Compensating method of the height anomalies for ultra-high Earth's gravity field model

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ABSTRACT: An ultra-high order Earth's gravity field model,such as EIGEN-6C4 and EGM2008, the order and degree of its expansion are limited, there will be a certain truncation error. RTM residual height anomaly can by high-resolution SRTM data and DTM2006.0 data using prism integral method is calculated. The experiment results show that (1) RTM residual height anomaly before fitting makes all the standard deviation of the EIGEN-6C4 and EGM2008(2190 order) model to be increased by 5.7 mm, the contribution rate of the truncation error of 10% and 7% respectively (2) RTM residual height anomaly after fitting makes their standard deviation to be increased 1 cm and 3.3 cm respectively, the contribution rate of truncation error of the model were 2% and 6% respectively.(3) RTM height anomaly changes with the increase of integral radius, its value tends to be stable, converge to 2 cm, EIGEN-6C4 and EGM2008 model elevation changes within 110 km radius of integral and are within 105 km of centimeter level. (4) RTM technology is a kind of an effective compensation method for ultra-high order Earth's gravity field model accuracy, help to improve the accuracy of the quasigeoid.

INTRODUCTION

EIGEN-6C4 (European Improved Gravity model of the Earth by New techniques)Earth's gravity field model is an ultra-high order gravity field model, it is released in august 2014 by GFZ German Research Center for Geoscience, this model is made up of GOCE(Gravity Field and Stead-state Ocean Circu-lation Explore)data, GRACE(Gravity Recove-ry and Climate Experiment) and LAGEOS（LAser GEOdynamic Satellite)data such as satellite gravity data and gravity data decoding on land or sea, is a kind of new high resolution earth gravity field model, its completely spheric harmonic order num-ber 2190 order.

The EGM2008 earth gravitational model, released by the US National Geospatial Agency in April 2008
, is a state-of-the-art thigh-degree global geopoten-tial model(GGM) of the Earth's external gravity fi-eld. It has got from GRACE data, CHAMP data and land gravity data. It is complete to spherical harmo-nic degree and order 2159 and provides some additional spherical harmonic coefficients to degree 2190. EIGEN-6C4 and EGM2008 these both models are ultra-high Earth's gravity field models. Their spatial resolution reaches 5 arc minutes. They are now the newest earth gravity field models.

The quasigeoid determined from ultra-high-gravi-ty field models even though can reach in scope of dm in global and our country(ZHANG Chuanyin, et al,2009;LIJiancheng,2012), but these models only represent the medium and long wave signals of the gravity fields, can not represent their high frequency signals, when computing the height anomalies the terrain effects of the high resolution still is conside-red. However, in case of no considering the terrain effects quasigeoid heights and other gravity field quantities computed solely by an ultra-high-gravity field model are always subject to the signal truncation errors. The truncation error comprises high-frequency gravity field signals that cannot be represented by a truncated spherical harmonic series expansion, i.e., all gravity field features occurring at scales finer than the spatial resolution of ultra-high-gravity field models(LI Jiancheng,et al,2013;Yahya ALLAHTAVA-KOLI, et al, 2015; ZHAI Changzhi,et al,2015).

There are two methods to compensate the omissi-on errors of ultra-high-gravity field models. (1)the commonly first methodology is the remove-restore approach known from regional quasigeoid modeling via Stokes's or Molodensky's integrals. EIGEN6C4-implied gravity anomalies are subtracted from a set terrestrial gravity observations, yielding residual gravity anomalies. These residual gravity anomalies are transformed to residual height anomalies using long-wavelength height anomalies. (2)the second method is RTM technique approach. In medium-elevated and rugged terrain, the residual terrain mo-del(RTM) data may be used for source-modelling of high-frequency gravity field signals(Z-HANG Xingfu, et al, 2012; M.S.Filmer, et al, 2010). In RTM modeling, a digital terrain model(DTM)is representing Earth's topography by prisms—is referred to a longwave-length reference surface. This step removes the low-frequency components from the DTM already imp-lied by the EIGEN-6C4. The transformation of RTM elevations to RTM height anomalies is accomplished using forward-modelling gravitational potential for-mulas for prisms.

In regions with sufficient terrestrial gravity data coverage, the first method generally allows more ac-curate modeling of the gravity field's fine structure than the RTM approach alone. This is because the RTM technique (second method)is usually based on simplifications of the distribution of mass-densities inside the topography. Often, a standard rock density is used uniformly for the complete RTM, thus neglecting the impact of any local mass-density variations. In regions with insufficient distribution or scarce availability of gravity data, the local gravime-tric refinement of the quasigeoid through the first method is of limited use or sometimes even impossible. Particularly in mountainous terrain, the second method represents a simple and promising alternative.

In order to improve the accuracy of local gravi-metric quasigeoid, no matter how much to improve the accuracy and resolution of some Earth's gravity field model, there always exists the models truncation error, this truncation that is composed of high-frequency components cannot be represented in the expansion of spherical harmonic synthesis in the Earth's gravity field model with limiting degree and order, and also need to condensate the high-freque-ncy components for Earth's gravity field model usi-ng RTM technique, particularly in mountainous regions.

The research and application of EIGEN-6C2 glob-al gravity field model also had been investigated and experienced for example the literature(FAN Hongt-ao,et al,2015;FAN Hongtao,2014), and compared with EGM2008 model, the EI-GEN-6C2 model is better than EGM2008 model. In theory applying RTM method can improve each component of global gravity field with high frequency including the gravity anomalies, the height anomalies, the vertical deviation and the ellipsoid height so on. For example the literature(Hirt C.,Feathertone,et al,2010; ZHAI Changzhi,et al, 2015;ZHANG Chuanyin,et al, 2009) had researched the improvement of height anomalies for EGM2008 model, the literature(Yahya ALLAH-TAVAKOLI1, et al,2015) had researched the impro-vement of gravity anomalies for EGM2008 model combining with near-surface mass-density anomali-es, the literature(ZHAIChangzhi, et al,2015; ZHA-NG Chuanyin,et al,2009; Z.UZANA,et al, 2016; M.C.DE Lacy, et al, 2001;L.E.Sjöberg, 2005;A LI K ILIÇOĞLU,et al,2010; Ilias N. Tziavos, et al, 2010;
H.Denker,et al, 1987) had researched the influence on EGM2008 model applying Stokes and Moloden-sky integration methods. In this paper the influences applying RTM technique on EIGEN-6C4 model height anomalies are discussed but also are com-pared with EGM2008 model, and the changes of height anomalies along with integral radius are dis-cussed.

