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To Solve the Problem of the Path Dynamic Interpolation and Jitter in the Simulation of Aircraft Flight Installed Beidou

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***Abstract*—After estimated the state data of the current and future a certain number of times by the moment and a few moments before position data transferred from the aircraft equipped with Beidou navigation, to ensure that the display effect in flight simulation is needed to solve the problem of smooth interpolation and data jitter. Proposed piecewise three smoothing interpolation- ensure the first order, two order derivative of the connecting point between two segments of the front and back was exist, at the same time, the sum of three derivative difference's square was minimum. By used by this method, the flight data reached the requirements of light as far as possible. And the derivation process was given. Flight simulation experiments showed that the aircraft is stable, and no major jitter, meet the visual requirements.**

***Keywords*-Dynamic Interpolation; Cubic Interpolation; Smooth Curver**

I. Introduction

In order to realize the machine navigation and ground monitoring, so, some aircrafts are equipped with Beidou navigation equipment. In the operation process, ground equipment receives Beidou aircraft position information data, tries to restore the aircraft's aerial flight state through these data of points in the ground monitors, just as real-time observation of aircraft in the air with the eyes, which requires the received data to the processing. By some discrete points recovers real flight routes, need to interpolate the data and because the data is a group of a group received, there between a group and the other group data convergence problem, at present the research results are not satisfactory.

In this paper, the piecewise three spline curve is used, because between the groups third derivative of the cut-off point is jumping, it is necessary to find ways to reduce the jump, the boundary point is as far as possible smooth between the group and the other group. Therefore, it is necessary to add some boundary conditions. In this paper, a set of curves and the next set of curves at the boundary point‘s the first derivative is equal, their two - order derivative is equal, and their three-order derivative square sum $\sum_{i=2}^{n-1}\left(p_{i+0}^{(3)}-p_{i-0}^{(3)}\right)^2$ is minimal. This perturbation kinetic energy $\int\left\|s^{(3)}(t)\right\|dt$ also is minimal.

II. The Specific Algorithm

Firstly, the first 4 points (1th point, 2th point, 3th point, 4th point) are interpolated by Lagrange, and the first curve is obtained:

$$p_3(t)=\frac{(t-t_4)(t-t_2)(t-t_3)}{(t_1-t_4)(t_1-t_2)(t_1-t_3)}y_1+\frac{(t-t_1)(t-t_4)(t-t_3)}{(t_2-t_1)(t_2-t_4)(t_2-t_3)}y_2+\frac{(t-t_1)(t-t_2)(t-t_4)}{(t_3-t_1)(t_3-t_2)(t_3-t_4)}y_3+\frac{(t-t_1)(t-t_2)(t-t_3)}{(t_4-t_1)(t_4-t_2)(t_4-t_3)}y_4 \quad (1)$$

The next interpolation with parameters a_4 and b_4 used the followed 4th and 5th point, such as formula (2), the value of the parameters a_4 and b_4 can be obtained through the equation (3) (4) (5), and then come to the second section of the curve.

$$p_4(t)=\frac{t-t_4}{t_5-t_4}y_5+\frac{t-t_5}{t_4-t_5}y_4+\left(a_4\frac{t-t_4}{t_5-t_4}+b_4\right)\frac{(t-t_4)(t-t_5)}{(t_5-t_4)(t_4-t_5)},$$
$$\text{let}\quad p_4(t_4)=p_3(t_4) \quad (2)$$

If the curve is smooth, it is necessary to make the first derivative of the connecting point of the first and the second sections to be equal at the 4th point:

$$p_4'(t_4) = \frac{y_5 - y_4}{t_5 - t_4} + b_4 \frac{1}{t_5 - t_4}$$, let $p_4'(t_4) = p_3'(t_4)$ (3)

Solution (3): $b_4 = (t_5 - t_4)\, p_3'(t_4) - y_5 + y_4$

In order to make the two connection curve smooth, the two - order derivative of the curve is linear at the 4th point:

$$p_4''(t_4) = 2\left(\frac{a_4}{(t_5 - t_4)^2} + \frac{b_4}{(t_5 - t_4)(t_4 - t_5)}\right)$$, let

$$p_4''(t_4) = 2k_4 p_3''(t_4) \qquad (4)$$

Solution (4): $a_4 = b_4 + (t_5 - t_4)^2 k_4 p_3''(t_4)$

In order to have the minimum rebound, the square of the jump value $L(p_4(t)) = [p_3^{(3)}(t_4) - p_4^{(3)}(t_4)]^2$ is minimal.

