multimedia services like online HD music, online HD videos, as well as real-time game playing. Besides, with the appearance of LTE technologies, voip services are getting more and more popular and even are preferred than traditional calls and messages in many situations. With the explosive growth of the number of users, the competition between service providers becomes more and more violent, which require new technologies and optimized network infrastructure to ensure the best quality of service. As a result, traffic forecasting is a key factor and an efficient tool in achieving better network management and optimization.

Traffic modeling and forecasting has been studied for years through statistical procedure that based on Box-Jenkins method like ARIMA and GARCH models. These models have been applied and proved to be useful in modeling and forecasting time series in different areas like short-time traffic flow in [1], day-ahead electricity prices in [2], etc. For internet traffic, ARIMA models were used to model and predict the

network flow data series by several researchers such as Li et al. [3] and Chen et al. [4]. Besides, Zhou et al. [5] figured out that conditional variance property of GARCH processes can capture and explain burstiness that causes network traffic behavior self-similarity and long range dependence (LRD). In addition, ARIMA/GARCH was proposed to predict internet traffic and showed better results compared with Minimum Mean Square Average (MMSE) and Fractional ARIMA (FARIMA). After that, based on RMSE criterion, Kim et al. [6] indicated that seasonal AR-GARCH outperformed seasonal ARIMA in predicting internet traffic. In terms of modeling and predicting mobile communication traffic, seasonal ARIMA models were presented by Shu et al. in [7], Guo et al. in [8] and Miao et al. in [9]. However, volatility characteristics in mobile traffic series were not considered in these studies that may result in loss of efficiency. In our prior study [10], an ARIMA/GARCH combination model was built to deal with EVN mobile traffic data of Vietnam Electricity, in which volatility was found but has an insignificant impact to the forecasting results. Therefore, even the estimate criteria evaluation showed that the chosen ARIMA/GARCH combination model was a better choice compared with the ARIMA, forecasting performance evaluation did not.

In this study, we collect a mobile traffic series during the New Year period in which the volatility is supposed to have a significant impact. By making important modifications to the former procedure, we proposed a novel procedure to build a multiplicative seasonal ARIMA/GARCH method to model and forecast mobile communication traffic data. A multiplicative seasonal ARIMA is firstly constructed to capture the conditional mean of the series. However, in ARIMA models, because the variance is assumed to be constant while in practice the non-constant variance should be considered, we implement the heteroskedasticity tests on the residuals to show that the volatility clustering does appear in the series. Hence, a GARCH model is applied to deal with the conditional heteroskedasticity based on the achieved ARIMA model. We proposed to make use of several essential criteria combination and multi-level validations to improve the accuracy of choosing model. Following our proposed procedure, a multiplicative seasonal ARIMA/GARCH-M is introduced and the results indicate that this combination model outperforms ARIMA model in term of forecast performance.

The rest of this paper is organized as follow. Section II introduces multiplicative seasonal ARIMA/GARCH models.

The proposed procedure to build the ARIMA/GARCH combination model is presented in section III. In section IV, experimental results and performance evaluation are discussed. Finally, the work is concluded in section V.

## II. MULTIPLICATIVE SEASONAL ARIMA/GARCH MODELS OVERVIEW

The description of a multiplicative seasonal ARIMA model can be derived from [7] as in (1).

$$\phi_p(B)\Phi_P(B^s)\nabla^d\nabla_s^D X_t = \theta_q(B)\Theta_Q(B^s)a_t(1)$$

Where B is the backward-shift operator,  $\nabla$  is the differencing operator,  $X_t$  is an ARIMA process,  $a_t$  is a white noise  $WN(0, \sigma^2)$  with zero mean and variance  $\sigma^2$ .  $\phi_p(B)$ ,  $\Phi_P(B^s)$ ,  $\theta_q(B)$ ,  $\Theta_Q(B^s)$  are polynomials in B and  $B^s$  of degrees p, P, q and Q respectively. In which, p, P, q and Q are added to distinguish the orders of the various operators.

