Improved ESPRIT Algorithm Based DOA Estimation for Multi-target Tracking

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Abstract. ESPRIT algorithm is one of the main algorithms for DOA estimation, which is a basic problem in multi-target tracking. This paper proposes an improved ESPRIT algorithm for multi-target tracking, and takes simulation analysis of the performance of DOA estimation of the SNR and sub-array spacing, then compares the improved ESPRIT algorithm with the MUSIC algorithm. The simulation results show that the estimation error is larger when the SNR is low and the sub-array spacing is small and the estimation error of the improved algorithm is slightly larger than that of the MUSIC algorithm, but the running time is much smaller than the MUSIC algorithm. The research of this paper can provide theoretical support for the follow-up study.

Introduction

A fundamental problem in multi-target tracking is direction of arrival (DOA) estimation for spatial signal, the typical algorithm of which contains Capon algorithm, Multiple Signal Classification(MUSIC) and Estimating Signal Via Rotational Invariance Techniques(ESPRIT). Among them, the MUSIC algorithm and ESPRIT algorithm are the most widely used. MUSIC algorithm can get a relatively high accuracy of the parameters estimation, but the amount of computation is too large, while ESPRIT algorithm can effectively overcome the large amount of calculation of music algorithm. The basic idea of the ESPRIT algorithm is: array in the structure is divided into two identical sub-arrays, and the distance of the two sub-array corresponding element offset is equal, then the array elements are divided into pairs of the form, and between each pair has the same translational distance. This incident angle in the two sub-arrays differ only by a rotation invariant factor, which contains all of the incident signal direction of arrival information, so by solving a generalized eigen value equation of the incident signal DOA can be obtained. ESPRIT algorithm does not need to know the geometric structure of the array, so the calibration requirements for the array is relatively low, and now the ESPRIT algorithm has become one of the main DOA estimation algorithm.

Improved ESPRIT Algorithm

By a composition of m dipole for the antenna array with K array element, the elements corresponding to two sub-arrays get equal sensitivity model and the same offset displacement volume d and consist of D narrowband signal sources with the center frequency ω_0, which incident to the array, then the received signal of the corresponding element of two sub-array groups can be expressed as follow:

$$x_i(t) = \sum_{k=1}^{D} s_k(t) a_i(\theta_k) + n_{xi}(t) \tag{1}$$

$$u_i(t) = \sum_{k=1}^{D} s_k(t) e^{j\omega_0 d \sin\theta_k/c} a_i(\theta_k) + n_{ui}(t) \tag{2}$$

In the formula, θ_k indicates the incident direction of the k signal source and the received signal of each sub-array is represented as a vector form:

$$\mathbf{x}(t) = \mathbf{A}(\theta)\mathbf{S}(t) + \mathbf{n}_x(t) \tag{3}$$

$$\mathbf{u}(t) = \mathbf{A}(\theta)\Phi\mathbf{S}(t) + \mathbf{n}_u(t) \tag{4}$$

In the formula, $\mathbf{x}(t), \mathbf{u}(t) \in \pounds^{m \times 1}$ is the data vector with noise, $\Phi = diag\{e^{j\omega_0 d \sin\theta_1/c}, L, e^{j\omega_0 d \sin\theta_D/c}\}$ stand for the phase delay between the two arrays, which is also known as the rotation invariant factor, $\mathbf{n}_u(t) = [n_{u1}(t), L, n_{um}(t)]^T$ represent the additive noise vector.

The receiving vector of the whole array is defined as $\mathbf{z}(t)$, the vector of the sub-array of which is represented as follows::

$$\mathbf{z}(t) = \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix} = \overline{\mathbf{A}}\mathbf{S}(t) + \mathbf{n}_z(t) \tag{5}$$

In the formula,

$$\overline{\mathbf{A}} = \begin{bmatrix} \mathbf{A} \\ \mathbf{A}\Phi \end{bmatrix}, \mathbf{n}_z(t) = \begin{bmatrix} \mathbf{n}_x(t) \\ \mathbf{n}_u(t) \end{bmatrix} \tag{6}$$

