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A Robust Adaptive Beamforming Algorithm based on Subspace Complement

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Abstract. A novel algorithm for robust adaptive beamforming is put forward in this paper. This algorithm use the characteristic of complement of desired signal subspace complement to build constrained model and solve the supremum, then the solution based on convex programming are given in the paper .The two characteristics of the proposed algorithm are: on one hand, this algorithm does not need imprecise steering vector of desired signal and exact antenna array geometry; on the other hand, the presented algorithm has robustness for steering vector mismatches, antenna element displacement errors and other errors, and has better performance than existing algorithms. The simulation results show the effectiveness of the proposed algorithm. Due to the little prior information is needed; therefore, it is helpful to engineering application.

Introduction

Robust adaptive beamforming(RAB) arouse much and deep study among scientists in recent years due to main two reasons. First, adaptive beamforming is widely used in radar, sonar, wireless communication, microphone arrays, medical imaging and other fields[1-2]; Second, there is an urgent demand of the robustness of the algorithm in the process of practical application. Early robust adaptive beamforming methods such as diagonal loading method (DL)[3] and eigen-subspace method (ESB)[4] have their own drawbacks, the former drawback is that the loading factor is difficult to select, while the latter results in poor performance in the case of low signal to noise ratio. After 2000, a kind of algorithm with very clear theoretical meaning, namely robust beamforming algorithm based on Uncertainty Set constrains, has been proposed. The main idea of the algorithm is to constrain the steering vector of the desired signal to various uncertain sets, including spherical uncertainty set [5,6], ellipsoidal uncertainty set [7], and diamond uncertainty set[8]. The aim is to ensure that the beamformer can still maintain the ideal performance when the steering vector error varies within a certain range. Besides, the robust algorithm against big pointing error is studied in [9,10]. In [11], the situation when the vector mismatch occurs and the training sample is contaminated by the target signal meanwhile is discussed, all of above are the improvement and supplement to the uncertain set algorithms.

In another sense, most of the algorithms of the uncertainty set are robust adaptive beamforming methods (RAB) based on the minimum variance distortionless response (MVDR) criterion. The typical algorithms can be summarized as: (i) worst case performance optimization (Worst Case) [5]; (ii) doubly constrained robust beamformer[7]; (iii) probabilistic constrained robust beamformer [12]; (iv) a robust beamformer (EOD) based on steering vector mismatch orthogonal decomposition [13].

The above four robust adaptive beamforming methods can solve the performance degradation problem which result from the steering vector mismatch well, but it is still necessary to know the exact azimuth direction of the assumed steering vector in advance, and other factors are based on the hypothetical perfect array model . Once the exact orientation of the steering vector can not be known, or there are other array model errors such as array geometry deviation, local scattering and other common errors, whether the above algorithms are still robust is an unknown problem.

In this paper, a new robust adaptive beamforming algorithm based on desired signal subspace complement is proposed. The algorithm only needs little prior information such as less accurate array shape and approximate arrival azimuth interval. It is not necessary to know the exact incoming wave steering vector like above algorithms. Simulation experiments show that the proposed algorithm requires very little prior information and is not only robust to steering vector mismatch but also robust to array errors such as array pitch errors, and is more robust than existing current several robust adaptive beamforming algorithms.

Array Signal Model

Let us consider an uniformly linear array composed of M elements and receive narrow-band signals, the received signal at time t is:

$$\mathrm{x(t)=x_s(t)+x_i(t)+n(t)}\,. \tag{1}$$

Where $\mathrm{x_s(t)}$, $\mathrm{x_i(t)}$ and $\mathrm{n(t)}$ are statistically independent of the desired signal, interference and noise Respectively. Here, $\mathrm{x_s(t)=s(t)a}$, $\mathrm{s(t)}$ represents the desired signal waveform and a is the actual steering vector of the desired signal.

So the output of the adaptive beamformer is:

$$\mathrm{y(t)} = \mathbf{w}^{\mathrm{H}}\mathrm{x(t)}\,. \tag{2}$$

Where $\mathbf{w} = [w_{1,}...w_M]^T \in C^M$ is the beamformer weight complex vector, $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$ represents the transpose and conjugate transpose of the matrix, respectively.

The MVDR criterion was first proposed by J. CAPON, so also called CAPON beamformer, it is described as:

$$\min_{w} \mathbf{w}^{\mathrm{H}}\mathbf{R_x}\mathbf{w}$$

$$\text{s.t.} \qquad \mathbf{w}^{\mathrm{H}}\mathrm{a} = 1$$

(3a)

(3b)

The solution given is:

$$\mathbf{w}_{capon} = \frac{\mathbf{R}_{i+n}^{-1}\mathrm{a}}{\mathrm{a}^{\mathrm{H}}\mathbf{R}_{i+n}^{-1}\mathrm{a}}\,. \tag{4}$$

where $\mathbf{R}_{i+n}$ is the interference plus noise covariance matrix which can not be obtained in the actual processing , so $\mathbf{R}_{i+n}$ is usually replaced by the estimated value which is the average of the sampling of $\mathrm{X(i)X^H(i)}$, here N is the number of snapshot, the estimated value of $\mathbf{R}_x$ is defined as:

$$\mathrm{R}_{\mathbf{x}} = \frac{1}{\mathrm{N}}\sum_{i=1}^{\mathrm{N}} \mathrm{X(i)X^H(i)}\,\cdot \tag{5}$$