At the 4th point $p_4(t)$'s the three order derivative is $p_4^{(3)}(t_4) = -\frac{6a_4}{(t_5 - t_4)^3}$.

Let $\frac{\partial L}{\partial k_4} = 0$, Launch:

$$p_3^{(3)}(t_4) = -\frac{6(b_4 + (t_5 - t_4)^2 k_4 p_3''(t_4))}{(t_5 - t_4)^3} \qquad (5)$$

So $(p_3^{(3)}(t_4)(t_5 - t_4)^3 + 6b_4) / (6p_3''(t_4)(t_5 - t_4)^2) = -k_4$, then we get $k_4 = -\frac{[p_3^{(3)}(t_4)(t_5 - t_4)^3 + 6b_4]}{6p_3^{(2)}(t_4)(t_5 - t_4)^2}$, as long as $p_3(t)$'s two and three order derivatives can be obtained at the 4th point t_4 .

Seeking derivative through (1), we get the result as follow:

$$p_3'(t_4) = \frac{(t_4 - t_2)(t_4 - t_3)}{(t_1 - t_4)(t_1 - t_2)(t_1 - t_3)} y_1 + \frac{(t_4 - t_1)(t_4 - t_3)}{(t_2 - t_1)(t_2 - t_4)(t_2 - t_3)} y_2 + \frac{(t_4 - t_1)(t_4 - t_2)}{(t_3 - t_1)(t_3 - t_2)(t_3 - t_4)} y_3 + \frac{1}{(t_4 - t_1)} y_4 + \frac{1}{(t_4 - t_2)} y_4 + \frac{1}{(t_4 - t_3)} y_4$$

$$p_3''(t) = \frac{(t - t_3)}{(t_1 - t_4)(t_1 - t_2)(t_1 - t_3)} y_1 + \frac{(t - t_2)}{(t_1 - t_4)(t_1 - t_2)(t_1 - t_3)} y_1$$

$$+ \frac{(t - t_4)}{(t_3 - t_1)(t_3 - t_2)(t_3 - t_4)} y_3 + \frac{(t - t_2)}{(t_3 - t_1)(t_3 - t_2)(t_3 - t_4)} y_3$$

$$+ \frac{(t - t_4)}{(t_3 - t_1)(t_3 - t_2)(t_3 - t_4)} y_3 + \frac{(t - t_1)}{(t_3 - t_1)(t_3 - t_2)(t_3 - t_4)} y_3$$

$$+ \frac{(t - t_2)}{(t_3 - t_1)(t_3 - t_2)(t_3 - t_4)} y_3 + \frac{(t - t_1)}{(t_3 - t_1)(t_3 - t_2)(t_3 - t_4)} y_3$$

$$+ \frac{(t - t_3)}{(t_4 - t_1)(t_4 - t_2)(t_4 - t_3)} y_4 + \frac{(t - t_2)}{(t_4 - t_1)(t_4 - t_2)(t_4 - t_3)} y_4$$

$$+ \frac{(t - t_3)}{(t_4 - t_1)(t_4 - t_2)(t_4 - t_3)} y_4 + \frac{(t - t_1)}{(t_4 - t_1)(t_4 - t_2)(t_4 - t_3)} y_4$$

$$+ \frac{(t - t_2)}{(t_4 - t_1)(t_4 - t_2)(t_4 - t_3)} y_4 + \frac{(t - t_1)}{(t_4 - t_1)(t_4 - t_2)(t_4 - t_3)} y_4$$

$$p_3''(t_4) = 2\frac{(t_4 - t_3)}{(t_1 - t_4)(t_1 - t_2)(t_1 - t_3)} y_1 + 2\frac{(t_4 - t_2)}{(t_1 - t_4)(t_1 - t_2)(t_1 - t_3)} y_1$$

