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FDTD Method for Obliquely Incident Electromagnetic Wave in Time-Varied Plasma

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***Abstract*—One-dimensional FDTD (Finite Different Time Domain) method for electromagnetic (EM) wave in obliquely incident time-varied plasma is introduced in this paper. By eliminating a field value in two-dimensional Maxwell equation, it can be converted to one-dimensional Maxwell equation with time-varied value. In this way, the calculating efficiency can be enhanced and the computing source can be saved. Based on piecewise linear current density recursive convolution (PLJERC) FDTD method, the method for EM wave in obliquely incident time-varied plasma is derived and used to calculate transmission response. The calculated results are compared to measurement results to verify the method, and the influence of EM wave in plasma under different incident angle is analyzed.**

Keywords: Obliquely Incident, Time-varied, FDTD Method.

I. INTRODUCTION

Currently, obliquely incident EM wave in plasma and EM wave in time-varied plasma are discussed separately in most literature. [1] And [2] have used FDTD method to analysis EM wave in time-varied plasma without consider obliquely incident EM wave. [3] Has analyzed the reflection, absorption and transmission of obliquely incident EM wave in time-invariant plasma. [4] And [5] have utilized two dimensional FDTD method to analysis infinite layered time-invariant plasma, however, one-dimensional FDTD method is enough to analysis in this condition. [6] Has derived one-dimensional FDTD method for obliquely incident EM wave in plasma, however, the time-varied plasma problem is not considered. In this paper, the obliquely incident EM wave in plasma and EM wave in time-varied plasma are considered together using one-dimensional FDTD method, it can enhance the calculating efficiency and computing source is saved.

Based on PLJERC FDTD method in [7], one-dimensional FDTD equation for obliquely incident EM wave in time-varied plasma is derived, and anisotropic perfectly matched layer (UMPL) is modified. The transmission response of EM wave in time-varied plasma is calculated and compared with measurement results, in addition, the influence of EM wave in plasma under different incident angle is analyzed.

II. ONE-DIMENSIONAL METHOD FOR OBLIQUELY INCIDENT EM WAVE IN TIME-VARIED PLASMA

The diagram of obliquely incident EM wave in plasma is shown in Fig. 1, medium 1 and medium n+1 in the figure is vacuum, and medium 2~n are layered plasma. The incident angle of EM wave is θ, and k is wave number.

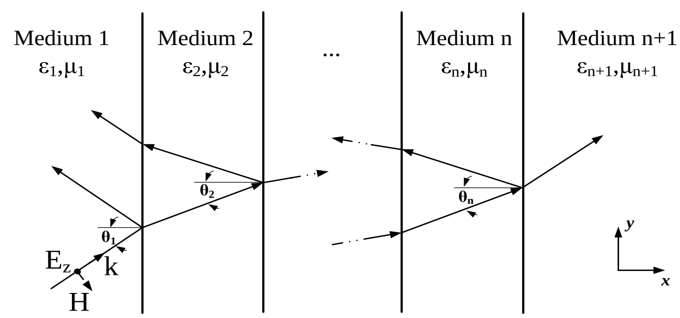


Figure 1. Diagram of obliquely incident TE wave in plasma.

Two-dimensional Maxwell equation for unmagnified TE wave in plasma is

$$\left.\begin{aligned} \frac{\partial E_z}{\partial y} &= -\mu_0 \frac{\partial H_x}{\partial t} \\ \frac{\partial E_z}{\partial x} &= \mu_0 \frac{\partial H_y}{\partial t} \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} &= \varepsilon_0 \frac{\partial E_z}{\partial t} + J_z \\ \frac{\partial J_z}{\partial t} + \nu J_z &= \varepsilon_0 \omega_p^2 E_z \end{aligned}\right\}$$

J_z Is current density. Take Fourier transform for 3rd and 4th equations in Eq. 1

$$\frac{\partial \boldsymbol{H}_y}{\partial x}-\frac{\partial \boldsymbol{H}_x}{\partial y}=j\omega\varepsilon_0\boldsymbol{E}_z+\boldsymbol{J}_z \quad (2)$$

$$\boldsymbol{J}_z=\varepsilon_0\frac{\omega_p^2}{j\omega+\nu}\boldsymbol{E}_z \quad (3)$$

Take Eq. 3 into Eq. 2

$$\frac{\partial \boldsymbol{H}_y}{\partial x}-\frac{\partial \boldsymbol{H}_x}{\partial y}=j\omega\varepsilon_0\left(1+\frac{\sigma_n(\omega)}{j\omega\varepsilon_0}\right)\boldsymbol{E}_z=j\omega\varepsilon_n\boldsymbol{E}_z \quad (4)$$

ε_n Is relative permittivity of the nth layer plasma, $\sigma(\omega)=\varepsilon_0\frac{\omega_p^2}{j\omega+\nu}$. Take differential and Fourier transform for 1st equation in Eq. 1,

$$\frac{\partial^2\boldsymbol{E}_z}{\partial y^2}=-k_{ny}^2\boldsymbol{E}_z=-j\omega\mu_0\frac{\partial \boldsymbol{H}_x}{\partial y} \quad (5)$$