And similar in case there are two periodicities  $s_1$  and  $s_2$ :

$$\phi_p(B)\Phi_{P_1}(B^{s_1})\Phi_{P_2}(B^{s_2})\nabla^d\nabla_{s_1}^{D_1}\nabla_{s_2}^{D_2}X_t = \theta_q(B)\Theta_{Q_1}(B^{s_1})\Theta_{Q_2}(B^{s_2})a_t(2)$$

We define:

$$W_t = \phi_p^{-1}(B)\Phi_{P_1}^{-1}(B^{s_1})\Phi_{P_2}^{-1}(B^{s_2})\theta_q(B)\Theta_{Q_1}(B^{s_1})\Theta_{Q_2}(B^{s_2})a_t(3)$$

then (2) can be rewritten as:

$$W_t = \nabla^d\nabla_{s_1}^{D_1}\nabla_{s_2}^{D_2}X_t(4)$$

The multiplicative process is then written in the form of  $(p, d, q) \times (P_1, D_1, Q_1) \times (P_2, D_2, Q_2)$ .

The conditional variance  $\sigma_t^2$  of  $a_t$  is then processed by a GARCH(u, v) process [5][11][12], which is given by (5) in our case.

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^v \alpha_i a_{t-i}^2 + \sum_{i=1}^u \beta_i \sigma_{t-i}^2(5)$$

Where  $u \geq 0, v > 0, \alpha_0 > 0, \alpha_i \geq 0, \beta_i \geq 0$ .

GARCH model can be extended to allow the conditional variance to affect the mean. This extended model is defined as GARCH-M in [13], and can be expressed as in

$$Y_t = X_t'\theta + \lambda\sigma_t^2 + a_t(6)$$

In addition to the standard GARCH model, several other variance models include TARCH [14], EGARCH [15] and PARCH [16] are also introduced and can be applied to time series data analysis.

## III. A PROPOSED PROCEDURE TO BUILD A MULTIPLICATIVE SEASONAL ARIMA/GARCH MODEL

In this work, a procedure to model and forecast mobile communication traffic using ARIMA/GARCH combination

model is proposed. Based on the 4 basic steps of building ARMA models, GARCH models building steps are added together with several restrictions to make the proposed procedure more convinced. The details are described in the flow chart of Figure 1 which includes the following ten steps:

- (1) Implement spectrum analysis to find out the seasonal factor  $s$  exists in the series.
- (2) Identify whether the series is stationary or not, then determine the order of non-seasonal differencing  $d$  and seasonal differencing  $D$  needed to make the series stationary.
- (3) ARIMA model identification is implemented to find out  $p, q, P$  and  $Q$ .
- (4) Estimate ARIMA model to obtain the parameters  $\phi_p, \theta_q, \phi_{PP}, \theta_{QQ}$ .
- (5) Forecast based on the training data. The best model must be chosen based on the estimation criteria include AIC and SIC, residual correlogram diagnostics, as well as in-sample forecast parameters evaluation. If the model does not meet the requirements of all these conditions, then go back to step 3 and repeat until finding the suitable model.
- (6) Implement heteroskedasticity tests on the residual of the chosen ARIMA model. These tests include correlogram of residuals squared and ARCH test. Once ARCH effect is found in the residual series, go to the next step.
- (7) Estimate the conditional variance for the errors using GARCH models, based on the mean equation with AR and MA terms achieved after step 5.
- (8) GARCH models are now found based on the same constraints include AIC, SIC, in-sample forecast parameters like in step 5. But in this case, the white noise achieved in correlogram of residuals squared are also included and considered to be a critical factor. Repeat steps 7 & 8 until finding the suitable models.
- (9) The chosen models are used to compare to the ARIMA model that achieved after step 5 in terms of the difference between the actual and forecast values determined by in-sample forecast descriptive statistics such as mean, medium, etc. The lower difference values are preferred. The model that observes the highest number of lower difference values outperforming ARIMA and other models is chosen as the final ARIMA/GARCH model.
- (10) Implement out-of-sample forecast based on the training data using the ARIMA model obtained in step 5 and the ARIMA/GARCH combination model obtained after step 9 then compare the forecasted values with the actual values to validate the chosen ARIMA/GARCH combination model.