PRINCIPLE AND METHOD

Computing height anomaly for gravity field model

In order to compute the height anomaliesζ^{EIGEN} from the set of EIGEN-6C4 fully normalized spherical harmonic coefficients $\overline{C}_{nm}$, $\overline{S}_{nm}$, according to Bru-ns equation, the standard series expansion is written: $z^{EIGEN} = \frac{GM}{Rg}\sum_{n=2}^{n_{max}^{EIGEN}}(\frac{a}{r})^n \times \sum_{m=0}^{n}(\overline{C}_{nm}\cos ml + \overline{S}_{nm}\sin ml)\overline{P}_{nm}(\cos q)$ (1)

In equation (1)are fully normalized associated Leg-endre functions. GM is geocentric gravitational con-stant, indicating the maximum degree(2190) of EI-GEN-6C4 spherical harmonic expansion, the coordi-nate tripler(r,q,l)denotes the geocentric polar coor-dinate of radius, geocentric co-latitude and longitude, which are computed from the geodetic coordinates that is geodetic latitude, geodetic longitude and geo-detic height, a are EIGEN-6C4 scaling parame-ters(semi major axis), g is normal gravity on the surface of the reference ellipsoid.

The use of the spherical harmonic series expan-sion(Eq.1) poses the problem of signal truncation because of the truncation at the maximum expansion degree,their series expansion with wavelengths and with spatial resolution may be written as:

$l = 360/n_{max}^{EIGEN}$ or $\Delta x = 180/n_{max}^{EIGEN}$ （2）

The maximum degree of EIGEN-6C4 is 2190 de-gree, according to Eq.(2) the computed minimum wavelengths are about 10 arc minutes (18km),accor-ding to Eq.(2) the computed minimum spatial reso-lution are about 5.0 arc minutes(9km).As such any gravity field structures at spatial scales shorter than 5.0 arc minutes are not represented by the EIGEN-6C4 degree 2190 series expansion. In fact, at moun-tain area with big fluctuation height anomalies from a degree 2190 expansion may be affected by signal truncation errors of several cm up to the dm order (Hirt et al, 2014).

RTM height anomalies computing

RTM data is applied to representing the differences values between actual terrain surface and local ave-rage terrain surfaces. Usually the actual terrain sur-faces are represented by DTM(Digital Terrain Model) data with high resolutions, local average terrain sur-faces are represented by the DTM data with low re-solutions, the local average terrain surface is got fro-m the actual terrain surface DTM data through the smooth treatment. At mountain area with big fluctu-ation the short wavelength information of the gravity field model is difficultly modeling when the degree-2190 EIGEN-6C4 (resolution 9km) and the limited amounts of GPS leveling data are only used, the sho-rt wavelength information which is caused by the te-rrain change can be represented modeling when the RTM technique are used, that is ,can compensate the signal truncation errors of gravity field model.

This paper uses the prism integral forward-mode-lling method to compute RTM height anomalies, the one means that the local terrains are divided into se-veral prisms, each a grid terrain point is replaced by a single rectangular prism of constant density and of constant height, the RTM height anomalies are the residual height anomalies, the gravitational potential of each prism mass is(Hirt et al, 2014):

$$V = Gr_0 \Big| \Big| \Big| xy\ln(z+r) + yz\ln(x+r) + zx\ln(y+r) - \frac{x^2}{2}\tan^{-1}(\frac{yz}{xr}) - \frac{y^2}{2}\tan^{-1}(\frac{xz}{yr}) - \frac{z^2}{2}\tan^{-1}(\frac{xy}{zr}) \Big|_{x_1}^{x_2} \Big|_{y_1}^{y_2} \Big|_{z_1}^{z_2} \quad (3)$$

Where r is the distance between the origin of the coordinate system and (x_1, y_1, z_1) the point , G is the gravitational constant, (x_1, y_1, z_1) and (x_2, y_2, z_2) are the coordinates of side corner points of a prism, $r_0 = 2670 kg/m^3$ is the topographic mass-density of the prism. We use $z_1 = 0, z_2 = z_{RTM} = z^{PRISM}$, the prism height z^{PRISM} represents the residual eleva-tions. A variant of Bruns equation is applied, the co-rresponding height anomalies of the prism gravita-tional potential is got, its equation is written:

$$z^{PRISM} = \frac{V}{g_Q} \tag{4}$$

Where is normal gravity on quasigeoid, V is the prism's potential, z^{PRISM} is the prism height anoma-lies. The height anomaly z^{RTM} of all prisms forming the RTM is then obtained as the sum of the height anomalies z^{PRISM} implied by all single prism:

$$z^{RTM} = \sum_{i=1}^{n} z^{PRISM}(i) \tag{5}$$

With n denoting the number of prisms within so-me radius R around the computation point. As RTM height anomalies z^{RTM} possess spectral power beyo-nd the maximum degree of EIGEN-6C4, they repre-sent our estimates of the EIGEN-6C4 height anoma-ly truncation error. We acknowledge that the RTM height anomalies do not rigorously augment the EI-GEN-6C4 spectral content. This is because the gra-vitational potential of the topography is a nonlinear function of the height.

In order to reduce computation time in actual ap-plication, it is common practice to work with high-resolution inner zones($R \le R_1$)and low-resolution coarser outer zones ($R_1 \le R \le R_2$)around the compu-tation point. The inner zones closer to the computa-tion point, the SRTM data with high-resolution is generally used for computing the residual terrain he-ight anomalies, the outer zones is far away to the co-mputation point, the SRTM/DTM2006.0 data with low-resolution is generally used for computing the residual terrain height anomalies. For the inner zo-nes, we use $3'' \times 3''$ SRTM data for constructing RTM data. For outer zones, we use $2.5' \times 2.5'$ SRTM data and $5' \times 5'$ DTM data for constructing a coarser RTM data, the $2.5' \times 2.5'$ SRTM data is obtained from $3'' \times 3''$ SRTM data via the smooth processing.