$$+2\frac{(t_4 - t_3)}{(t_2 - t_1)(t_2 - t_4)(t_2 - t_3)} y_2 + 2\frac{(t_4 - t_1)}{(t_2 - t_1)(t_2 - t_4)(t_2 - t_3)} y_2$$

$$+2\frac{(t_4 - t_2)}{(t_3 - t_1)(t_3 - t_2)(t_3 - t_4)} y_3 + 2\frac{(t_4 - t_1)}{(t_3 - t_1)(t_3 - t_2)(t_3 - t_4)} y_3$$

$$+2\frac{1}{(t_4 - t_1)(t_4 - t_2)} y_4 + 2\frac{1}{(t_4 - t_1)(t_4 - t_3)} y_4 + 2\frac{1}{(t_4 - t_2)(t_4 - t_3)} y_4$$

$$p_3^{(3)}(t_4) =$$

$$\frac{6}{(t_1 - t_4)(t_1 - t_2)(t_1 - t_3)} y_1 + \frac{6}{(t_2 - t_1)(t_2 - t_4)(t_2 - t_3)} y_2$$

$$+ \frac{6}{(t_3 - t_1)(t_3 - t_2)(t_3 - t_4)} y_3 + \frac{6}{(t_4 - t_1)(t_4 - t_2)(t_4 - t_3)} y_4$$

This has been the second curve, that is, fourth point and fifth point are connected. Next, the curve between fifth and sixth point is established by using the same method as the above which conclude the parameters a_5 *and* b_5 , the curve is like (6).

$$p_5(t) = \frac{t - t_5}{t_6 - t_5} y_6 + \frac{t - t_6}{t_5 - t_6} y_5 + \left(a_5 \frac{t - t_5}{t_6 - t_5} + b_5\right) \frac{(t - t_5)(t - t_6)}{(t_6 - t_5)(t_5 - t_6)} \qquad (6)$$

When t=t5, $p_5'(t_5) = \frac{y_6 - y_5}{t_6 - t_5} + b_5 \frac{1}{t_6 - t_5}$ Let: $p_5'(t_5) = p_4'(t_5)$

Get: $b_5 = (t_6 - t_5)\, p_4'(t_5) - y_6 + y_5$

$$p_5''(t_5) = 2\left(\frac{a_5}{(t_6 - t_5)^2} + \frac{b_5}{(t_6 - t_5)(t_5 - t_6)}\right)$$,

Let: $p_5''(t_5) = 2k_5 p_4''(t_5)$,

Get: $a_5 = b_5 + (t_6 - t_5)^2 k_5 p_4''(t_5)$.

Because $p_5^{(3)}(t_5) = -\dfrac{6a_5}{(t_6 - t_5)^3}$.

when $L(p_5(t)) = [p_4^{(3)}(t_5) - p_5^{(3)}(t_5)]^2$, Let: $\dfrac{\partial L}{\partial k_5} = 0$,

That is

$$2\left(p_4^{(3)}(t_5) + \frac{6\left(b_5 + (t_6 - t_5)^2 k_5 p_4''(t_5)\right)}{(t_4 - t_5)^3}\right)\frac{6p_4''(t_5)}{t_6 - t_5} = 0$$

Conclude $p_4^{(3)}(t_5) = -\dfrac{6\left(b_5 + (t_6 - t_5)^2 k_5 p_4''(t_5)\right)}{(t_6 - t_5)^3}$

$$(p_4^{(3)}(t_5)(t_6 - t_5)^3 + 6b_5)/(6p_4''(t_5)(t_6 - t_5)^2) = -k_5$$

Then, $k_5 = -\dfrac{[p_4^{(3)}(t_5)(t_6 - t_5)^3 + 6b_5]}{6p_4''(t_5)(t_6 - t_5)^2}$

$$p_4(t) = \frac{t - t_4}{t_5 - t_4} y_5 + \frac{t - t_5}{t_4 - t_5} y_4 + \left(a_4 \frac{t - t_4}{t_5 - t_4} + b_4\right)\frac{(t - t_4)(t - t_5)}{(t_5 - t_4)(t_4 - t_5)}$$