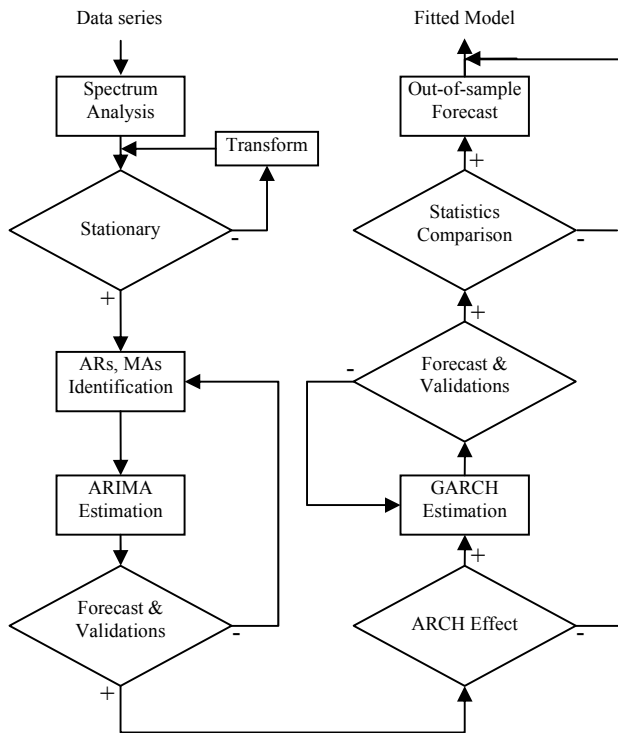


FIGURE 1. PROPOSED ALGORITHM TO BUILD ARIMA/GARCH COMBINATION MODELS

The main distributions of this study lie in steps 5, 8 and 9. Normally, the fitted model is chosen only based on the estimation criteria i.e. AIC and BIC. However, the validation steps in our work also consider in-sample forecast parameters evaluation, and residual correlogram diagnostics to find out the suitable models. Correlogram diagnostics which show the serial correlation through displaying autocorrelation and partial autocorrelation functions become important in this case. In step 5, the residual correlogram has to be as closed to white noise as possible. Otherwise, the correlogram of residual squared must be white noise in step 8. In addition, the most suitable model is chosen by in-sample forecast descriptive statistics comparison in step 9. In this proposed procedure, by using multiple validation levels, the accuracy in choosing fitted model is improved.

#### IV. EXPERIMENTAL RESULTS AND PERFORMANCE EVALUATION

##### A. Data Sources and Tools

The mobile communication data used in this experiment is the 2G/3G traffic collected from Mobifone, which is one of the best mobile communication network service providers in Vietnam. The data series is hourly recorded in MSC Hanoi from December 9th, 2015 till January 8th, 2016. This 31-day series include 744 observations in total. In our work, 30-day 720 observations are used as the training data, and the 31st day 24 observations are used to compare with the forecasted values.

These data is processed using Eviews 9.0, which is a useful tool in time series analysis.

##### B. Experimental Results and Evaluation

The proposed procedure is applied for 720-observation mobifone traffic series as mentioned above. The series include more than 4-week data points. By spectrum analysis as discussed in [7], the two periods of  $s1 = 24$ , i.e. a day and  $s2 = 168$ , i.e. a week in mobifone traffic series are found.

The series is stationary after taking the 1st order difference, i.e.  $d = 1$ , and a seasonal difference, i.e.  $D = 1$  as our experience. In the next section of our procedure which includes steps 3, 4 and 5, the ARIMA model is formed by several strict verifications. In this way, not only AIC and SIC included in the model identification step that take effect, but also the residual correlogram as well as the forecast performance information by the models are considered.

The chosen model must be the harmonious combination of the satisfaction for all of these parameters' constraint conditions. In this case, the comparisons are made among the estimated models. The parameters of the two best models are listed in Table 1.

In Table 1, the smaller AIC and BIC of the first model show better results compare to second model in term of estimated criterions.

Besides, the forecast performance information includes Root Mean Square Error (RMSE), Mean Absolute Error (MAE) as well as Theil Inequality Coefficient (TIC), Bias Proportion, Variance Proportion and Covariance Proportion that are defined as in (7), (8) and (9).

$$RMSE = \sqrt{\sum_{t=T+1}^{T+h} (\hat{y}_t - y_t)^2 / h} \quad (7)$$

$$MAE = \sum_{t=T+1}^{T+h} |\hat{y}_t - y_t| / h \quad (8)$$

$$TIC = \frac{\sqrt{\sum_{t=T+1}^{T+h} (\hat{y}_t - y_t)^2 / h}}{\sqrt{\sum_{t=T+1}^{T+h} \hat{y}_t^2 / h} + \sqrt{\sum_{t=T+1}^{T+h} y_t^2 / h}} \quad (9)$$

Where,  $y_t, \hat{y}_t$  are actual and forecasted values in period  $t$ ,  $j = T+1, T+2, \dots, T+h$  is forecast sample.