Integral radius and RTM constructing

We know that the RTM effect on the quasigeoid is given as the approximate expression(Omang & For-sberg, 2000):

$$z^{RTM} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{h_{ref}}^{h} \frac{1}{r} dx dy dz \tag{6}$$

Where h is the height of the topography, h_{ref} is the height of smooth local average terrain surface, r is the distance between original point and compu-ting point. When RTM effect on quasigeoid to be small, the term $1/r$ can be written as

$$\frac{1}{r} = \frac{1}{r_0} - \frac{1}{2}\frac{(h - h_{ref})^2}{r_0^3} + \cdots\cdots \tag{7}$$

Where r_0 is the planar distance. If a planar distan-ce gives sufficient accuracy a linear approximation might be assumed and the first term on the right-hand side of Eq.(7) is sufficient. The RTM effect on q-asigeoid is then expressed as

$$z^{RTM} = \frac{Gr(h-h_{ref})}{g}\int_{x_1}^{x_2}\int_{y_1}^{y_2}\frac{1}{r_0}dydx \qquad (8)$$

It should be noted that the above expression(8) is approximation, but the higher-order terms are relati-vely small. The above linear expression is readily evaluated by FFT methods, also by prisms integra-tion method of Eq.(5). The neglecting of the higher order terms in Eq.(7) is an error source in the pure RTM method. By increasing the resolution on the lo-cal average terrain height surface the overall RTM effects z^{RTM} can be made small, but this might inva-lidate the approximation of Eq.(6). In the present ca-se we used a resolution of the local average terrain height surface of 60 km,which is by experience a good compromise value.

The key of computing the height anomalies using the RTM method is how to construct the RTM data. Now the $3''\times3''$SRTM data, the $1''\times1''$ DTM data of global ASTER GDEM and the local high-resolution DTM data and so on can be used for constructing the RTM data. Now it is difficult to obtain the local high-resolution DTM data in china, then SRTM data and GDEM data are obtained free, the accuracy of the former is higher than the later. Therefore The method for constructing the RTM data in this paper is that the free $3''\times3''$SRTM data represented actual terrain height surface(H^{SRTM}), from which the high-resolution and low-resolution DTM data are obtain-ed. The local average terrain surface（$H^{DTM2006.0}$）can be obtained from the smooth processing of $3''\times3''$SRTM data and from reducing their resolutions, that is, the DTM data based on DTM2006.0 model represented the local average terrain surface height. Thus the RTM data model can be expressed as

$$z_{RTM} = z_2 - z_1 = H^{SRTM} - H^{DTM\,2006.0} \qquad (9)$$

DTM2006.0 serves as high-pass filter, removing the long wavelength features from the SRTM data. It comprises about 2.4 million pairs of fully normalis-ed height coefficients $\overline{HC}_{nm}, \overline{HS}_{nm}$ that give any poi-nt p on the ground elevation using(Yahya et al, 201-5):

$$H_P^{DTM\,2006.0}(q,l) = \sum_{n=0}^{n_{\max}^{DTM}}\sum_{m=0}^{n}(\overline{HC}_{nm}\cos ml + \overline{HS}_{nm}\sin ml)$$

$$\overline{P}_{nm}(\cos q)$$

(10)

Where n_{MAX}^{DTM} is the maximum degree of the heig-ht model, (q,l) is the spherical coordinates of the P point. The RTM data, that is constructed from SRT-M/DTM2006.0 data, can represent the high freque-ncy part of EIGEN-6C4 in the spatial resolutions.

Combination of EIGEN-6C4 model and RTM model

Applying the computation Eq.(1) of EIGEN-6C4 can compute the height anomalies of the gravity field model for GPS points, applying Eq.(5) can compute the RTM height anomalies , the overall height anomalies are expressed as

$z = z^{EIGEN} + z^{RTM} + z^{RES}$ (11) where z is the height anomalies at the surface of the topography, z^{RTM} is the height anomalies (including the direct effect and the indirect effect)at the local average terrain surface(smoothing surface), z^{EIGEN} is the height anomalies at the gravity field reference surface, z^{RES} is the residual height anomalies cause-d from the differences of above three surfaces. In pr-actice computation the model height anomalies, the z^{EIGEN} could first be obtained via Eq.(1) computed and then the RTM height anomalies z^{RTM} was obta-ined via Eq.(5), (10) and (11), the residual height anomalies z^{RES} was obtained via a subtraction of the model height anomalies z^{EIGEN} and the RTM hei-ght anomalies z^{RTM} from the

GPS leveling height anomalies, finally the residual height anomalies for unknown GPS points could be computed via a quad-ratic polynomial fitting method.

A CASE COMPUTATION AND RESULTS ANALYSIS

In order to validate the method provided in this pa-per, now we selected the GPS levelling data in the mountainous area with certain terrain fluctuation as testing data.

Selecting gravity field model

Now in the world there are many global Earth's gravity field model used to compute for the gravime-tric quasigeoid, including the famous EGM series models, GGM series models, EGM2008 model, and EIGEN series models and so on. For selecting some global gravity field models as reference gravity field model there are no any uniform optimal model. To built national and local quasigeoid models, the most comfortable model as reference gravity field models should be taken. for different regions, this is because global and regional GPS levelling geoids include different error components and different amount levels, need to correspond with different reference gravity field models(Zhang & Liu, 2012). The elevation accuracy of EGM2008 model in China in total is about 20 cm(FANHongtao,et al,2015), the accuracy of EIGEN-6C2 model in China Coastal Zo-ne comparing with GNSS levelling is close to centi-meter level. EGM-2008(2190 order) model, EIGEN-6C2(1949) model, EIGEN-6C3STAT(1949)model and EIGEN6C4(2190)model had be applied in USA, German,Canada,Australia,Europe and Japan so on. The average geoid elevation that was computed by these models had respectively achieved to 15.31 cm, 15.55 cm, 15.28 cm and15.25 cm in these countrie-s(http://icgem.gfzpotsdam.de/ICGEM/documents/FoersteetalEIGEN6C3stat.pdf;http://icgem.gfzpotsdam.de/ICGEM/documents/Foerste-et-alEIGEN6C4.pdf).

Due to EIGEN-6C4 model is priority to EIGEN-6C3STAT model and others. Therefore this paper ta-kes the EIGEN-6C4 model as the experiment and compares with the EGM2008 model with the same order and degree.

Testing data preparing

To satisfy the practice computation requirement, this paper searched some mountainous area with the cer-tain fluctuation as a testing area(see Fig.1). The size of the testing area is $2^\circ \times 2^\circ$ extent.

