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A Computational Algorithm for the Inverse of a Sevendiagonal Matrix

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***Abstract*—In this article, we investigate the inverse of the sevendiagonal matrix. Under certain conditions, the efficient algorithm which is suitable for implementation using computer algebra systems, is proposed. Also, the inverse of an anti-sevendiagonal matrix is obtained based on the special relation between the sevendiagonal matrix and the anti-sevendiagonal matrix. A numerical example is provided to further illustrate the validity of the corresponding algorithm.**

Keywords-a sevendiagonal matrix; inverse matrix; determinant; computer algebra system

I. INTRODUCTION

A large number of banded matrices are appeared in the process of numerical analysis and numerical calculation, such as tridiagonal, pentadiagonal and sevendiagonal matrices. They play an important role in theory, and they are applied in spline approximation, parallel computing, partial differential equations(PDE) and boundary value problems(BVP). For a long time, the inverses of tridiagonal and pentadiagonal matrices have been researched[1-11]. Recently, the inverses of Toeplitz tridiagonal and pentadiagonal matrices are also investigated[12]. And the inverses of sevendiagonal and generalized sevendiagonal matrices have been studied[13-15]. Many of them gave algorithms to obtain the inverse matrices by LU factorization[3-5,9,10,13-15]. A few by dividing blocks of diagonal matrices[6,12]. And Ran Rui-sheng[7] used the Doolittle factorization to get the inverse matrix of a general tridiagonal matrix.

In this paper, based on the numerical algorithms given in [1,13], we present a fast algorithm for computing the inverse of the sevendiagonal matrix. Moreover, the inverse of anti-sevendiagonal matrix is also investigated.

The rest of this paper is organized as follows. In Section 2, we present an efficient algorithm for the inverse of an n-by-n sevendiagonal matrix, and the inverse of the anti-sevendiagonal matrix is also given. One numerical example is provided to illustrate the proposed algorithm in Section 3. Finally, some conclusions of the work are given.

II. INVERSE OF A SEVENDIAGONAL MATRIX

In this section, we mainly focus on giving some recurrence formulas for the inverse of an n-by-n sevendiagonal matrix in terms of their columns.

Without loss of generality, a sevendiagonal matrix with order n can be given as

$$\mathbf{T}=\begin{pmatrix} a_1 & b_1 & c_1 & d_1 & 0 & \cdots & \cdots & 0 \\ \alpha_2 & a_2 & b_2 & c_2 & d_2 & \ddots & & \vdots \\ \beta_3 & \alpha_3 & a_3 & b_3 & c_3 & d_3 & \ddots & \vdots \\ \gamma_4 & \beta_4 & \alpha_4 & a_4 & \ddots & \ddots & \ddots & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & d_{n-3} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & c_{n-2} \\ \vdots & & \ddots & \ddots & \ddots & \ddots & \ddots & b_{n-1} \\ 0 & \cdots & \cdots & 0 & \gamma_n & \beta_n & \alpha_n & a_n \end{pmatrix} \tag{1}$$

where $a_i, b_i, c_i, d_i, \alpha_i, \beta_i$ and $\gamma_i (i=1,2,\cdots,n)$ are finite sequences of numbers such that $d_i \neq 0 (i=1,2,\cdots,n-3)$, $d_{n-2}=d_{n-1}=d_n=1, \alpha_1=\beta_1=\beta_2=0, \gamma_1=\gamma_2=\gamma_3=0$, and $b_n = c_{n-1} = c_n = 0$.

Suppose that the sevendiagonal matrix $\mathbf{T}$ is non-singular and the inverse is

$$\mathbf{T}^{-1}=(C_1, C_2, \ldots, C_n),$$

where C_j denote the jth column of $\mathbf{T}^{-1}$, $j=1,2,\cdots,n$. Here C_j also can be written as

$$C_j=(C_1, C_2, \cdots, C_n)E_j,$$

where $E_j=(\delta_{1j}, \delta_{2j}, \cdots, \delta_{nj})^t$, $j=1,2,\cdots,n$, and δ_{ij} is the Kronecker symbol.