$$p_4'(t) = \frac{1}{t_5 - t_4} y_5 + \frac{1}{t_4 - t_5} y_4 + \left(a_4 \frac{1}{t_5 - t_4}\right)\frac{(t - t_4)(t - t_5)}{(t_5 - t_4)(t_4 - t_5)}$$
$$+\left(a_4 \frac{t - t_4}{t_5 - t_4} + b_4\right)\frac{(t - t_5)}{(t_5 - t_4)(t_4 - t_5)} + \left(a_4 \frac{t - t_4}{t_5 - t_4} + b_4\right)\frac{(t - t_4)}{(t_5 - t_4)(t_4 - t_5)}$$

$$p_4'(t_5) = \frac{1}{t_5 - t_4} y_5 + \frac{1}{t_4 - t_5} y_4 + (a_4 + b_4)\frac{1}{(t_4 - t_5)}$$

$$p''_4(t) = \left(a_4 \frac{1}{t_5 - t_4}\right)\frac{(t - t_5)}{(t_5 - t_4)(t_4 - t_5)} + \left(a_4 \frac{1}{t_5 - t_4}\right)\frac{(t - t_4)}{(t_5 - t_4)(t_4 - t_5)}$$
$$+\left(a_4 \frac{1}{t_5 - t_4}\right)\frac{(t - t_5)}{(t_5 - t_4)(t_4 - t_5)} + \left(a_4 \frac{t - t_4}{t_5 - t_4} + b_4\right)\frac{1}{(t_5 - t_4)(t_4 - t_5)}$$
$$+\left(a_4 \frac{1}{t_5 - t_4}\right)\frac{(t - t_4)}{(t_5 - t_4)(t_4 - t_5)} + \left(a_4 \frac{t - t_4}{t_5 - t_4} + b_4\right)\frac{1}{(t_5 - t_4)(t_4 - t_5)}$$

$$p''_4(t_5) = \left(a_4 \frac{1}{t_5 - t_4}\right)\frac{1}{(t_4 - t_5)} + (a_4 + b_4)\frac{1}{(t_5 - t_4)(t_4 - t_5)}$$
$$+\left(a_4 \frac{1}{t_5 - t_4}\right)\frac{1}{(t_4 - t_5)} + (a_4 + b_4)\frac{1}{(t_5 - t_4)(t_4 - t_5)}$$
$$= \frac{-4a_4}{(t_4 - t_5)^2} + \frac{-2b_4}{(t_5 - t_4)^2}$$

$$p_4^{(3)}(t_5) = \left(a_4 \frac{1}{t_5 - t_4}\right)\frac{1}{(t_5 - t_4)(t_4 - t_5)} + \left(a_4 \frac{1}{t_5 - t_4}\right)\frac{1}{(t_5 - t_4)(t_4 - t_5)}$$
$$+\left(a_4 \frac{1}{t_5 - t_4}\right)\frac{1}{(t_5 - t_4)(t_4 - t_5)} + \left(a_4 \frac{1}{t_5 - t_4}\right)\frac{1}{(t_5 - t_4)(t_4 - t_5)}$$
$$+\left(a_4 \frac{1}{t_5 - t_4}\right)\frac{1}{(t_5 - t_4)(t_4 - t_5)} + \left(a_4 \frac{1}{t_5 - t_4}\right)\frac{1}{(t_5 - t_4)(t_4 - t_5)}$$
$$= \frac{-6a_4}{(t_5 - t_4)^3}$$

The curve of the parameters a_i and b_i is established between the first i point and the second point i+1:

$$p_i(t) = \frac{t - t_i}{t_{i+1} - t_i} y_{i+1} + \frac{t - t_{i+1}}{t_i - t_{i+1}} y_i + \left(a_i \frac{t - t_i}{t_{i+1} - t_i} + b_i\right)\frac{(t - t_i)(t - t_{i+1})}{(t_{i+1} - t_i)(t_i - t_{i+1})}$$

$$p_i'(t_i) = \frac{y_{i+1} - y_i}{t_{i+1} - t_i} + b_i \frac{1}{t_{i+1} - t_i} = p_{i-1}'(t_i)$$

$$b_i = (t_{i+1} - t_i)\, p_{i-1}'(t_i) - y_{i+1} + y_i \qquad (7)$$

$$p_i''(t_i) = 2\left(\frac{a_i}{(t_{i+1} - t_i)^2} + \frac{b_i}{(t_{i+1} - t_i)(t_i - t_{i+1})}\right) = 2k_i p_{i-1}''(t_i)$$