The scope is from north latitude $40^\circ 30'$ to $42^\circ 30'$,from east longitude $113^\circ 30'$ to $115^\circ 30'$.The minimum value of GPS leveling height is 597.04 m , the maximum value is 1513.438 m the maximum height difference value is 916.398 m , the average height value is 1253.785m, including the red point represents the situations of GPS leveling points.

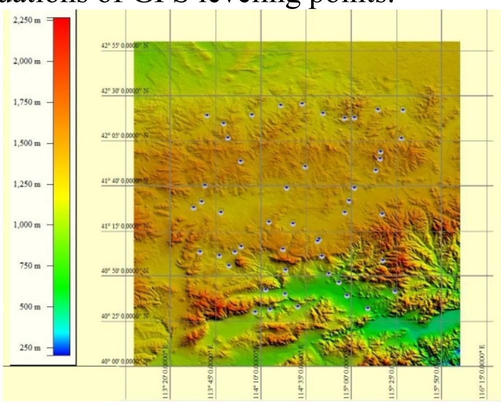

Fig.1 SRTM Terrains and GPS Leveling Points in Test Area.

(1) The $3'' \times 3''$ SRTM data is directly got from the SR-TM official website and represents the detail ter-rain surface;

(2) The $2.5' \times 2.5'$ SRTM data is obtained from smoo-thing $3'' \times 3''$ SRTM data and represents the coar-ser terrain surface;

(3) The local average terrain DTM data is computed from Eq.(10) according to DTM2006.0 model;

(4) GPS leveling data have a total surveying data of 49 points that belong to CGCS2000 geodetic co-ordinates system, the accuracy of geodetic heigh-ts are upon C order, the leveling accuracy are upon third order.

A case first applying the Eq.(10) computes the DTM2006.0 height data, then the $2.5' \times 2.5'$ SRTM data is obtained from smoothing $3'' \times 3''$ SRTM data, finally the RTM data is obtained from the computa-tion of Eq.(5). In the testing area the average values of residual height anomalies are 0.022m, the maxi-mum value is 0.085m, the minimum value is 0.040m
, the STD value is 0.014m. The RTM residual height anomalies contributions situation of all grids points distanced from central points in the testing area is shown as Fig.2.

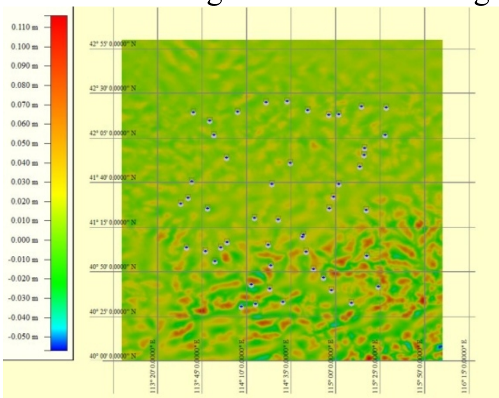

Fig.2 Residual Height Anomalies in Test Area.

The results in Fig.1 show that even though in the testing area heights of all points are high(above 1000m), but in the north of altitude 41°15′ the topograp-hic undulation is smaller(about 300m), so the RTM residual height anomalies also are not big(see Fig.2), amounts of GPS points(28 points) are located in this region with smaller RTM elevations, the points dis-tribution also is sparse, there are almost no the cont-rol points in many regions. In the south of altitude of 41°15′ the topographic undulation is bigger, the RTM elevations are also bigger(see Fig.2),only a small amount of GPS points(21points) is there in this region with bigger RTM elevations, the points distribution also is sparse. These facts explains that the topographic data in this testing area is unchara-cteristic, especially the GPS leveling points distri-bution is sparse. Therefore analyzing the influence of the RTM residual height anomalies on the height anomalies of the ultra-high-gravity field model, the testing area should be divided into two subranges with the RTM bigger elevations and the RTM small-er elevations.

Radius analysis

According to Eq.(8) it is known that the RTM resi-dual height anomalies are a function of the radius r_0. To investigate the change case of the RTM residual height anomalies along with the integral radius, the central point in the testing area is determined at latitude 41.5º and longitude 114.5º. The change si-tuation of the RTM residual height anomalies in the scope 120km around the central point is given in the Fig.3.

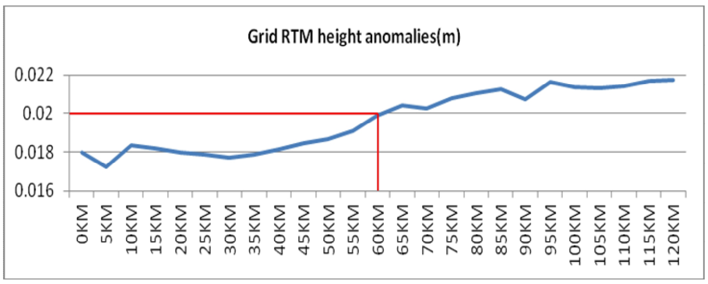

Fig.3 RTM Residual Height Anomalies in integral radius in test area.

The change situation of the RTM grids height anomalies along with the integral radius is given in the Fig.3. Taking the central point in the testing area as a center, it shows the RTM residual height ano-maly values of all grids with 2.5 arc minutes that are made of all 2401 grid points. The total wave shape of the RTM height anomalies is more stable, the maximum value of the wave is 0.022m,.the mini-mum value is 0.017m, the maximum change value is 5mm. It is known from the Fig.3 that the RTM height anomalies generally change not big in the scope of 120km.

To compute the RTM height anomalies of all GPS leveling points spend very much times. To raise computation efficiency the detail grids DTM data and the coarser grids DTM data should be distinguished. The RTM height anomalies of a GPS point in the inner area are used for the detail grids and The RTM height anomalies of a GPS point in the outer area are used for the coarser grids According to the RTM height anomalies change in the Fig.3 it is known that when the radius is 60km, the corresponding height anomalies value is within 2mm. Taking 2mm as the boundary we can divide the whole integral radius in the testing area into two integral areas, that is, the first area is the scope from 0 km to 60km, the second area is from 60 km to 129 km..Thus it taken the first area as the details grids and the second area as the coarser grids. This paper takes the integral radius R_1=60 km, R_2=120 km.