Through the relation $\mathbf{T}^{-1}\mathbf{T}=I_n$ (here I_n is the $n\times n$ identity matrix), we get the relations as

$$\begin{cases} C_{n-3}=(E_n-c_{n-2}C_{n-2}-b_{n-1}C_{n-1}-a_nC_n)/d_{n-3} \\ C_{n-4}=(E_{n-1}-c_{n-3}C_{n-3}-b_{n-2}C_{n-2}-a_{n-1}C_{n-1}-\alpha_nC_n)/d_{n-4} \\ C_{n-5}=(E_{n-2}-c_{n-4}C_{n-4}-b_{n-3}C_{n-3}-a_{n-2}C_{n-2}-\alpha_{n-1}C_{n-1}-\beta_nC_n)/d_{n-5} \\ C_{j-3}=(E_j-c_{j-2}C_{j-2}-b_{j-1}C_{j-1}-a_jC_j-\alpha_{j+1}C_{j+1}-\beta_{j+2}C_{j+2}-\gamma_{j+3}C_{j+3})/d_{j-3} \\ 4\leq j\leq n-3 \end{cases} \tag{2}$$

where $E_j = (\delta_{1j}, \delta_{2j}, \cdots, \delta_{nj},)^t$, $j = 1,2,\cdots,n$, and δ_{ij} is the Kronecker symbol.

It follows from (2) that if we get the last three columns C_n, C_{n-1} and C_{n-2}, then we can recursively determine all other columns $C_{n-3},\ldots,C_1$. Now we shall give the recurrence formulas for computing C_n, C_{n-1} and C_{n-2}.

Consider the three finite sequences of numbers A_i, B_i and D_i $(i = 0,1,\cdots,n+2)$ with the initial conditions of $A_0 = A_1 = B_0 = B_2 = D_1 = D_2 = 0$ and $A_2 = B_1 = D_0 = 1$, which satisfy

$$\begin{pmatrix}\mathbf{T}_{n\times n} & \mathbf{T}'_{n\times 3}\end{pmatrix}\begin{pmatrix}\mathbf{A}_{n\times 1}\\ \mathbf{A}'_{3\times 1}\end{pmatrix} = \mathbf{0}_{n\times 1}, \tag{3}$$

$$\begin{pmatrix}\mathbf{T}_{n\times n} & \mathbf{T}'_{n\times 3}\end{pmatrix}\begin{pmatrix}\mathbf{B}_{n\times 1}\\ \mathbf{B}'_{3\times 1}\end{pmatrix} = \mathbf{0}_{n\times 1}, \tag{4}$$

$$\begin{pmatrix}\mathbf{T}_{n\times n} & \mathbf{T}'_{n\times 3}\end{pmatrix}\begin{pmatrix}\mathbf{D}_{n\times 1}\\ \mathbf{D}'_{3\times 1}\end{pmatrix} = \mathbf{0}_{n\times 1}, \tag{5}$$

where $\mathbf{T}' = \begin{pmatrix}\mathbf{0}_{(n-3)\times 3} & \mathbf{I}_{3\times 3}\end{pmatrix}^t$, $\mathbf{A} = [A_0, A_1, \ldots, A_{n-1}]^t$, $\mathbf{B} = [B_0, B_1, \ldots, B_{n-1}]^t$, $\mathbf{D} = [D_0, D_1, \ldots, D_{n-1}]^t$, $\mathbf{A}' = [A_n, A_{n+1}, A_{n+2}]^t$, $\mathbf{B}' = [B_n, B_{n+1}, B_{n+2}]^t$, $\mathbf{D}' = [D_n, D_{n+1}, D_{n+2}]^t$. Here $\mathbf{I}_{3\times 3}$ is the 3×3 identify matrix.