TABLE 1. COMPARISONS BETWEEN THE TWO BEST ARIMA MODELS

Parameters	ARIMA(2,1,2)x(1,1,0) <sub>24</sub> x(1,0,0) <sub>168</sub>	ARIMA(4,1,2)x(0,1,1) <sub>24</sub> x(0,0,1) <sub>168</sub>
AIC	15.05094	15.18598
SIC	15.09670	15.23175
Root Mean Square Error	423.3	472.1
Mean Absolute Error	261.1	289.7
Theil Inequality Coefficient	0.019857	0.022381
Bias Proportion	0.000003	0.000000
Variance Proportion	0.000001	0.000001
Covariance Proportion	0.999995	0.999999

RMSE and MAE are used as relative measures to compare forecasts of different models on the same series. The smaller errors of the first model show its better forecasting ability. TIC is scale invariant and always lies between zero and one. The smaller TIC indicates that the first model is a little bit fitter to forecast the series than the second model.

$$BiasProportion = \frac{(\sum \hat{y}_t/h - \bar{y})^2}{\sum (\hat{y}_t - y_t)^2/h} \quad (10)$$

$$VarianceProportion = \frac{(s_{\hat{y}} - s_y)^2}{\sum (\hat{y}_t - y_t)^2/h} \quad (11)$$

$$CovarianceProportion = \frac{2(1-r)s_{\hat{y}}s_y}{\sum (\hat{y}_t - y_t)^2/h} \quad (12)$$

Where,  $\sum \hat{y}_t/h, \bar{y}, s_{\hat{y}}, s_y$  are the means and (biased) standard deviation of  $\hat{y}_t$  and  $y$ ; and  $r$  is the correlation between  $\hat{y}_t$  and  $y$ .

The bias proportion as in (10) is the difference between the mean of the forecast and the mean of the actual series. On the other hands, the variance proportion as in (11) is the difference between the variation of the forecast and the variation of the actual series. The covariance proportion as in (12) measures the remaining unsystematic forecasting errors. These three parameters add up to one. The results in Table 1 show that the variance proportions of the two models are the same. However, bias proportion and covariance proportion of the second model show better results than the first model.

Finally, the decision is made based on the diagnostic of residual correlogram which needed to be white noise or as closed to white noise as possible. In this case, the significant peaks of -0.164 and -0.212 at ACF and PACF of the first model (Table 2) leads us to decide that the second model ARIMA(4,1,2)x(0,1,1)<sub>24</sub>x(0,0,1)<sub>168</sub> is the chosen model.

The residuals of ARIMA(4,1,2)x(0,1,1)<sub>24</sub>x(0,0,1)<sub>168</sub> is now used to test for ARCH effect. The peaks of the ACFs and PACFs in the correlogram of residual squared (Figure 2) indicate that heteroskedasticity does occur in the series. Moreover, the significant statistic with high probability (Figure 3) once again proves the existence of ARCH effect in the series.

TABLE II. RESIDUAL CORRELOGRAM OF THE TWO ARIMA MODELS

Lag	ARIMA(2,1,2)x(1,1,0) <sub>24</sub> x(1,0,0) <sub>168</sub>	ARIMA(4,1,2)x(0,1,1) <sub>24</sub> x(0,0,1) <sub>168</sub>		
	ACFs	PACFs	ACFs	PACFs
...	...	...	...	...
15	-0.015	-0.024	-0.021	-0.010
16	-0.069	-0.091	-0.110	-0.088
17	0.005	0.020	0.026	0.042
18	0.025	0.000	0.040	0.017
19	0.115	0.132	0.107	0.120
20	-0.002	-0.059	-0.015	-0.077
21	-0.055	-0.052	-0.027	-0.048
22	0.057	0.008	0.028	-0.055
23	-0.027	-0.067	-0.002	-0.061
24	0.164	-0.212	-0.065	-0.137
25	0.122	0.053	0.117	0.064
...	...	...	...	...