Testing data computing and analyzing

In the testing area reducing from the height anoma-lies of 49 GPS leveling points to the height anomali-es of EIGEN-6C4 that are got from the Eq.(1) the results are expressed as the EIGEN-6C4 residual height anomalies in the Fig.4. According to different topographic undulation this testing area is divided into two subranges: the first subrange is in altitude 41°15′~42°30′ and in longitude 113°30′~115°30′, the second subrange is in altitude 40°30′~41°15′ and in longitude 113°30′~115°30′, in the two subranges the height anomalies residuals of various GPS points are respectively computed by EIGEN-6C4 model and EGM2008 model.

In the first subranges there are altogether 28 GPS points, their topographic undulation is not big, the maximum value of their elevations is 1514.061m, the minimum value is 1225.651m, difference betwe-en the maximum and the minimum values is 288.41 m. The residual results of various GPS points height anomalies, that are respectively computed by EIGE-N-6C4 model and EGM2008 model via above met-hods, are shown in Fig.4. For EIGEN-6C4model the maximum, minimum, average and STD values res-pectively are 0.239m, 0.002m, 0.111m, and 0.058m. For EGM2008 model the maximum, minimum, ave-rage and SDT values are respectively 0.418m, -0.05-2m, 0.136m and 0.133m.

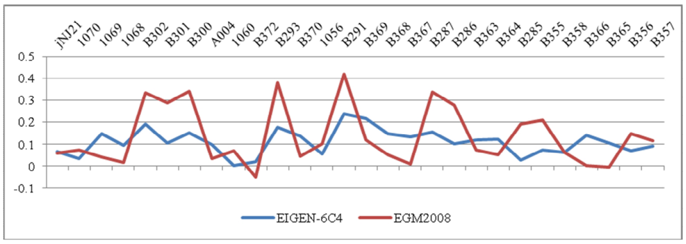

Fig.4 the height anomaly residuals of EIGEN-6C4 and EG-M2008 for different GPS levelling points in first subrange.

Fig.4 shows that in the first subrange the STD value of the height anomaly residuals for EIGEN-6C4 is 5.8cm, and for EGM2008 is 13.3cm, a syste-matic difference between both models is 2.5cm. The accuracy of EIGEN-6C4 model is improved 7.5cm than EGM2008 model. From views of gravity field frequency spectrum, the frequency spectrum of EI-GEN-6C4 model is smaller than EGM2008 model.

In the second subranges there are altogether 21 GPS points, their topographic undulation is bigger, the maximum value of their elevations is 1525.087 m, the minimum value is 608.159m, difference betw-een the maximum and the minimum values is 916.928m, is three times than the biggest elevation difference in the first subrange. The residual results of various GPS points height anomalies, that are respectively computed by EIGEN-6C4 model and EGM2008 model, are shown in Fig.5. For EIGEN-6C4 model the maximum, minimum ,average and STD values respectively are 0.211m, -0.008m, 0.106m, and 0.060m. For EGM2008 model the max-imum, minimum, average and SDT values are respe-ctively 0.223m, -0.070m, 0.062m and 0.080m.

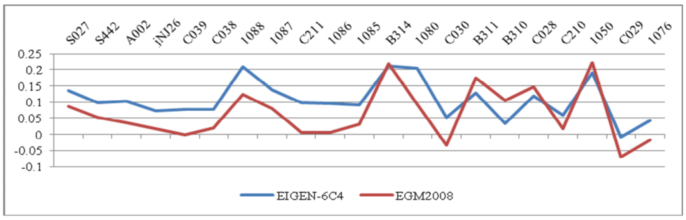

Fig.5 the height anomaly residuals of EIGEN-6C4 and EGM2-008 for different GPS levelling points in second subrange.

Fig.5 shows that in the second subrange the STD value of the height anomaly residuals for EIGEN-6C4 is 6.0cm, and for EGM2008 is 8.0cm, a system-atic difference between both models is 4.4cm. The accuracy of EIGEN-6C4 model is improved 2.0cm than EGM2008 model. From views of gravity field frequency spectrum, the frequency spectrum of EIG-EN-6C4 model is smaller than EGM2008 model.Co-mparing with the first subrange, the spectrum wave-form features of height anomaly residuals of both two models tend to be more consistent that is their frequencies tend to be more consistent.

Even though applying EIGEN-6C4 model or EGM2008 model in any global gravity field models there certainly are the truncation errors, these models do not represent the high frequency signals of the gravity field. In order to overcome this defects this paper uses the RTM technique that is above (2) met-hod. The RTM height anomaly values of all GPS levelling points are computed using Eq.(5), their co-mputed results are shown in the Fig.6.The RTM res-idual height anomaly values are in the scope of +4cm～-2cm, the average value is 1.0 cm, the maxi-mum value is 3.6 cm, the minimum value is -1.7 cm, the STD value is 1.0 cm. It is shown in the Fig.6 that the RTM residual height anomaly values are within 6 cm, such values is not neglected for reducing the truncation errors of ultra-high-gravity field model.

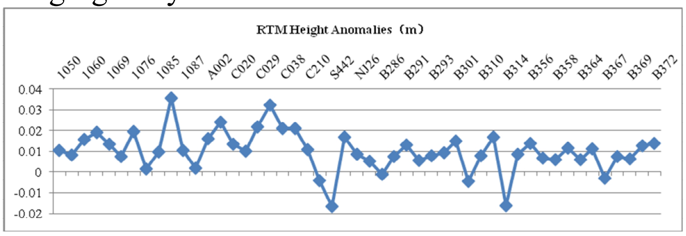

Fig.6 RTM Residual Height Anomalies for GPS Leveling Poi-nts (SRTM-DTM2006.0).

To investigate the effects of the RTM height ano-malies on the truncation errors of EIGEN-6C4 and EGM2008, reducing from the EIGEN-6C4 and EG-M2008 residual height anomalies to the corresponsi-ve RTM residual height anomalies the their differen-ces are shown in the Fig.7,Fig.8,Fig.9 and Fig.10

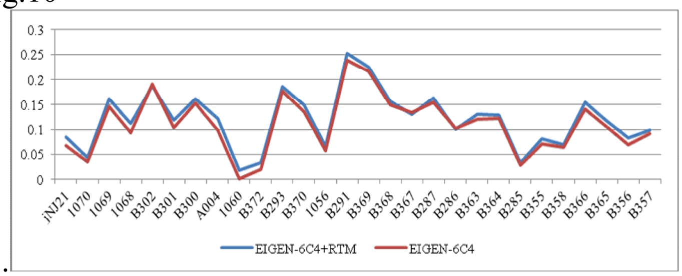

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Fig.7 Height Anomalies Residuals of EIGEN-6C4+RTM and EIGEN-6C4 for Different GPS Leveling Points in First Subrange .