On the basis of (3), (4) and (5) ,we can get the Eqs

$$\begin{cases} a_1A_0 + b_1A_1 + c_1A_2 + d_1A_3 = 0 \\ \alpha_2A_0 + a_2A_1 + b_2A_2 + c_2A_3 + d_2A_4 = 0 \\ \beta_3A_0 + \alpha_3A_1 + a_3A_2 + b_3A_3 + c_3A_4 + d_3A_5 = 0 \\ \gamma_{i-1}A_{i-5} + \beta_{i-1}A_{i-4} + \alpha_{i-1}A_{i-3} + a_{i-1}A_{i-2} + b_{i-1}A_{i-1} + c_{i-1}A_i + d_{i-1}A_{i+1} = 0 \\ i = 5,6,\cdots,n-2 \\ \gamma_{n-2}A_{n-6} + \beta_{n-2}A_{n-5} + \alpha_{n-2}A_{n-4} + a_{n-2}A_{n-3} + b_{n-2}A_{n-2} + c_{n-2}A_{n-1} = -A_n \\ \gamma_{n-1}A_{n-5} + \beta_{n-1}A_{n-4} + \alpha_{n-1}A_{n-3} + a_{n-1}A_{n-2} + b_{n-1}A_{n-1} = -A_{n+1} \\ \gamma_nA_{n-4} + \beta_nA_{n-3} + \alpha_nA_{n-2} + a_nA_{n-1} = -A_{n+2} \end{cases} \tag{6}$$

$$\begin{cases} a_1B_0 + b_1B_1 + c_1B_2 + d_1B_3 = 0 \\ \alpha_2B_0 + a_2B_1 + b_2B_2 + c_2B_3 + d_2B_4 = 0 \\ \beta_3B_0 + \alpha_3B_1 + a_3B_2 + b_3B_3 + c_3B_4 + d_3B_5 = 0 \\ \gamma_{i-1}B_{i-5} + \beta_{i-1}B_{i-4} + \alpha_{i-1}B_{i-3} + a_{i-1}B_{i-2} + b_{i-1}B_{i-1} + c_{i-1}B_i + d_{i-1}B_{i+1} = 0 \\ i = 5,6,\cdots,n-2 \\ \gamma_{n-2}B_{n-6} + \beta_{n-2}B_{n-5} + \alpha_{n-2}B_{n-4} + a_{n-2}B_{n-3} + b_{n-2}B_{n-2} + c_{n-2}B_{n-1} = -B_n \\ \gamma_{n-1}B_{n-5} + \beta_{n-1}B_{n-4} + \alpha_{n-1}B_{n-3} + a_{n-1}B_{n-2} + b_{n-1}B_{n-1} = -B_{n+1} \\ \gamma_nB_{n-4} + \beta_nB_{n-3} + \alpha_nB_{n-2} + a_nB_{n-1} = -B_{n+2} \end{cases} \tag{7}$$

$$\begin{cases} a_1D_0 + b_1D_1 + c_1D_2 + d_1D_3 = 0 \\ \alpha_2D_0 + a_2D_1 + b_2D_2 + c_2D_3 + d_2D_4 = 0 \\ \beta_3D_0 + \alpha_3D_1 + a_3D_2 + b_3D_3 + c_3D_4 + d_3D_5 = 0 \\ \gamma_{i-1}D_{i-5} + \beta_{i-1}D_{i-4} + \alpha_{i-1}D_{i-3} + a_{i-1}D_{i-2} + b_{i-1}D_{i-1} + c_{i-1}D_i + d_{i-1}D_{i+1} = 0 \\ i = 5,6,\cdots,n-2 \\ \gamma_{n-2}D_{n-6} + \beta_{n-2}D_{n-5} + \alpha_{n-2}D_{n-4} + a_{n-2}D_{n-3} + b_{n-2}D_{n-2} + c_{n-2}D_{n-1} = -D_n \\ \gamma_{n-1}D_{n-5} + \beta_{n-1}D_{n-4} + \alpha_{n-1}D_{n-3} + a_{n-1}D_{n-2} + b_{n-1}D_{n-1} = -D_{n+1} \\ \gamma_nD_{n-4} + \beta_nD_{n-3} + \alpha_nD_{n-2} + a_nD_{n-1} = -D_{n+2} \end{cases} \tag{8}$$