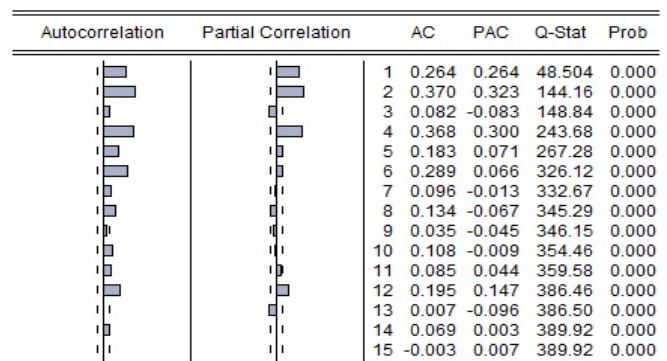


FIGURE II. CORRELOGRAM OF RESIDUAL SQUARED BY ARIMA(4,1,2)X(0,1,1)<sub>24</sub>X(0,0,1)<sub>168</sub> MODEL

Heteroskedasticity Test: ARCH

F-statistic	23.46523	Prob. F(10,674)	0.0000
Obs*R-squared	176.8958	Prob. Chi-Square(10)	0.0000

FIGURE III. ARCH TEST RESULTS BY ARIMA(4,1,2)X(0,1,1)<sub>24</sub>X(0,0,1)<sub>168</sub> MODEL

TABLE III. COMPARISONS AMONG BEST ESTIMATED MODELS

Parameters	GARCH	GARCH-M	EGARCH	EGARCH-M	PARCH	PARCH-M
AIC	-25.36	-18.39	-63.82	-51.30	-53.79	-2.009
SIC	-25.30	-18.33	-63.76	-51.23	-53.71	-1.931
RMSE	638.5076	608.7993	601.0217	582.1337	576.7703	593.9334
MAE	366.7524	323.8669	326.9939	320.8214	318.5804	322.4654
TIC	0.030268	0.028854	0.028478	0.027589	0.027334	0.028147
Bias Proportion	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Variance Proportion	0.000003	0.000131	0.000684	0.000175	0.000265	0.000275
Covariance Proportion	0.999997	0.999869	0.999316	0.999825	0.999735	0.999725

TABLE IV. COMPARISON BETWEEN CHOSEN ARIMA AND ARIMA/GARCH MODELS IN TERM OF DIFFERENCES IN DESCRIPTIVE STATISTICS OF ACTUAL AND FORECAST VALUES

Descriptive Statistics	ARIMA	GARCH(1,1)-M
Mean	0.106	0.001
Median	62.501	136.35
Maximum	365.3	265.96
Minimum	646.6553	510.0505
Std. Dev.	0.497	6.987
Skewness	0.001273	0.000904
Kurtosis	0.002314	0.001224
Jarque-Bera	0.14715	0.0686
Probability	0	0
Sum	73	1
Sum Sq. Dev.	0	0

GARCH models are now used to estimate the conditional variance for the errors of the obtained ARIMA model. Again, the constraints of choosing ARIMA model are now applied in this section. The model is chosen based on AIC, SIC, in-sample forecast parameters and residual correlogram. The comparisons again are made among the estimated models, and the best results are listed in Table 3.

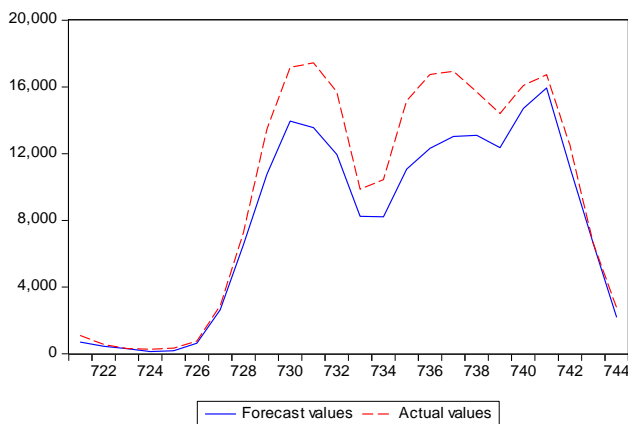
All of these models have the relatively good results of estimate and forecast performance parameters and meet the requirements of white noise in the correlogram of residual



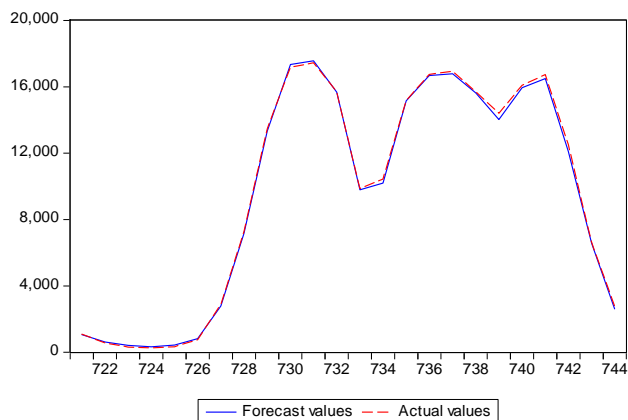
squared. The comparisons of differences between actual and in-sample forecast descriptive statistics values are now made for these combination models. The comparative results achieved by these models are now compared to those results achieved by ARIMA(4,1,2)x(0,1,1)<sub>24</sub>x(0,0,1)<sub>168</sub> model.