Firstly it is discussed in the first subrange the inf-luences of the RTM height anomalies on both mode-ls. Fig.7 shows the comparing results of the height anomalies for EIGEN-6C4 and EIGEN-6C4+RTM in the first subrange. The maximum value of the hei-ght anomalies for EIGEN-6C4+RTM is 0.252m, its minimum value is 0.018m, its average value is 0.120

m, the STD value is 0.057m. It is shown in Fig,7 that in the first subrange (elevation differences in 300m) the STD of height anomalies for EIGEN-6C4+RTM after reducing the RTM height

anomalies is 5.7cm, the STD for EIGEN-6C4 is 5.8cm, the STD for EIG-EN-6C4+RTM only improved 1mm than for EIGEN
-6C4. The spectrum waveform of EIGEN6C4+RTM with EIGEN-6C4 almost is same. The RTM corre-ctions on the model accuracy of EIGEN-6C4 contri-buted 2%(see table 1) and on the truncation errors are almost without any compensation, so the RTM corr-ections make little sense.

Fig.8 shows the comparing results of the height anomalies for EGM2008 and EGM2008+RTM in the first subrange. The maximum value of the height anomalies for EGM2008+RTM is 0.431m, its mini-mum value is -0.038m, its average value is 0.145m, the STD value is 0.136m. It is shown in Fig.8 that in the first subrange (elevation differences in 300m) the STD of height anomalies for EGM2008+RTM after reducing the RTM height anomalies is 13.2cm, the STD for EGM2008 is 13.3cm, the STD for EGM-2008+RTM only improved 1mm than for EGM2008. The spectrum waveform of EGM2008+RTM with EGM2008 almost is same. The RTM corrections on the model accuracy of EGM2008 contributed1%(see
Table1) and on the truncation errors are almost wi-thout any compensation, so the RTM corrections make little sense. This is because the RTM correc-tions mainly depend on the change of height anomalies, the differences of various height anoma-lies not only are related with the horizontal distance, the vertical deviation and the gravity anomaly, but also are related with the elevation differences. We know the changes of the vertical deviations and the gravity anomalies are not big in a flat terrain area. If in some area the elevation differences is not big, then in this area the height anomalies differences usually are also not big. Due to in the first subrange the elevation differences are not big, so their height anomaly differences still are not big, that is, the RTM corrections make little sense.

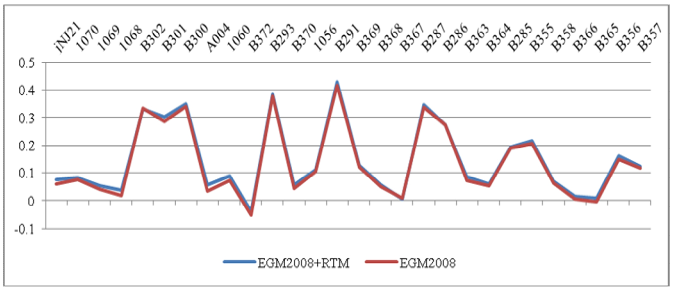

Fig.8 Height Anomalies Residuals of EGM2008+RTM and EGM2008 for Different GPS Leveling Point in First Subrange.

It is shown in Fig.7 and Fig.8 that in not big un-dulation mountain (in the first subrange) the spec-trum waveform of EIGEN-6C4+RTM is 23.4cm, the spectrum waveform of EGM2008-+RTM is 46.9cm, the spectrum waveform of EIGEN-6C4+RTM is half of EGM2008-+RTM, the EIGEN-6C4 model accu-racy improved 7.5cm than the EGM2008 model. This fact explains the EIGEN-6C4 model is the priority of the EGM2008 model.

The following to analyze the situation in second subrange. Fig.9 shows the results of the height ano-malies residuals for EIGEN-6C4 and EIGEN-6C4+-RTM . The maximum value of the height anomaly residuals for EIGEN-6C4+RTM is 0.212m, their mi-nimum value, average value and STD value is respe-ctively 0.013m, 0.116m and 0.054m. Fig.9 shows that in the not big undulation mountain the STD of the height anomaly residuals for EIGEN-6C4+RTM model is 5.4cm, the STD for EIGEN-6C4 is 6.0cm, the EIGEN-6C4+RTM model accuracy is improved 6mm than EIGEN-6C4 model. The spectrum wave-form of both models are different. The RTM correc-tions contributed 10% to the height anomaly impro-vement of EIGEN-6C4 model(see the table 1), this is because the RTM compensating action improved the waveform of EIGEN-6C4 model, so the RTM correction results are the better.

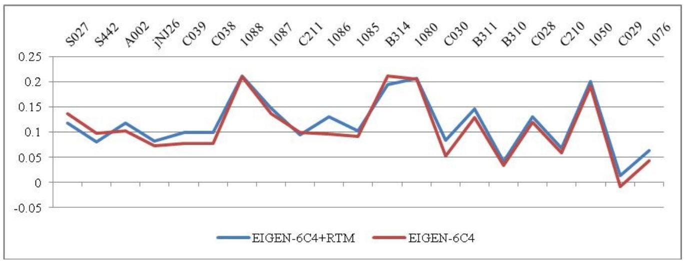

Fig.9 Height Anomalies Residuals of EIGEN-6C4+RTM and EIGEN-6C4 for Different GPS Leveling Points in second area

Fig.10 shows the results of the height anomaly residuals for EGM2008+RTM and EGM2008 mode-ls. The maximum value, the minimum value, the average value and the STD value for EGM2008+-RTM are respectively 0.233m, 0.049m, 0.072m and 0.075m. It is shown in Fig.10 that in the not big undulation terrain mountain the STD for EGM2008 model is 7.5cm, the STD for EGM2008 model is 8.0
cm, the accuracy of EGM2008+RTM model is imp-roved 5mm than the EGM2008 model. The spect-rum waveforms for EGM2008+RTM model and EGM2008 model are different, the RTM corrections contributed 7%(see Table 1) to the accuracy impro-vement of EGM2008 model. This is because the RTM compensating action improves the waveform of EGM2008 model, so the RTM correction results are the better.