From the recurrence equations (6), (7) and (8), we can give a matrix form as

$$\mathbf{TA} = -A_nE_{n-2} - A_{n+1}E_{n-1} - A_{n+2}E_n, \tag{9}$$

$$\mathbf{TB} = -B_nE_{n-2} - B_{n+1}E_{n-1} - B_{n+2}E_n, \tag{10}$$

$$\mathbf{TD} = -D_nE_{n-2} - D_{n+1}E_{n-1} - D_{n+2}E_n. \tag{11}$$

Multiply (9),(10),(11) by Q_{11}, Q_{12}, Q_{13} respectively to get the following equations

$$\mathbf{TA}Q_{11} = -A_nQ_{11}E_{n-2} - A_{n+1}Q_{11}E_{n-1} - A_{n+2}Q_{11}E_n, \tag{12}$$

$$\mathbf{TB}Q_{12} = -B_nQ_{12}E_{n-2} - B_{n+1}Q_{12}E_{n-1} - B_{n+2}Q_{12}E_n, \tag{13}$$

$$\mathbf{TD}Q_{13} = -D_nQ_{13}E_{n-2} - D_{n+1}Q_{13}E_{n-1} - D_{n+2}Q_{13}E_n, \tag{14}$$

multiply (9),(10),(11) by Q_{21}, Q_{22}, Q_{23} respectively to get the following equations

$$\mathbf{TA}Q_{21} = -A_nQ_{21}E_{n-2} - A_{n+1}Q_{21}E_{n-1} - A_{n+2}Q_{21}E_n, \tag{15}$$

$$\mathbf{TB}Q_{22} = -B_nQ_{22}E_{n-2} - B_{n+1}Q_{22}E_{n-1} - B_{n+2}Q_{22}E_n, \tag{16}$$

$$\mathbf{TD}Q_{23} = -D_nQ_{23}E_{n-2} - D_{n+1}Q_{23}E_{n-1} - D_{n+2}Q_{23}E_n, \tag{17}$$

multiply (9),(10),(11) by Q_{31}, Q_{32}, Q_{33} respectively to get the following equations

$$\mathbf{TA}Q_{31} = -A_nQ_{31}E_{n-2} - A_{n+1}Q_{31}E_{n-1} - A_{n+2}Q_{31}E_n, \tag{18}$$

$$\mathbf{TB}Q_{32} = -B_nQ_{32}E_{n-2} - B_{n+1}Q_{32}E_{n-1} - B_{n+2}Q_{32}E_n, \tag{19}$$

$$\mathbf{TD}Q_{33} = -D_n Q_{33} E_{n-2} - D_{n+1} Q_{33} E_{n-1} - D_{n+2} Q_{33} E_n, \quad (20)$$

where

$Q_{11} = B_{n+2}D_{n+1} - B_{n+1}D_{n+2}, Q_{12} = A_{n+1}D_{n+2} - A_{n+2}D_{n+1}, Q_{13} = A_{n+2}B_{n+1} - A_{n+1}B_{n+2},$

$Q_{21} = B_{n+2}D_n - B_nD_{n+2},\ Q_{22} = A_nD_{n+2} - A_{n+2}D_n,\ Q_{23} = A_{n+2}B_n - A_nB_{n+2},$

$Q_{31} = B_{n+1}D_n - B_nD_{n+1},\ Q_{32} = A_nD_{n+1} - A_{n+1}D_n,\ Q_{33} = A_{n+1}B_n - A_nB_{n+1}.$

