Theoretical Analysis and Numerical Simulation of Concrete D-K Model

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Abstract. According to the characteristics of the concrete fracture process zone, a damage and fracture model for concrete, which combines damage mechanics and fracture mechanics is proposed. This model takes the initial fracture damage threshold as the incipient cracking criterion, and takes the unstable fracture toughness as the unstable fracture criteria of crack propagation. With this model, the whole damage and fracture process from initial damage, macro-crack forming to crack unstably propagating can be analysed. Based on the unified strength theory and damage theory, the boundary equations of microcrack generation zone of concrete mode I crack are discussed respectively. Assuming the critical lengths of microcrack generation zone on its propagation direction are equal with the two methodes above, the expression of initial fracture damage threshold is derived. Using numerical method, the crack growth processes of three-point bending concrete beams with and without original cracks are simulated. The feasibility and validity for numerical solving method with damage and fracture model are verified.

Description of Concrete Damage Fracture Model

Studies of fracture behavior about concrete have shown that there is a fracture process zone in the front of the concrete macro crack. In this region, strain softening phenomenon can be produced. Hillerborg etc. [1]put forward the fictitious crack model (FCM).This model has been widely application and development in the study of fracture mechanics of concrete. The analytical solution of the stable propagation length of concrete crack cannot be obtained by FCM, so it need to combined use of finite element method to solution. Shilang Xu and Reinhardt[2-3] proposed a double-K fracture model of concrete by combined cohesive model of the fracture process zone of FCM with stress intensity factor using the method of equivalent elastic. In this model, the initial fracture toughnessis used as the cracking control parameter and the unstable fracture toughness as the control of critical instability, meanwhile the analytical solution of the double-K fracture parameters is given.

Concrete is a kind of heterogeneous no uniform composite material with initial defects. In fracture mechanics, a macroscopic crack that can be identified in the material is required, so it is difficult to analyze the influence of the development of micro defects on the performance deterioration of concrete before the formation of macroscopic cracks, but damage mechanics can make up for this defect. Typical concrete damage models are Loland model[4], Mazars model [5] and so on.

In order to overcome the limitations of concrete crack research with single mechanics, the damage fracture model (hereinafter referred to as the D-K model) is proposed based on double-K fracture model proposed by Professor Shilang Xu and combined with concrete damage mechanics. This model can be used for analysis the whole process from the initial injury until fracture about the concrete with and without initial cracks. The model of control parameter is derived. By using this model, the crack propagation process of three point bending beam with and without initial crack are simulated and compared.

D-K model divided the whole failure process of concrete from the initial damage to the macro cracks appear, and then crack growth to failure into three stages.

In the first stage, damage leads to the forming of micro crack formation zone. Because there are a large number of initial micro defects, damage can be caused by a small tensile stress. When the tensile stress exceeds the limit of the tensile proportional limit of the concrete, a damage concentration area will gradually produce in the vicinity of the weak point of the structure, called for the micro crack generation zone CGZ (Zone OA in Fig 1 (a),). Along with the increase of load, the range of CGZ is extended, and the tensile stress also increases gradually and tends to the tensile strength of concrete i.e. ft. When the damage variable D of concrete in the CGZ reaches a certain value, CGZ has been considered developed to the plump state, and its length r0 does not change. The D value at this time is called initiation damage threshold D_{IG}, which is used as the criterion of crack initiation (Fig. 1 (a)).

The second stage is micro crack propagation stage. The load continues to increase, the concrete appears to soften. The damage area outside the CGZ is called the micro crack development zone CDZ (Zone OO' in Fig. 1 (b)).CDZ can be regarded as an equivalent crack, the length of which is r_p. The closing forces are distributed on the two sides of the CDZ equivalent crack, their values are determined by the tensile softening curve of concrete. If there is an initial macro crack which length is a_0, the initial crack and the CDZ are use as the equivalent crack, its length is equal to a_0+r_p. When the r_p is extended to the critical crack length, it will cause the cracks instability growth. In this model the unstable fracture toughness in Xu Shilang double K fracture model is taken as the instability criterion K_{IC}. (Fig. 1 (b)).