The autocorrelation matrix of the received vector $\mathbf{z}(t)$ of the antenna array is:

$$\mathbf{R}_{zz} = E\{\mathbf{z}(t)\mathbf{z}^H(t)\} = \overline{\mathbf{A}}\mathbf{R}_{ss}\overline{\mathbf{A}}^H + \sigma_n^2 \Sigma_n \tag{7}$$

Given $D \leq 2m$, then the smallest $2m - D$ generalized eigenvalue of $(\mathbf{R}_{zz}, \Sigma_n)$ is equal to σ_n^2, while the characteristic vector $\mathbf{E}_s$ corresponding to the D maximum generalized eigenvalue is according to:

$$Range\{\mathbf{E}_s\} = Range\{\overline{\mathbf{A}}\} \tag{8}$$

In the formula, $Range\{g\}$ represents the space of the vector in the matrix is. The existence and uniqueness of the non-singular matrix is according to:

$$\mathbf{E}_s = \overline{\mathbf{A}}\mathbf{T} \tag{9}$$

Because of the rotation invariant structure of the array, $\mathbf{E}_s$ can be decomposed into $\mathbf{E}_x \in \pounds^{m \times D}$ and $\mathbf{E}_u \in \pounds^{m \times D}$.

$$\mathbf{E}_s = \begin{bmatrix} \mathbf{E}_x \\ \mathbf{E}_u \end{bmatrix} = \begin{bmatrix} \mathbf{AT} \\ \mathbf{A\Phi T} \end{bmatrix} \tag{10}$$

As $\mathbf{E}_x$ and $\mathbf{E}_u$ share a column space, the rank of the $\mathbf{E}_{xu} = \begin{bmatrix} \mathbf{E}_x \mid \mathbf{E}_u \end{bmatrix}$ is D, then:

$$Range\{\mathbf{E}_x\} = Range\{\mathbf{E}_u\} = Range\{\mathbf{A}\} \tag{11}$$

This indicates that there exists a unique matrix $\mathbf{F} \in \pounds^{2D \times D}$ with rank D,

$$0 = \begin{bmatrix} \mathbf{E}_x \mid \mathbf{E}_u \end{bmatrix} \mathbf{F} = \mathbf{E}_x \mathbf{F}_x + \mathbf{E}_u \mathbf{F}_u = \mathbf{ATF}_x + \mathbf{A\Phi TF}_u \tag{12}$$

Define:

$$\Psi = -\mathbf{F}_x \mathbf{F}_u^{-1} \tag{13}$$

Handle (13) into type (12), then,

$$\mathbf{AT\Psi} = \mathbf{A\Phi T} \Rightarrow \mathbf{AT\Psi T}^{-1} = \mathbf{A\Phi} \tag{14}$$

If the direction of the incident signal is different, then the array $\mathbf{A}$ is full of rank, then it can be obtained:

$$\mathbf{T\Psi T}^{-1} = \Phi \tag{15}$$

Obviously, the eigenvalues of Ψ must be equal to the diagonal elements of the diagonal matrix Φ, while the column vectors $\mathbf{T}$ are the eigenvectors of Ψ.

ESPRIT algorithm avoids the search process of some DOA estimation method, which greatly reduces the amount of computation and the storage requirements for hardware. Compared to MUSIC algorithm, ESPRIT algorithm does not need to know exactly the popular vector array, therefore, the requirements of the array calibration is not very strict.

Implementation Process of the Improved ESPRIT Algorithm Based DOA Estimation

The basic idea of the improved ESPRIT algorithm is to use a norm square as the smallest perturbation ΔU_{s2} to interfere with the signal subspace U_{s2}, the purpose of which is to correct the noise in the U_{s2}. While the total least squares ESPRIT (TLS-ESPRIT) idea is to interfere both U_{s1} and U_{s2}, so the norm of the perturbation can be minimized, and the noise of U_{s1} and U_{s2} can be corrected simultaneously. The improved ESPRIT algorithm is an improved ESPRIT algorithm, which transforms a larger dimension morbid generalized characteristic value problem into a smaller dimension eigenvalue problems without sick generalized by singular value decomposition and the total least squares. The steps of TLS-ESPRIT algorithm to solve the DOA algorithm are as follows:

Step 1: the data covariance matrix is obtained by the data of two sub-arrays.

Step two: the signal subspace U_s of the two data matrix is obtained by the $\{R, R_N\}$ characteristics decomposition of the matrix.