Let (12)+(13)+(14), (15)+(16)+(17)and(18)+(19)+(20), thus

$$\mathbf{TX} = -X_n E_{n-2}, \quad (21)$$

$$\mathbf{TY} = -Y_{n+1} E_{n-1}, \quad (22)$$

$$\mathbf{TZ} = -Z_{n+2} E_n, \quad (23)$$

where $\mathbf{X} = [X_0, X_1, \ldots, X_{n-1}]^t$, $\mathbf{Y} = [Y_0, Y_1, \ldots, Y_{n-1}]^t$, $\mathbf{Z} = [Z_0, Z_1, \ldots, Z_{n-1}]^t$, and for $0 \le i \le n+2$

$$X_i = \det\begin{pmatrix} A_{n+2} & A_{n+1} & A_i \\ B_{n+2} & B_{n+1} & B_i \\ D_{n+2} & D_{n+1} & D_i \end{pmatrix}, \quad (24)$$

$$Y_i = \det\begin{pmatrix} A_{n+2} & A_n & A_i \\ B_{n+2} & B_n & B_i \\ D_{n+2} & D_n & D_i \end{pmatrix}, \quad (25)$$

$$Z_i = \det\begin{pmatrix} A_{n+1} & A_n & A_i \\ B_{n+1} & B_n & B_i \\ D_{n+1} & D_n & D_i \end{pmatrix}. \quad (26)$$

We can easily get $X_n = -Y_{n+1} = Z_{n+2}$ by (24), (25) and (26).

Lemma 2.1. If $X_n = 0$, then the matrix $\mathbf{T}$ is singular.

Proof. We have

$$X_0 = A_{n+2}B_{n+1} - A_{n+1}B_{n+2}, Y_0 = A_{n+2}B_n - A_nB_{n+2}, Z_0 = A_{n+1}B_n - A_nB_{n+1}$$

If $A_{n+2} \ne 0, B_{n+1} \ne 0$ and $A_{n+1} = 0$, then $\mathbf{X}$ is non-null and $\mathbf{TX} = 0$.

If $A_{n+1} \ne 0, B_n \ne 0$ and $A_n = 0$, then $\mathbf{Z}$ is non-null and $\mathbf{TZ} = 0$.

If $A_n \ne 0, B_{n+2} \ne 0$ and $A_{n+2} = 0$, then $\mathbf{Y}$ is non-null and $\mathbf{TY} = 0$.

If $A_{n+2} = A_{n+1} = A_n = 0,$ then $\mathbf{A}$ is non-null and $\mathbf{TA} = 0$. The proof is completed.

Theorem 2.2. Suppose that $X_n \ne 0$, then $\mathbf{T}$ is invertible and

$$\begin{aligned} C_n &= [-Z_0 / Z_{n+2}, -Z_1 / Z_{n+2}, \cdots, -Z_{n-1} / Z_{n+2}]^t \\ C_{n-1} &= [-Y_0 / Y_{n+1}, -Y_1 / Y_{n+1}, \cdots, -Y_{n-1} / Y_{n+1}]^t \\ C_{n-2} &= [-X_0 / X_n, -X_1 / X_n, \cdots, -X_{n-1} / X_n]^t \end{aligned} \quad (27)$$

Proof. According to the determinant of tridiagonal and pentadiagonal matrix (see [1]), we can get $\det(\mathbf{T}) = (-1)^n (\prod_{k=1}^{n} d_k) X_n \ne 0$, so $\mathbf{T}$ is invertible. From the (21), (22), (23) and (27), we have $\mathbf{T}C_n = E_n$, $\mathbf{T}C_{n-1} = E_{n-1}$ and $\mathbf{T}C_{n-2} = E_{n-2}$. The proof is completed.

Algorithm 2.3. To find the $n \times n$ inverse matrix of the sevendiagonal matrix in (1) by using the relations (2)-(27).

INPUT: Order of the matrix n and the components a_i, b_i, c_i , d_i , α_i , β_i , γ_i , $i = 1,2,\cdots,n$, where $d_{n-2} = d_{n-1} = d_n = 1$, $c_{n-1} = c_n = b_n = \alpha_1 = \beta_1 = \beta_2 = \gamma_1 = \gamma_2 = \gamma_3 = 0$.

OUTPUT: inverse matrix $\mathbf{T}^{-1}$.

Step 1: If $d_k = 0$ for any $k = 1,2,\cdots,n-3$, then set $d_k = t$ (t is just a symbolic name).