From phase match in layered medium theory, the tangential value of wave vector in different layer is equal, so $k_{ny}=k_{1y}=k_1\sin\theta$, take Eq. 4 into Eq. 5 and introduce incident angle,

$$\frac{\partial \boldsymbol{H}_y}{\partial x}=j\omega\varepsilon_n\boldsymbol{E}_z-j\omega\varepsilon_0\varepsilon_1\sin^2\theta\boldsymbol{E}_z \quad (6)$$

Take Fourier transform for Eq. 6

$$\left.\begin{array}{l}\frac{\partial E_z}{\partial x}=\mu_0\frac{\partial H_y}{\partial t}\\ \frac{\partial H_y}{\partial x}=(cf\cdot\varepsilon_0)\frac{\partial E_z}{\partial t}+J_z\\ \frac{\partial J_z}{\partial t}+\nu J_z=\varepsilon_0\omega_p^2E_z\end{array}\right\} \quad (7)$$

In Eq. 7, $cf=1-\varepsilon_1\sin^2\theta$, and introduce time-varied value $\omega_p(t)$ into 3rd equation in Eq. 7,

$$\frac{\partial J_z}{\partial t}+\nu J_z=\varepsilon_0\omega_p^2(t)E_z \quad (8)$$

$$\ddot{E}(t)=\omega_p^2(t)E_z(t)$$

By taking Eq. 8 into Eq. 7, the one-dimensional equation for EM wave in time-varied plasma can be shown

$$\left.\begin{array}{l}\frac{\partial E_z}{\partial x}=\mu_0\frac{\partial H_y}{\partial t}\\ \frac{\partial H_y}{\partial x}=cf\cdot\varepsilon_0\frac{\partial E_z}{\partial t}+J_z\\ \frac{\partial J_z}{\partial t}+\nu J_z=\varepsilon_0\ddot{E}_z\end{array}\right\}$$

Take Fourier transform for 3rd equation in Eq. 10, and then take inverse Fourier transform under the assumption of $\ddot{E}(t)$ is linear

$$J_z^{n+1}+J_z^n=(\sigma^0-\xi^0)\ddot{E}_z^{n+1}+\xi^0\ddot{E}_z^n+\sum_{m=0}^{n-1}\Big[\ddot{E}_z^{n-m}(\sigma^m+\sigma^{m+1})+(\ddot{E}_z^{n-m-1}-\ddot{E}_z^{n-m})(\xi^m+\xi^{m+1})\Big]$$

In Eq. 11

$$\sigma^m=\int_{m\Delta t}^{(m+1)\Delta t}\sigma(\tau)d\tau=\frac{\varepsilon_0}{\nu}\big[1-\exp(-\nu\Delta t)\big]\exp(-m\nu\Delta t)$$

$$\xi^m=\int_{m\Delta t}^{(m+1)\Delta t}(\tau-m\Delta t)\sigma(\tau)d\tau=\frac{\varepsilon_0}{\nu^2\Delta t}\big[1-(1+\nu\Delta t)\exp(-\nu\Delta t)\big]\exp(-m\nu\Delta t)$$

$$\psi_z^n=\sum_{m=0}^{n-1}\Big[E_z^{n-m}(\sigma^m+\sigma^{m+1})+(E_z^{n-m-1}-E_z^{n-m})(\xi^m+\xi^{m+1})\Big]$$

So Eq. 11 can be written as

$$J_z^{n+1}+J_z^n=(\sigma^0-\xi^0)\ddot{E}_z^{n+1}+\xi^0\ddot{E}_z^n+\psi_z^n$$

From Eq. 12, Eq. 13 and Eq. 14

$$\psi_z^n=(\sigma^0+\sigma^1-\xi^0-\xi^1)\ddot{E}_z^n+(\xi^0+\xi^1)\ddot{E}_z^{n-1}+\exp(-\nu\Delta t)\psi_z^{n-1}$$

$$\psi_z^n(i)=(\sigma^0+\sigma^1-\xi^0-\xi^1)(\omega_p^n)^2E_z^n(i)+(\xi^0+\xi^1)(\omega_p^{n-1})^2E_z^{n-1}(i)+\exp(-\nu\Delta t)\psi_z^{n-1}(i)$$

Take difference discrete for 2nd equation in Eq. 10

$$(\nabla\times H)_z^{n+1/2}=\frac{H_y^{n+1/2}\left(i+\frac{1}{2}\right)-H_y^{n+1/2}\left(i-\frac{1}{2}\right)}{\Delta x}=cf\cdot\varepsilon_0\frac{E_x^{n+1}\left(i+\frac{1}{2},j,k\right)-E_x^{n}\left(i+\frac{1}{2},j,k\right)}{\Delta t}+\frac{J_x^{n+1}\left(i+\frac{1}{2}\right)+J_x^{n}\left(i+\frac{1}{2}\right)}{2} \tag{18}$$

Take Eq. 9 and Eq. 15 into Eq. 18, the electric field intensity can be written as

$$E_z^{n+1}(i)=\frac{1}{1+\frac{\Delta t}{2cf\cdot\varepsilon_0}\left(\sigma^0-\xi^0\right)\cdot\left(\omega_p^{n+1}\right)^2}\left\{\left(1-\frac{\Delta t}{2cf\cdot\varepsilon_0}\xi^0\cdot\left(\omega_p^{n}\right)^2\right)E_z^n(i)+\frac{\Delta t}{cf\cdot\varepsilon_0}(\nabla\times H)_z^{n+1/2}-\frac{\Delta t}{2cf\cdot\varepsilon_0}\psi_z^n(i)\right\} \tag{19}$$

The magnetic field intensity can be derived similarly.