$$a_i = b_i + (t_{i+1} - t_i)^2 k_i p_{i-1}''(t_i) \qquad (8)$$

$$L(p_i(t)) = [p_{i-1}^{(3)}(t_i) - p_i^{(3)}(t_i)]^2$$

$$p_i^{(3)}(t_i) = -\frac{6a_i}{(t_{i+1} - t_i)^3}$$

Let: $\dfrac{\partial L}{\partial k_i} = 0$, that is:

$$2\left(p_{i-1}^{(3)}(t_i) + \frac{6\left(b_i + (t_{i+1} - t_i)^2 k_i p_{i-1}''(t_i)\right)}{(t_{i-1} - t_i)^3}\right)\frac{6p_{i-1}''(t_i)}{t_{i+1} - t_i} = 0$$

Push off: $p_i^{(3)}(t_i) = -\dfrac{6\left(b_i + (t_{i+1} - t_i)^2 k_i p_{i-1}''(t_i)\right)}{(t_{i+1} - t_i)^3}$

$$(p_{i-1}^{(3)}(t_i)(t_{i+1} - t_i)^3 + 6b_i)/(6p_{i-1}''(t_i)(t_{i+1} - t_i)^2) = -k_i$$

$$k_i = -\frac{[p_{i-1}^{(3)}(t_i)(t_{i+1} - t_i)^3 + 6b_i]}{6p_{i-1}''(t_i)(t_{i+1} - t_i)^2} \qquad (9)$$

$$p_{i-1}(t) = \frac{t - t_{i-1}}{t_i - t_{i-1}} y_i + \frac{t - t_i}{t_{i-1} - t_i} y_{i-1} + \left(a_{i-1} \frac{t - t_4}{t_i - t_4} + b_{i-1}\right)\frac{(t - t_{i-1})(t - t_i)}{(t_i - t_{i-1})(t_4 - t_i)}$$

$$p_{i-1}'(t) = \frac{1}{t_i - t_{i-1}} y_i + \frac{1}{t_{i-1} - t_i} y_{i-1} + \left(a_{i-1} \frac{1}{t_i - t_{i-1}}\right)\frac{(t - t_{i-1})(t - t_i)}{(t_i - t_{i-1})(t_{i-1} - t_i)}$$
$$+\left(a_{i-1} \frac{t - t_{i-1}}{t_i - t_{i-1}} + b_{i-1}\right)\frac{(t - t_i)}{(t_i - t_{i-1})(t_{i-1} - t_i)} + \left(a_{i-1} \frac{t - t_{i-1}}{t_i - t_{i-1}} + b_{i-1}\right)\frac{(t - t_{i-1})}{(t_i - t_{i-1})(t_{i-1} - t_i)}$$

$$p_{i-1}'(t_i) = \frac{1}{t_i - t_{i-1}} y_i + \frac{1}{t_{i-1} - t_i} y_{i-1} + (a_{i-1} + b_{i-1})\frac{1}{(t_{i-1} - t_i)} \qquad (10)$$

$$
\begin{aligned}
&=\left(a_{i-1}\frac{1}{t_i-t_{i-1}}\right)\frac{(t-t_i)}{(t_i-t_{i-1})(t_{i-1}-t_i)}+\left(a_{i-1}\frac{1}{t_i-t_{i-1}}\right)\frac{(t-t_{i-1})}{(t_i-t_{i-1})(t_{i-1}-t_i)}\\
&+\left(a_{i-1}\frac{1}{t_i-t_{i-1}}\right)\frac{(t-t_i)}{(t_i-t_{i-1})(t_{i-1}-t_i)}+\left(a_{i-1}\frac{t-t_{i-1}}{t_i-t_{i-1}}+b_{i-1}\right)\frac{1}{(t_i-t_{i-1})(t_{i-1}-t_i)}\\
&+\left(a_{i-1}\frac{1}{t_i-t_{i-1}}\right)\frac{(t-t_{i-1})}{(t_i-t_{i-1})(t_{i-1}-t_i)}+\left(a_{i-1}\frac{t-t_{i-1}}{t_i-t_{i-1}}+b_{i-1}\right)\frac{1}{(t_i-t_{i-1})(t_{i-1}-t_i)}
\end{aligned}
$$