Step 2: Set $A_0 = A_1 = 0$, $A_2 = 1$, then for $i = 3,\cdots,n+2$, compute the components A_i by using (6).

Step 3: Set $B_0 = B_2 = 0$, $B_1 = 1$, then for $i = 3,\cdots,n+2$, compute the components B_i by using (7).

Step 4: Set $D_0 = 1, D_1 = D_2 = 0$, then for $i = 3,\cdots,n+2$, compute the components D_i by using (8).

Step 5: For $0 \le i \le n+2$, compute the components X_i, Y_i and Z_i by using (24), (25) and (26).

Step 6: Substitute the actual value $t = 0$, and compute $\det(\mathbf{T}) = (-1)^n (\prod_{k=1}^{n} d_k) X_n$. If the matrix $\mathbf{T}$ is singular , then OUTPUT ('Singular Matrix'); Stop.

Step 7: Compute the components C_{n-2}, C_{n-1} and C_n by using (27).

Step 8: Compute the components C_{n-3}, C_{n-4} and C_{n-5} by using the first three equations in (2).

Step 9: For $j = n-3:-1:4$, compute the components C_{j-3} by using the last equation in (2).

Remark 2.4. Here the algorithm complexity of algorithm 2.3 will be computed.

The algorithm complexity in step 1 is $n-3$, in each step 2-4 is $n+3$, in step 5 is $3n+9$, in the step 6-8 are $n, 3n, 3n$ respectively, and in the last step is n^2-6n. So the algorithm complexity of algorithm 2.3 is $n^2+8n+15$.

Here we will discuss the inverse of $n\times n$ anti-sevendiagonal matrix, the anti-sevendiagonal matrix of $\mathbf{T}$ is denoted by $\mathbf{H}$, take the form

$$\mathbf{H}=\begin{pmatrix} 0 & \cdots & \cdots & 0 & d_1 & c_1 & b_1 & a_1 \\ \vdots & & \iddots & d_2 & c_2 & b_2 & a_2 & \alpha_2 \\ \vdots & \iddots & d_3 & c_3 & b_3 & a_3 & \alpha_3 & \beta_3 \\ 0 & \iddots & \iddots & \iddots & a_4 & \alpha_4 & \beta_4 & \gamma_4 \\ d_{n-3} & \iddots & \iddots & \iddots & \iddots & \iddots & \iddots & 0 \\ c_{n-2} & \iddots & \iddots & \iddots & \iddots & \iddots & \iddots & \vdots \\ b_{n-1} & \iddots & \iddots & \iddots & \iddots & \iddots & & \vdots \\ a_n & \alpha_n & \beta_n & \gamma_n & 0 & \cdots & \cdots & 0 \end{pmatrix} \tag{28}$$

To do this we take into account the fact that for the $n\times n$ matrix $\mathbf{R}=(r_{ij})$ given by

$$\mathbf{R}=\begin{pmatrix} 0 & \cdots & \cdots & 0 & 1 \\ \vdots & & \iddots & 1 & 0 \\ \vdots & \iddots & \iddots & \iddots & \vdots \\ 0 & \iddots & \iddots & & \vdots \\ 1 & 0 & \cdots & \cdots & 0 \end{pmatrix}=\mathbf{R^t} \tag{29}$$

From [1] we can get

$$\mathbf{H}=\mathbf{TR}. \tag{30}$$

Moreover, if the matrix $\mathbf{T}$ is invertible and its inverse is $\mathbf{T}^{-1}$, then we obtain

$$\mathbf{H}^{-1}=\mathbf{R}^{-1}\mathbf{T}^{-1}=\mathbf{RT}^{-1}. \tag{31}$$

III. ONE ILLUSTRATIVE EXAMPLE

In this section we are going to give one illustrative example.