In the third stage, the fracture process zone reaches its maximum size. The fracture process zone of concrete constitutes the entire damage area including the CGZ and CDZ, and its length l_f equal to r_p+r_0.When the closing force of equivalent crack CDZ is covered with concrete softening curve length, the CDZ extension to the full state, r_p reaches its maximum value r_{pu}, fracture process zone length l_f is also reaches its maximum value l_{fu}, and the end of CDZ begins to develop macroscopic cracks. Then CGZ and CDZ have been moving forward, but their length remains constant during the crack propagation (Fig. 1 (c)). It is assumed that concrete outside the damage zone is always remaining linearly elasticity.

In summary, the damage and fracture criterion of D-K model can be summarized as follows:

When $0<D<D_{IG}$, concrete does not crack initiation;

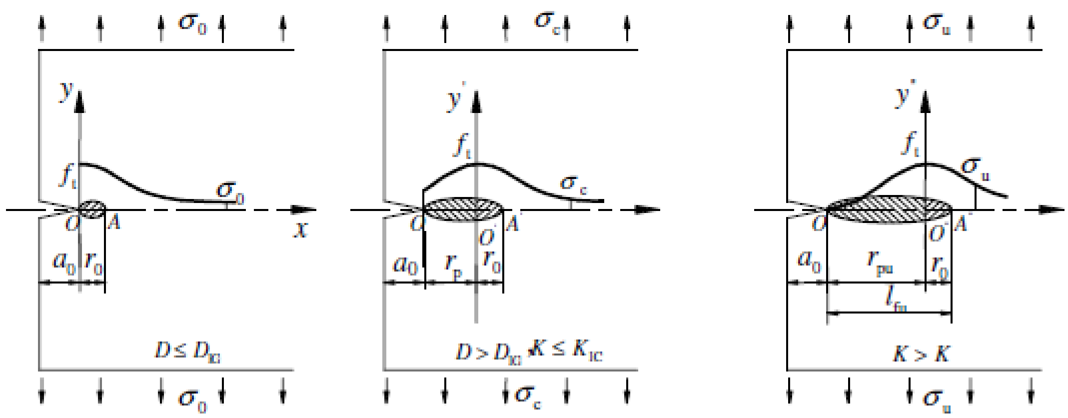
When $D= D_{IG}$, equivalent cracks appear and begin to stable propagation;

When $D> D_{IG}$, and $K<K_{IC}$, Cracks are in stage of stable propagation;

When $K=K_{IC}$, crack starts to unstable propagation;

When $K>K_{IC}$, the cracks are in the stage of unstable propagation.

By the criterion of crack initiation in D-K model, the crack initiation position of concrete without initial cracks can be determined. The analysis models of concrete members with and without initial cracks are unified.



(a)Microcrack generation (b)Reach the critical crack length (c)Full development of fracture process zone

Fig1. Extension process of fracture process area

Determination of Threshold Value of Crack Initiation Damage in D-K Model

There are two criterions in D-K model, i.e. initiation damage threshold and instability criterion.In this paper, the concrete micro crack shape obtained by the solutions of elastic stress field of mode I crack tip based on the unified strength theory, as well as the boundary equations of micro crack zone described by the damage variable were discussed respectively, and the expression of D_{IG} is derived by using the consistency of the two methods.

The Shape of the Microcrack Generation Zone Based on the Unified Strength Theory for Concrete

The D-K model, takes the damage threshold of the micro crack generation zone develops to full as the criterion of crack initiation, the shape of micro crack generation zone should be studied firstly. It is thought that the macro crack has not been developed beforemicro crack generation zone reaching to its full shape, linear elastic fracture mechanics and the traditional strength theory can be used to analyze its shape. The model is solved by using the solution of the elastic stress field of the mode I crack tip and the unified strength theory of concrete proposed by Yu Maohong[6].