In this case, GARCH(1,1)-M is chosen due to outperforming ARIMA in most of descriptive statistics that listed in Table 4. The out-of-sample forecast is now implemented for both achieved ARIMA model and ARIMA/GARCH model to confirm our choice.

Figure 4 and Figure 5 clearly indicate that ARIMA/GARCH outperforms ARIMA model in out-of-sample forecasting of this mobile communication traffic series. This means that GARCH model is capable to process well the volatility in the series, while ARIMA model still meets the problem of heteroskedasticity, so the out-of-sample forecast values do not fit well to the actual values. This out-of-sample forecast result is an evidence to prove that our proposed procedure can work well in order to build an ARIMA/GARCH model that outperforms ARIMA in term of capture and forecast mobile communication traffic data.



(A) ARIMA(4,1,2)x(0,1,1)<sub>24</sub>x(0,0,1)<sub>168</sub>



(B) ARIMA(4,1,2)x(0,1,1)<sub>24</sub>x(0,0,1)<sub>168</sub>/GARCH(1,1)-M

FIGURE IV. OUT-OF-SAMPLE FORECAST RESULTS COMPARISON BETWEEN ARIMA AND ARIMA/GARCH

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob*
1	-0.005	-0.005	0.0162	0.899	
2	-0.004	-0.004	0.0248	0.988	
3	-0.005	-0.006	0.0458	0.997	
4	-0.004	-0.004	0.0589	1.000	
5	-0.005	-0.005	0.0763	1.000	
6	0.002	0.002	0.0786	1.000	
7	-0.004	-0.004	0.0922	1.000	
8	0.034	0.034	0.9092	0.999	
9	-0.005	-0.005	0.9267	1.000	
10	-0.005	-0.005	0.9469	1.000	
11	-0.005	-0.005	0.9630	1.000	
12	-0.001	-0.001	0.9636	1.000	
13	-0.005	-0.005	0.9832	1.000	
14	-0.005	-0.005	0.9993	1.000	
15	-0.005	-0.005	1.0201	1.000	
16	-0.005	-0.007	1.0403	1.000	
17	-0.005	-0.004	1.0552	1.000	
18	-0.005	-0.005	1.0737	1.000	
19	-0.005	-0.005	1.0931	1.000	
20	-0.004	-0.004	1.1029	1.000	

FIGURE V. WHITE NOISE ACHIEVED IN CORRELOGRAM OF RESIDUAL SQUARED BY ARIMA(4,1,2)x(0,1,1)<sub>24</sub>x(0,0,1)<sub>168</sub>/GARCH(1,1)-M MODEL

## V. CONCLUSIONS

In this study, we improved our earlier research and proposed a novel procedure to build an ARIMA/GARCH model to fit and forecast mobile communication traffic. Based on Box-Jenkin method, a series of constraints include AIC, SIC, in-sample forecast performance evaluation as well as residual correlogram diagnostics are applied to strictly examine models in order to firstly find the most suitable ARIMA model. GARCH models are then applied to deal with the volatility found in the series based on the achieved ARIMA model. In this step, AIC, SIC, in-sample-forecast performance evaluation parameters are deployed again, together with correlogram of residual squared. Finally, we figured out that descriptive statistics of the series can be used as a good tool to find out the best ARIMA/GARCH combination model which outperforms ARIMA model with the proof of out-of-sample forecast.

By applying our proposed procedure to the training data of mobifone 30-day hourly traffic series which has a significant impact of volatility, an ARIMA/GARCH-M model is found to be the best one in modeling and forecast our mobile communication traffic data after processing the 31st day out-of-sample forecast results. The achieved ARIMA/GARCH-M model shows a better result compared with ARIMA model, leading to the fact that our proposed procedure is feasible in modeling and forecasting mobile communication traffic data.

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