$$
\begin{aligned}
p''_{i-1}(t_i)&=\left(a_{i-1}\frac{1}{t_i-t_{i-1}}\right)\frac{1}{(t_{i-1}-t_i)}+(a_{i-1}+b_{i-1})\frac{1}{(t_i-t_{i-1})(t_{i-1}-t_i)}\\
&+\left(a_{i-1}\frac{1}{t_i-t_{i-1}}\right)\frac{1}{(t_{i-1}-t_i)}+(a_{i-1}+b_{i-1})\frac{1}{(t_i-t_{i-1})(t_{i-1}-t_i)}
\end{aligned}
$$

$$
p''_{i-1}(t)=\frac{-4a_{i-1}}{(t_{i-1}-t_i)^2}+\frac{-2b_{i-1}}{(t_i-t_{i-1})^2} \tag{11}
$$

$$
\begin{aligned}
p^{(3)}_{i-1}(t_i)&=\left(a_{i-1}\frac{1}{t_i-t_{i-1}}\right)\frac{1}{(t_i-t_{i-1})(t_{i-1}-t_i)}+\left(a_{i-1}\frac{1}{t_i-t_{i-1}}\right)\frac{1}{(t_i-t_{i-1})(t_{i-1}-t_i)}\\
&+\left(a_{i-1}\frac{1}{t_i-t_{i-1}}\right)\frac{1}{(t_i-t_{i-1})(t_{i-1}-t_i)}+\left(a_{i-1}\frac{1}{t_i-t_{i-1}}\right)\frac{1}{(t_i-t_{i-1})(t_{i-1}-t_i)}\\
&+\left(a_{i-1}\frac{1}{t_i-t_{i-1}}\right)\frac{1}{(t_i-t_{i-1})(t_{i-1}-t_i)}+\left(a_{i-1}\frac{1}{t_i-t_{i-1}}\right)\frac{1}{(t_i-t_{i-1})(t_{i-1}-t_i)}
\end{aligned}
$$

$$
p^{(3)}_{i-1}(t_i)=\frac{-6a_{i-1}}{(t_i-t_{i-1})^3} \tag{12}
$$

The interpolation of this section is obtained by solving the parameter a_i *and* b_i .

III. THE EXPERIMENTAL PROCESS AND RESULTS

Procedures used in the process, as shown in figure 1.

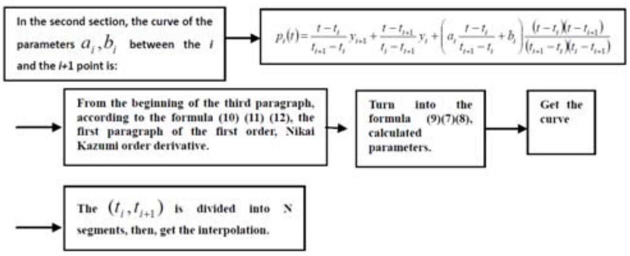

Fig.1 The flue of algorithm in this paper

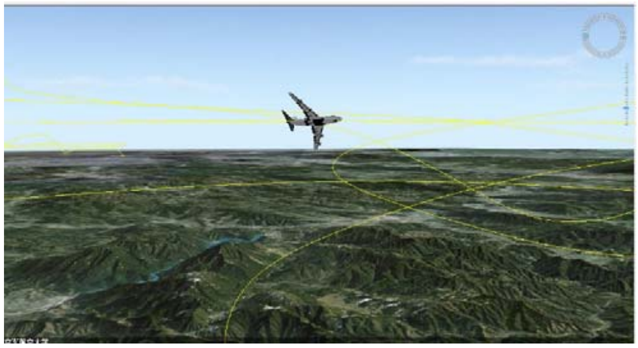

Fig.2.The route gotten by algorithm in this paper

The experiment is implemented using the method in this paper in c# programming language in the c# and skyline interface environment. From the result of the experiment the curve is fairing, very well realize the dynamic interpolation of the motion state of the aircraft, flying is stable in a flight simulation environment without chattering.

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Biography: Zhao Xiuying (1972_06), female, doctor, engaged in the field: digital signal processing.

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