By fracture mechanics, the elastic stress field at the tip of the mode I crack can be expressed with the principal stresses as follow.

$$\begin{cases} \sigma_1 = \dfrac{K_{\mathrm{I}}}{\sqrt{2\pi r}}\cos\dfrac{\theta}{2}\left(1+\sin\dfrac{\theta}{2}\right) \\ \sigma_2 = \dfrac{K_{\mathrm{I}}}{\sqrt{2\pi r}}\cos\dfrac{\theta}{2}\left(1-\sin\dfrac{\theta}{2}\right) \\ \sigma_3 = \begin{cases} 0 & \text{(plane stress)} \\ \dfrac{2vK_{\mathrm{I}}}{\sqrt{2\pi r}}\cos\dfrac{\theta}{2} & \textit{(plane strain)} \end{cases} \end{cases} \tag{1}$$

where σ_1, $\sigma_2$and $\sigma_3$are three principal stresses, K_{I} is the stress intensity factory of mode I crack, v for Poisson's ratio, $r$as the polar diameter and θ is the polar angle.

The mathematical expression of the unified strength theory is as follows:

$$\begin{cases} F = \sigma_1 - \dfrac{\alpha}{1+b}(b\sigma_2+\sigma_3) = \sigma_s,\ \sigma_2 \le \dfrac{\sigma_1+\alpha\sigma_3}{1+\alpha} \quad (a) \\ F' = \dfrac{1}{1+b}(\sigma_1+b\sigma_2) - \alpha\sigma_3 = \sigma_s,\ \sigma_2 \ge \dfrac{\sigma_1+\alpha\sigma_3}{1+\alpha} \ (b) \end{cases} \tag{2}$$

where α is the ratio of compressive to tensile strength of materials, b is the material strength parameters reflecting the influence degree on material failure of the intermediate principal shear stress and the normal stress on the corresponding surface.

Taking the plane stress problem as an example, the derivation procedure is as follows.

Hypothesizing$\theta_1$is the polar angle of intersection of (a) and (b) in formula (2)under plane stress type, then the boundary equation of the microcrack generation region in the plane stress state is as follows

$$\begin{cases} r_{\mathrm{CU}} = \dfrac{K_{\mathrm{I}}^2}{2\pi\sigma_s^2}\cos^2\dfrac{\theta}{2}[1-\dfrac{\alpha b}{1+b}+(1+\dfrac{\alpha b}{1+b})\sin\dfrac{\theta}{2}]^2 \quad (|\theta| \ge \theta_1)\,(a) \\ r_{\mathrm{CU}} = \dfrac{K_{\mathrm{I}}^2}{2\pi\sigma_s^2}\cos^2\dfrac{\theta}{2}[1+\dfrac{1-b}{1+b}\sin\dfrac{\theta}{2}]^2 \quad (|\theta| \le \theta_1) \qquad (b) \end{cases} \tag{3}$$

By the formula (3)(b), the critical length of microcrack generation zone in the X axis under plane stress state can be gotten.

$$r_{CU0}=\frac{K_{IC}^2}{2\pi\sigma_s^2} \tag{4}$$

Similar method is used in plane strain problem. The critical length of microcrack generation zone in the X axis under plane strain state can be gotten.

$$r_{CU0}=\frac{K_{IC}^2}{2\pi\sigma_s^2}(1-2\alpha\nu)^2 \tag{5}$$

Theα of ordinary strength concrete is equal to0.062~0.127[7], b is equal to0.5~1.0, ν is equal to0.15~0.25. Put the α,b and ν into the above formulas, the shape of microcracks generation zone under the condition of plane stress and the plane strain can be gotten.