Step three: U_{s12} is structured by the matrix U_s, and according to the type of $U_{s12}^H U_{s12} = E \sum E^H$ eigenvalue decomposition of the matrix E, which will be divided into 4 sub matrix according to the formula $E = \begin{pmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{pmatrix}$.

Step 4: matrix Ψ_{TLS} and the feature value decomposition is obtained by formula $\Psi_{TLS} = -E_{21}E_{22}^{-1}$, N eigenvalues can be obtained for N signal direction of arrival.

Simulation Experiment and Performance Analysis for Multi-target Tracking

Effect of SNR on the accuracy of DOA

Simulation array is uniform linear array, with 8 array element and 0.5 array element spacing. Set the 3 signal source, with DOA 10 degrees, 20 degrees and 30 degrees, while the sampling number is 500. The simulation results for the DOA estimation value under different SNR are shown in Figure 1, while the effect of the SNR to the estimation variance is shown in Figure 2.
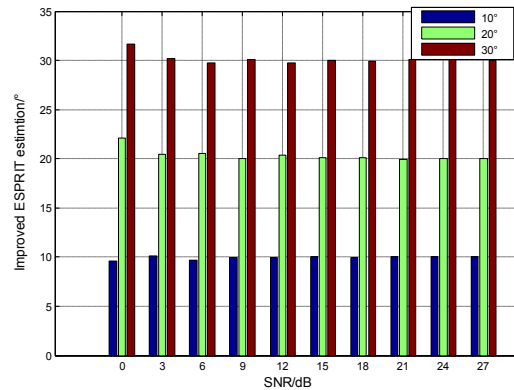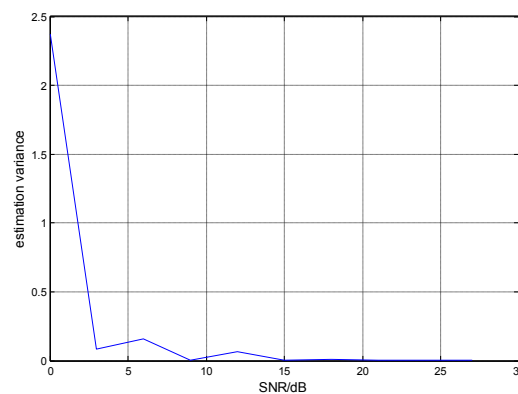

Fig. 1 Effect of SNR on the estimated value

Fig. 2 Effect of SNR on estimation variance

Figure 1 and Figure 2 show that SNR takes a certain impact on DOA estimation. When the SNR is greater than 10dB, the estimation error is small. However, the estimation error is large when the SNR is low.

Effect of Sub-array Spacing on the Accuracy of DOA

Simulation array is uniform linear array, with 8 array element and SNR 10dB. Set the 3 signal source, with DOA 10 degrees, 20 degrees and 30 degrees, while the sampling number is 500. The simulation results for the DOA estimation value under different sub-array spacing are shown in Figure 3, while the effect of the sub-array spacing to the estimation variance is shown in Figure 4.
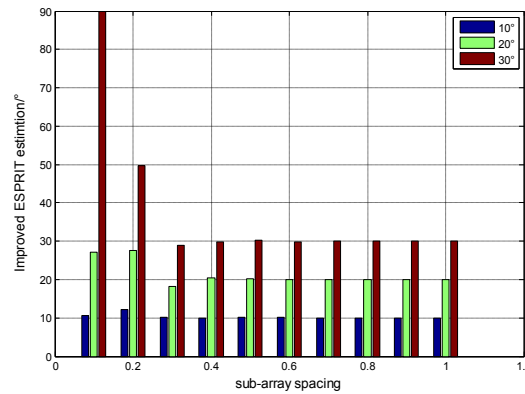
Fig. 3 Effect of sub-array spacing on the estimated value
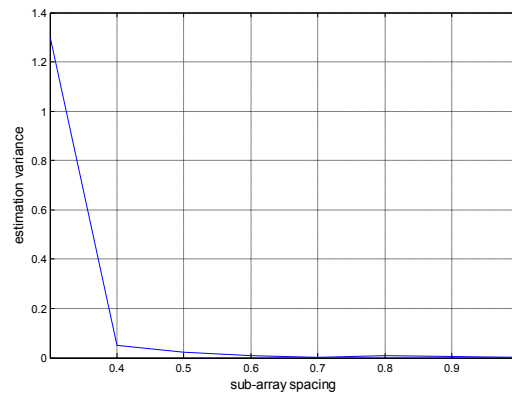
Fig. 4 Effect of sub-array spacing on estimation variance

Figure 3 and Figure 4 show that sub-array spacing takes a large impact on DOA estimation. When the sub-array spacing is less than 0.4, the estimation performance declines sharply.