Example 3.1. Consider the 6×6 matrices $\mathbf{T}$ and $\mathbf{H}$ given by

$$\mathbf{T}=\begin{pmatrix} 1 & 1 & -2 & 1 & 0 & 0 \\ 1 & 1 & -1 & -1 & 1 & 0 \\ 2 & 0 & 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & 2 & 1 & 1 \\ 0 & -1 & 1 & 2 & 1 & 4 \\ 0 & 0 & 1 & 1 & -1 & 1 \end{pmatrix}$$

$$\mathbf{H}=\begin{pmatrix} 0 & 0 & 1 & -2 & 1 & 1 \\ 0 & 1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & 0 & 2 \\ 1 & 1 & 2 & 1 & -1 & 1 \\ 4 & 1 & 2 & 1 & -1 & 0 \\ 1 & -1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

By applying the algorithm 2.3, we get

- $\mathbf{A}=[0,0,1,2,3,4]^t$ and $\mathbf{A}'=[-12,-24,-4]^t$, (step 2)
- $\mathbf{B}=[0,1,0,-1,-2,-3]^t$ and $\mathbf{B}'=[8,17,2]^t$, (step 3)
- $\mathbf{D}=[1,0,0,-1,-2,-5]^t$ and $\mathbf{D}'=[8,24,4]^t$, (step 4)
- $\mathbf{X}=[-20,0,-20,-20,-20,20]^t$ and $X_6=80, X_7=0, X_8=0$
  $\mathbf{Y}=[-8,-16,-16,-8,0,24]^t$ and $Y_6=0, Y_7=-80, Y_8=0$
  $\mathbf{Z}=[12,-96,-56,-28,0,4]^t$ and $Z_6=0, Z_7=0, Z_8=80$,
  (step 5)
- $\det(\mathbf{T})=(-1)^6\left(\prod_{k=1}^{6} d_k\right)X_6=80\neq 0$, (step 6)

$$C_6 = [-0.15, \quad 1.2, \quad 0.7, \quad 0.35, \quad 0, \quad -0.05]^t$$
$$C_5 = [\ -0.1, \ -0.2, \ -0.2, \ -0.1, \quad 0, \quad 0.3]^t$$
- $C_4 = [\ 0.25, \quad 0, \quad 0.25, \ 0.25, \ 0.25, \ -0.25]^t$,

(step 7)

$$C_3 = [\ 0.3, \ -0.4, \ -0.15, \ -0.2, -0.25, \quad 0.1]^t$$
$$C_2 = [\quad 0, \quad 1, \quad 0.5, \quad 0, \quad 0.5, \quad 0]^t$$
- $C_1 = [0.15, \ -0.2, \ -0.45, \ 0.15, \ -0.25, \ 0.05]^t$. (step 8)

So,

$$\mathbf{T}^{-1} = \frac{1}{20}\begin{pmatrix} 3 & 0 & 6 & 5 & -2 & -3 \\ -4 & 20 & -8 & 0 & -4 & 24 \\ -9 & 10 & -3 & 5 & -4 & 14 \\ 3 & 0 & -4 & 5 & -2 & 7 \\ -5 & 10 & -5 & 5 & 0 & 0 \\ 1 & 0 & 2 & -5 & 6 & -1 \end{pmatrix}.$$

Hence, using (31) gives

$$\mathbf{H}^{-1} = \frac{1}{20}\begin{pmatrix} 1 & 0 & 2 & -5 & 6 & -1 \\ -5 & 10 & -5 & 5 & 0 & 0 \\ 3 & 0 & -4 & 5 & -2 & 7 \\ -9 & 10 & -3 & 5 & -4 & 14 \\ -4 & 20 & -8 & 0 & -4 & 24 \\ 3 & 0 & 6 & 5 & -2 & -3 \end{pmatrix}.$$

IV. CONCLUSION

In this paper, we mainly give a fast algorithm for the inverse of a sevendiagonal matrix, and the fast algorithm is appropriate for implementation using computer algebra systems. What's more, in the light of algorithm 2.3, then we can obtain that the algorithm complexity is $n^2+8n+15$. Base on the illustrative example of section 3 which confirm the validity of this algorithm. The recent algorithm presented in [1] for inverting a tridiagonal and pentadiagonal matrix is a special case of this new algorithm.

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