The Damage Model of Subsection Power Function

In order to determine the concrete initiation damage threshold without initial macro cracks, on basis of concrete to existing damage models [8-10], D-K model proposed the damage model of subsection power function to describe the tensile damage behavior near peak value. The damage evolution equation of the damage model of subsection power function under uniaxial tension can be expressed as

$$D=\begin{cases} D_0\left(\dfrac{\varepsilon}{\varepsilon_0}\right)^m & (\varepsilon<\varepsilon_0) \\ 1-(1-D_0)\left(\dfrac{\varepsilon}{\varepsilon_0}\right)^n & (\varepsilon\geq\varepsilon_0) \end{cases} \tag{6}$$

Where ε is the tensile strain of concrete, and the ε_0 is the peak tensile strain. D_0、 m and n are material parameters.According to boundary conditions and the stress-strain curve of tensile testing proposed by Zhenhai Guo etc[11], the parameters of the model can be taken as D_0=1/5, m = 4, and n=-5/3. Assumed concrete is an isotropic material, so equivalent strain ε_e is defined to instead of strain ε in formula (6). The damage evolution equation of concrete under three-dimensional stress can be expressed as

$$D=\begin{cases} D_0\left(\dfrac{\varepsilon_e}{\varepsilon_0}\right)^m & (\varepsilon_e\leq\varepsilon_0)\text{(a)} \\ 1-(1-D_0)\left(\dfrac{\varepsilon_e}{\varepsilon_0}\right)^n & (\varepsilon_e>\varepsilon_0)\text{(b)} \end{cases} \tag{7}$$

Threshold Value of Concrete Crack Initiation Damage

The above analysis shows that the shape of concrete microcrack generation zone can be analyzed based on the traditional strength theory, and also can be described by the damage variable.

Because the shape of concrete microcrack generation zone reach to the critical state obtained by using the above two methods should be consistent,the threshold value of crack initiation damage can be deduced. Under the plane stress state, the main strain field can be expressed as

$$\begin{cases} \varepsilon_1=\dfrac{K_I}{E\sqrt{2\pi r}}\cos\dfrac{\theta}{2}[1-\nu+(1+\nu)\sin\dfrac{\theta}{2}] \\ \varepsilon_2=\dfrac{K_I}{E\sqrt{2\pi r}}\cos\dfrac{\theta}{2}[1-\nu-(1+\nu)\sin\dfrac{\theta}{2}] \\ \varepsilon_3=-2\nu\dfrac{K_I}{E\sqrt{2\pi r}}\cos\dfrac{\theta}{2} \end{cases} \tag{8}$$

Only the case of $\theta \le \theta_0 = 2\arcsin\frac{1-\nu}{1+\nu}$ is discussed, in this case, the $\varepsilon_1 \ge 0$, $\varepsilon_2 \ge 0$, and $\varepsilon_3 < 0$, then ε_e can be gotton.

$$\begin{aligned}\varepsilon_{\mathrm{e}} &= \sqrt{<\varepsilon_1>^2 + <\varepsilon_2>^2} \\ &= \frac{K_{\mathrm{I}}}{E\sqrt{\pi r}}\cos\frac{\theta}{2}\sqrt{(1-\nu)^2+(1+\nu)^2\sin^2\frac{\theta}{2}}\end{aligned} \tag{9}$$

Take the formula (9) into the formula (a) of formula (7), then Davailable.

$$D = \frac{D_0}{\varepsilon_0^m}\left(\frac{K_{\mathrm{I}}}{E\sqrt{\pi r}}\cos\frac{\theta}{2}\sqrt{(1-\nu)^2+(1+\nu)^2\sin^2\frac{\theta}{2}}\right)^m \tag{10}$$

By formula (10), the boundary equations of the micro crack generating region and the critical length on the X axis can be btained.

$$r = \frac{K_{\mathrm{I}}^2}{\pi E^2\varepsilon_0^2}\cos^2\frac{\theta}{2}\left[(1-\nu)^2+(1+\nu)^2\sin^2\frac{\theta}{2}\right]\left(\frac{D_0}{D}\right)^{\frac{2}{m}} \tag{11}$$

$$r_{\mathrm{D}} = \frac{(1-\nu)^2 K_{\mathrm{IC}}^2}{\pi E^2\varepsilon_0^2}\left(\frac{D_0}{D}\right)^{\frac{2}{m}} \tag{12}$$