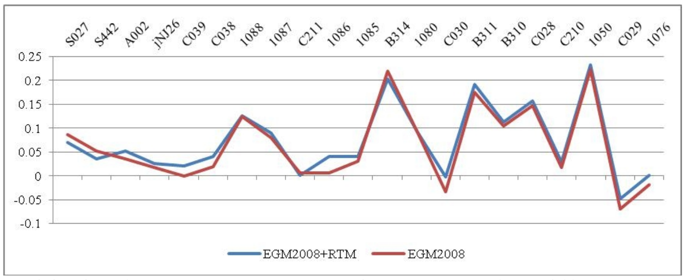

Fig.10 Height Anomalies Residuals of EGM2008+RTM and EGM2008 for Different GPS Leveling Points in second area

It is shown in Fig.9 and Fig.10 that in the big un-dulation terrain mountain the amplitudes of the spec-trum waveform for EIGEN-6C4+RTM model is19.9
cm, the amplitude for EGM2008+RTM model is28.2
cm, the STD accuracy of EIGEN-6C4+RTM model improves 2cm than EGM2008 model. This explains that in this such subrange the EIGEN-6C4 model still is priority to the EGM2008+RTM model.This is because the EIGEN-6C4 model has applied to a glo-bal land gravity data, but also applied the ocean gra-vity data, this model is complete to spherical har-monic degree and order 2190, the EGM2008 model has mainly applied to the land gravity data, and in south pole area is a blank area, it is complete to spherical harmonic degree and order 2159 and pro-vides some additional spherical harmonic coefficie-nts to degree 2190.

In order to analyze the spatial positions of corre-lation with the height anomalies of ultra-high Earth's gravity field models at same point in this area, the different height anomaly values at the central point(altitude 41.5^{o}, longitude 114.5^{o}) in this area in the scopes of different integral radius are interpolated by the grid height anomaly values(5km-×5km, 2.5′×2.5′）) via Eq.(8). The change situations of the height anomalies that respectively are compu-ted by applying EIGEN-6C4 and EGM2008 models along with the integral radius to the central point is shown in the Fig.11. The maximum value, the mini-mum value, the average value and the STD value for EIGEN-6C4 model respectively are -12.852m,-12.9-61m, -12.909m and 3.6 cm. In the scope of 110km their change values are 9.81cm, are in the cm order. The maximum value value, the minimum value, the average value and the STD value for EGM2008 model respectively are -12.842m, -12.956m, -12.89-9m and

3.7cm. In the scope of 105 km their changes are 9.55 cm, are in the cm order. It is shown in Fig.11 that all height anomalies for EIGEN-6C4 and EGM2008 models are changed along with the integral radius. The spectrum waves of both models are the same, peak at 60 km, and then gardually become smaller. At about 110 km the later their height ano-malies relative to the change of the peak achieve dm order.

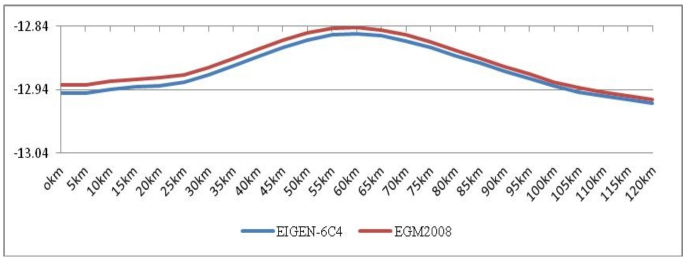

Fig.11 Height Anomalies of EIGEN-6C4and EGM2008 for Different Integration Radius to the Central Point

According to Eq.(11) computed the height anoma-ly residuals to the GPS levelling data are fitted by the quadratic polynomial, and the results before and after the fitting are compared. The uniform distribu-tions of the fitting points and the linear changes of their elevation values are mainly considered as far as possible. For the flat first subrange 50% points are selected that is 14 points as the fitting points. For the second subrange with the big terrain undulation 30% points are selected that is 6 points as the fitting points, and the fitting results after the fitting are shown in the Table 1 according to still above two subranges.

Table1 comparing of EIGEN-6C4 and EGM2008 residual height anomalies Unit：**cm**

First subrange（small terrain undulation）						
Fit	Models	Max	Min	Ave	STD	Contribution
Before	EIGEN-6C4	23.90	0.20	11.05	5.81	
	EIGEN-6C4+RTM	25.20	1.80	12.03	5.69	2%
	EGM2008	41.80	-5.20	13.56	13.33	
	EGM2008+RTM	43.10	-3.80	14.53	13.16	1%
After	EIGEN-6C4	17.80	3.70	9.85	4.13	
	EIGEN-6C4+RTM	17.50	4.30	10.74	4.02	3%
	EGM2008	30.50	1.00	12.55	10.10	
	EGM2008+RTM	31.40	2.20	13.45	9.87	2%
Second subrange（big terrain undulation）						
Before	EIGEN-6C4	21.10	-0.80	10.64	6.00	
	EIGEN-6C4+RTM	21.20	1.30	11.58	5.43	10%
	EGM2008	22.30	-7.00	6.25	8.03	
	EGM2008+RTM	23.30	-4.90	7.20	7.46	7%
After	EIGEN-6C4	15.40	0.90	8.31	4.75	
	EIGEN-6C4+RTM	15.50	3.00	7.72	4.66	2%
	EGM2008	10.70	-5.70	3.63	5.64	
	EGM2008+RTM	10.60	-5.60	4.59	5.31	6%

It is shown in the Table 1 that for the not big terrain undulation mountain before the fitting the STD values of EIGEN-6C4 and EIGEN-6C4+RTM are respectively 5.81cm and 5.69cm, only improved 1.2mm. This explains that firstly the accuracy of the height anomalies computed by EIGEN-6C4 model in this region is higher and achieved cm order, secondly adding the RTM corrections contribute to be very small to their height anomaly accuracy in this region and may be not added these corrections. Before the fitting the STD values of EGM2008 and EGM2008+RTM are respectively 13.33cm and13.16

cm, only improved 1.7mm. This explains that firstly the accuracy of the height anomalies computed by EGM2008 model in this region is achieved dm order and lower than EIGEN-6C4 model, secondly adding or not adding the RTM corrections makes little sense to their height anomaly accuracy in this region and should be not considered the RTM corrections. In this region after fitting the STD values of EIGEN-6C4 model and EIGEN6C4+RTM model are respec-tively 4.13cm and 4.02cm, the RTM contributes 1.1
mm, is the same before the fitting, and all STD values before the fitting and after the fitting are improved 29%. After the fitting the STD values of EGM2008 model and EGM2008+RTM model are respectively 10.10cm and 9.87cm, the RTM contri-butes 2.3mm that is improved comparing before the fitting, but all STD values before the fitting and after the fitting are increased about 25%. In addition it is also shown in the Table 1 that the STD values of EIGEN-6C4+RTM after fitting are more enhanced 1.79cm than EIGEN-6C4 before the fitting. the STD values of EGM2008+RTM after the fitting are more enhanced 3.46cm than EGM2008 before the fitting.