Compared with MUSIC Algorithm

Simulation array is uniform linear array, with 8 array element and 0.5 array element spacing. Set the 3 signal source, with DOA 10 degrees, 20 degrees and 30 degrees, while the snapshot number is 1000 and the search range is [-90°,90°]. Simulations for the MUSIC algorithm and the improved ESPRIT algorithm is taken respectively. The simulation results for the RMSE under different SNR are shown in Figure 5.
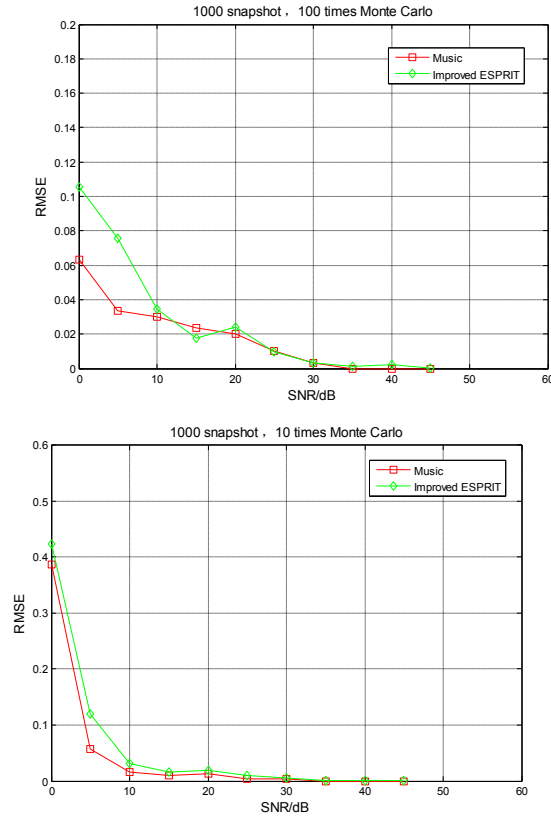
Fig. 5 Comparison of MUSIC algorithm and improved ESPRIT algorithm for estimating RMSE

In Figure 5, the number of Monte Carlo is 100 times on left chart and 10 times on right chart. It can be seen that with the increase of the number of Monte Carlo, the RMSE of MUSIC algorithm and the improved ESPRIT algorithm are significantly reduced. Under low SNR, the performance of the MUSIC algorithm is slightly better than that of the improved ESPRIT algorithm. Correspondingly, the time consuming is shown in Figure 6.

Fig. 6 Time consuming comparison between MUSIC algorithm and improved ESPRIT algorithm

Figure 6 shows that the improved ESPRIT algorithm is more than 1000 times faster than the MUSIC algorithm. With the increasing number of Monte Carlo, the efficiency of MUSIC algorithm is greatly reduced, while the time consuming of the improved ESPRIT algorithm is very little.

In general, the performance of the improved ESPRIT algorithm is less than MUSIC algorithm, but the improved ESPRIT algorithm has the advantage of real-time. As long as the two sub-array to meet the rotation invariance, the improved ESPRIT algorithm can be used, with the speed of which better than the MUSIC algorithm.

Conclusion

This paper firstly analyzes the basic principle of the Improved ESPRIT algorithm, then gives the DOA estimation implementation process and simulation and analysis of the effect on the performance of DOA estimation both for SNR and sub-array spacing. On basis of that, the improved ESPRIT algorithm is compared with the MUSIC algorithm. The simulation results show that the low SNR and small sub-array spacing may lead to large error; the estimation error of the improved ESPRIT algorithm is slightly larger than that of the MUSIC algorithm, but the running time is far less than that of the music

algorithm, with the increase of the operations number, the time consuming of the improved ESPRIT algorithm is not obviously increased with good resolution.

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