Supposing that the critical length of micro cracks generated on the X axis obtained from the damage theory is equal to that obtained by the unified strength theory, notice that $f_t = (1-D_0)E\varepsilon_0$, the damage factor of microcrack generation region when it is full can be obtained, that is, the crack damage threshold:

$$D_{\mathrm{IG}} = D_0\left[\sqrt{2}(1-\nu)(1-D_0)\right]^m \tag{13}$$

When m=4, V =0.2, its value is

$$D_{IG} = 1.64D_0\left(1-D_0\right)^4 \tag{14}$$

The crack initiation damage threshold under the plane strain can be got in the same way.

$$D_{\mathrm{IG}} = D_0\left[\frac{\sqrt{2(1+\upsilon)}(1-2\upsilon)(1-D_0)}{\sqrt{(1-\upsilon)}(1-2\alpha\upsilon)}\right]^m \tag{15}$$

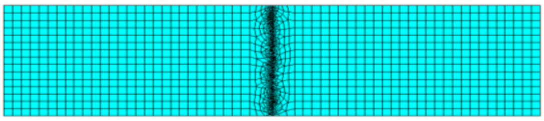
When m=4, V =0.2, and α=0.1, its value is

$$D_{IG} = 1.37D_0\left(1-D_0\right)^4 \tag{16}$$

Derivation of formula (14) and (16) respectively, when D_0=0.2, D_{IG}has extreme value 0.13 under the plane stress state and 0.11 under the plane strain. The threshold value of crack initiation damage under the plane strain state is slightly smaller than that of the plane stress state. The threshold value of crack initiation damage D_{IG} is smaller than the damage variable D_0 when stress reaches maximum value. It shows that the microcracks in the concrete have been cracked before its stress reaching the maximum value. This is consistent with the axial compression test of concrete. In addition, when the concrete material is determined, as d_0, m, ν, and α are all constants, then D_{IG}can be determined and can be used as material parameters, this indicated that take D_{IG} as cracking criterion of D-K concrete model is reasonable.

Using D-K Models to Simulate the Fracture Process of Three-Point Bending Concrete Beam

The D-K model can not only describe the crack initiation and failure state of concrete members in theory, but also can get the whole process of crack propagation through numerical simulation. In this paper, based on the ANSYS software, referring to the concrete propagation criterion of I type crack [12], the fracture process of three point bending beam with and without initial crack is simulated by using the D-K model. The crack propagation processes of the specimens simulated are shown in figure 2.



(a) finite element mesh

(b) crack initiation state

(c) Crack instability propagation state

(d)Finally failure state

Fig. 2. Typical crack propagation process of the three point bending beam with precast crack

For three-point bending beam without the initial fracture need the bending damage of the member to judge the initiation crack position. The judgment criterion is when damage factor of a point reach to crack damage threshold DIG, crack occurred at there, which is shown in Figure 3, the D_{IG} is also take for 0.13.

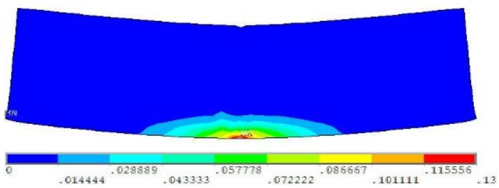


Fig. 3. Method for determining the initial fracture location of the three points bending beam without precast crack

Conclusions

According to the characteristics of the concrete fracture process zone, the fracture process zone is assumed to consist of the microcrack generation zone and the microcrack propagation zone, and the D-K model is proposed. In this model, through the damage initiation criterion D_{IG}, the location of concrete crack is determined, and it is applicable for with or without initial macrocracks concrete specimens. In order to achieve whole process analysis of concrete from initial microcrack to macrocrack formation and crack unstable propagation, take the fracture toughness K_{IC} as crack unstable propagation threshold.

The segmental power function damage model under axial tension is proposed. Boundary equations of mode I crack for concrete are derived on the basis of unified strength theory and damage theory respectively. Based on the condition that the critical lengths solved by the two methods on the X axis are equal, the expression of the threshold value of concrete crack initiation damage D_{IG} is derived.

Using the proposed concrete D-K model, numerical simulations of damage and fracture behavior of three-point bending concrete beams with and without initial cracks were carried out. It is feasible to simulate the whole process of mode I crack for concrete by using the numerical method.

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