It is shown in Table 1 that in the big terrain moun-tain the STD values of EIGEN-6C4 and EIGEN6C4-+RTM models before the fitting are respectively 6.00cm and 5.43cm, the RTM corrections contribute 5.7mm so that the accuracy of the height anomaly of EIGEN-6C4 model is increased 10%, the RTM corrections compensate more obvious and improve better to the truncation of an ultra-high Earth's gravi-ty field model. In addition this also explains that in this region the accuracy of the height anomaly com-puted by EIGEN-6C4 model is higher and is in sco-pe of cm. Besides the Table 1 explains that the STD values of EGM2008 and EGM2008+RTM in this region are respectively 8.03 cm and 7.46cm, the RTM corrections contribute 5.7mm that is the same with EIGEN-6C4 model, however the STD values are more improved than in the flat region, for the big undulation mountain the RTM corrections are more obvious improved to the height anomaly of EGM-2008 model, after the fitting the STD values of EI-GEN-6C4 and EIGEN-6C4+RTM are respectively 4.75cm and 4.66cm, the RTM corrections only con-tribute 1.5mm that is little lower than before the fit-ting, yet their STD before the fitting and after the fitting are respectively improved 21% and 14%, and are in the scope of cm order, after the fitting the STD values of EGM2008 and EGM2008+RTM are respectively 5.64cm and 5.31cm, the RTM correc-tions contribute 3.3mm that is little lower than befo-re the fitting and are more increased about 30% be-fore the fitting and after the fitting, and are in the scope of cm order. However it is seen from Table 1 that in the big undulation mountain the STD values of EIGEN-6C4+RTM after the fitting are more improved 1.34cm than EIGEN-6C4 before the fitting and the STD values of EGM2008+RTM after the fit-ting also are more improved 2.72cm than EGM2008 before the fitting.

CONCLUSION

The utilization of the RTM technique for reducing the truncation errors of ultra-high Earth's gravity field model, is a kind of effective methods. This paper used now the most advanced ultra-high Earth's gra-vity field models including EIGEN-6C4 model and EGM2008 model as well as combined SRTM/DTM-2006.0 data for constructing the RTM model, and that investigated the compensating action using the RTM technique for reducing EIGEN-6C4 model and EGM2008 model truncation errors, finally made a comparison of EIGEN-6C4 model with EGM2008 model and it is obtained the following results:

(1) There is a certain systematic deviation between the height datum followed by Foreign ultra-high Earth's gravity field models including EIGEN-6C4 and EGM2008 and the Chinese leveling hei-ght datum, this deviation values in the testing area are in the scope of fdm order. This result is consistent with the literature(see M.S.Filmer, W.E.Featherstone, M.Kuhn, 2010).
(2) The applying RTM model for correction to the truncation errors of ultra-high Earth's gravity field models can to a certain degree reduce the their truncation errors. This is because the ultra-high Earth's gravity field models with the limi-ted order and degree can not represent the high frequency signals of thhe gravity field, and the RTM residual height anomalies can express the

high frequency signals to have a certain compen-sation action, the performance of this compen-sation action is not obvious in the small undula-tion terrain mountains and is more significant in the big undulation terrain mountain. In the big undulation terrain testing region the contribu-tion rate of the RTM corrections to the truncation errors of EIGEN-6C4 model is about 10%, this is because the height anomalies mainly depend on the intermediate and low frequency signals, how-ever the RTM residual height anomalies only primarily represent the changes of the relative terrain undulation.

(3) Through a comparison between EIGEN-6C4 and EGM2008 models, on the side of the RTM compensation action, before the fitting the RTM corrections to EIGEN-6C4 model compensate to be more effective than to EGM2008 model, and after the fitting in the small undulation terrain testing region the RTM corrections to EIGEN-6C4 model compensate to be little effective than EGM2008 model, in the big undulation terrain testing region the RTM corrections to EGM2008 model compensate to be more effective than EIGEN-6C4 model, this is because in the case of no added RTM corrections the fitting effect to EGM2008 model is better than EIGEN-6C4 mo-del. On the side of the model STD, for the small undulation testing region before the fitting the accuracy of EGM2008 model is in the scope of dm order and after the fitting the accuracy close to in the scope of cm order, for the big undulation testing region the accuracy of EIGEN-6C4 model before the fitting and after the fitting always is in the scope of cm order, this explains that the EIGEN-6C4 model is priority to EGM2008 model, in the first subrange and seco-nd subrange the accuracy of EIGEN-6C4 model is more improved respectively 56% and 25% than EGM2008 model.

(4) Due to the RTM grid height anomalies are a non-linear function of the integral radius, this test al-so gives the change situation of the RTM grid height anomalies along with the integral radius. It is shown that in total the wave change of the RTM grid height anomalies along with the inte-gral radius is more smooth and is converge to 2 cm in the radius range of 120 km, still in the radius range of 15 km is small fluctuations. This is because the terrain undulation in total testing region is not big and the relative terrain undu-lation is more smooth. In addition the spectrum waveform of the height anomaly residuals change for EIGEN-6C4 model and EGM2008 model along with the integral radius is consiste-nt, including the height anomaly residuals chan-ge in the scope of cm order in the radius range of 110 km, the change of the height anomaly residuals for EGM2008 model is in the scope of cm order within the range of 105 km and beyond this range will be in the scope of dm order.

(5) I t is used to the combining method of the RTM residual height anomalies with the height anoma-lies of an ultra-high Earth's gravity field model for computation of the high accuracy geoid, the computation requirements are very low. It is mo-re important for its easy application, if the cost to get the gravity data would be high, then would not do any yield gravity surveying. Applying the method, that the RTM residual height anomalies are constructed by SRTM and DTM2006.0 data can reduce the truncation errors of ultra-high Earth's gravity field models, can be used as the supplementary means to improve the geoid accuracy.

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