III. Numerical Analysis and Validation

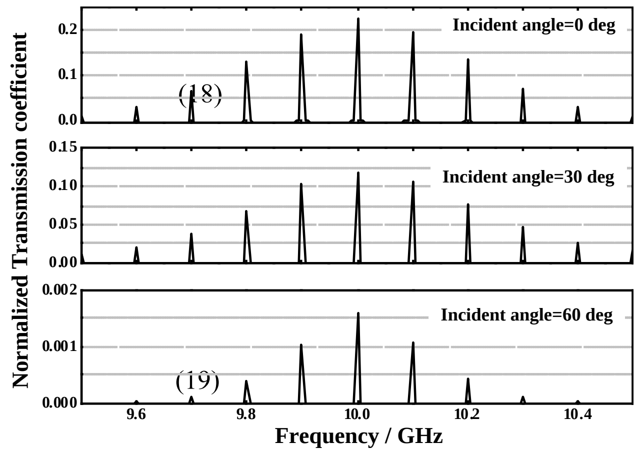
Example: Plasma thickness d=5cm, collision frequency v=1GHz, electron density $Ne=Ne_{Refer}\cdot\left[1+A\cdot\sin(2\pi f_D\cdot t)\right]$, and $Ne_{Refer}=1e18\,m^{-3}$, $A=0.15$, $f_D=100MHz$. The frequency of incident EM wave is 10GHz, and incident angle is 0° , 30° and 60° . The calculated results are shown in Fig. 2, and Fig. 2(a) is normalized transmission coefficient and Fig. 2(b) is power transmission coefficient.

It can be seen from Fig. 2 that, when electron density of plasma is periodic varied, there are some other frequency in spectrum in addition to the original frequency, and the spectrum of transmission wave can be written as

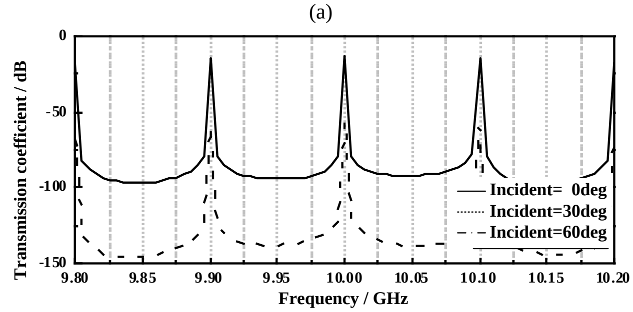
$$\mathcal{F}(ft)=A_0\mathcal{F}(fr)+\mathcal{F}(fr)*\left[A_1\cdot\mathcal{F}(f_D)+A_2\cdot\mathcal{F}(2f_D)+A_3\cdot\mathcal{F}(3f_D)+\dots\right] \tag{20}$$

$\mathcal{F}(ft)$ is spectrum of transmission wave, $\mathcal{F}(fr)$ is spectrum of incident wave, $\mathcal{F}(n\cdot f_D)$ is spectrum of sinusoidal wave with $n\cdot f_D$ frequency, and the coefficient A_0、A_1、A_2、A_3… are smaller than 1 and decrease progressively. The spectrum of transmission wave is similar as the sum of several modulation of incident wave.

We can also conclude from Fig. 2(a) that when the incident angle is large, the transmission distance of EM wave is plasma is large, the attenuation is large and the amplitude of transmission wave is small. It can be seen from Fig. 2(b) that the sub-frequencies in transmission wave are independent to the incident angle, the sub-frequencies is only related to electron density and the gap between sub-frequencies is equal to variation frequency of electron density.



(a)



(b)

Figure 2. Transmission spectrum of EM in plasma under different incident angle.

IV. Conclusion

Based on PLJERC FDTD method, one-dimensional FDTD equation for obliquely incident EM wave in time-varied plasma is derived. This method is used to calculate the transmission response of EM wave in plasma with variation of electron density, and good comparison is achieved between numerical results and measurement results.

The influence of EM wave in plasma under different incident angle is analyzed using this method, and it can be concluded that for slow variation plasma, the frequency shift and frequency broaden of transmission wave is large under large incident angle. For periodic variation plasma, sub-frequencies in transmission wave are independent to the incident angle, the sub-frequencies is only related to electron density and the gap between sub-frequencies is equal to variation frequency of electron density.

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