The beamformer output signal to noise ratio is:

$$\mathrm{SINR}=\frac{s_{\mathrm{s}}^{2}\,|\mathbf{w}^{\mathrm{H}}\mathrm{a}|^{2}}{\mathbf{w}^{\mathrm{H}}\mathbf{R}_{i+n}\mathbf{w}}\,. \tag{6}$$

Where s_{s}^{2} is the desired signal power.

Proposed Algorithm

Algorithm Description. It is easy to know from the definition of the steering vector,for the vector $\hat{a}$ to be estimated,there is $\|\hat{a}\|^2=M$, M is the number of the array elements. In order to avoid the convergence of the estimates flowing into the range of interfering signals or the linear combination of interference, we also need to give additional constraints. Suppose the specific direction of the assumed steering vector $a(q)$ can not be obtained, can only be roughly known in sector region $\Theta=[q_{\min},q_{\max}]$ which Θ can be obtained by low resolution direction finding methods. Assuming that the signal contained in the sector region Θ and the interfering signal can be distinguished significantly and Θ do not contain any interfering signals, we can assume that the sector Θ is a neighborhood centered on the assumed desired signal direction. So we construct the matrix

$$C=\int_{\Theta} g(q)g^{\mathrm{H}}(q)dq\,. \tag{7}$$

$$\tilde{C} @ \int_{\tilde{\Theta}} d(\tilde{q})d^{H}(q)dq\,. \tag{8}$$

Where $\tilde{\Theta}$ is the complement of the sector region Θ, which contains the range of interfering signals other than the possibly desired signal. And we can get eigenvectors and eigenvalues of $\tilde{C}$ by eigen decomposition method , U and Λ denote the the diagonal matrix containing eigenvectors and eigenvalues respectively. The eigenvalues in Λ can be arranged in the descending order as: $l_i \geq l_{i+1}, i=1,\ldots,M$, and further can be written as $\tilde{C}=\mathrm{U}_1\Lambda_1\mathrm{U}_1^{\mathrm{H}}+\mathrm{U}_2\Lambda_2\mathrm{U}_2^{\mathrm{H}}$, Λ_1 here is a $K\times K$ diagonal matrix containing K major eigenvalues of matrix $\tilde{C}$, U_1 is a set containing its corresponding eigen vector. while Λ_2 is a $(M-K)\times(M-K)$ diagonal matrix containing（$M-K$）non-major eigenvalues and U_2 is its corresponding eigenvector.

Since the matrix $\tilde{C}$ is computed from the complementary region of the desired signal, it can be concluded that the steering vector of the desired signal in the sector Θ and its complement $\tilde{\Theta}$ can be approximated as a linear combination of U_1 and U_2 for the properly chosen K ,that is,

$$d(q)\cong \mathrm{U}_2 v_2, \qquad q\in\Theta \tag{9}$$

$$d(q)\cong \mathrm{U}_1 v_1, \qquad q\in\tilde{\Theta} \tag{10}$$

Where v_1 and v_2 are the coefficient vectors.From the above equation, it is easy to know $\|v_1\|^2=M$,$\|v_2\|^2=M$, so we can get the next equations easily:

$$\begin{aligned}\mathrm{d}^{\mathrm{H}}(q)\tilde{C}\mathrm{d}(q) &\cong (\mathrm{U}_1\mathrm{v}_1)^{\mathrm{H}}\tilde{C}(\mathrm{U}_1\mathrm{v}_1)\\ &=\mathrm{v}_1^{\mathrm{H}}\Lambda_1\mathrm{v}_1, \qquad q\in\tilde{\Theta}\end{aligned} \tag{11}$$

$$\begin{aligned} d^H(q)\tilde{C}d(q) &\cong (U_2v_2)^H\tilde{C}(U_2v_2) \\ &= v_2^H\Lambda_2v_2, \qquad \theta\in\Theta \end{aligned} \tag{12}$$

Where Λ_1 is a diagonal matrix containing the K major eigenvalues of the matrix $\tilde{C}$, Λ_2 is the matrix containing the remaining minor eigenvalues. Then the quadratic constraint $d^H(q)\tilde{C}d(q)$ represents the majority of the value outside the desired sector, and the estimate $\hat{a}$ of the desired steering vector can be given some constraint to ensure that the estimate $\hat{a}$ does not converge to the region where the interfering signals are located. Next we look at the distribution curve of the value $d^H(q)\tilde{C}d(q)$ using an example.

Assuming a 10-element half-wavelength uniform linear array(ULA), Fig. 1 shows the values of the quadratic form $d^H(q)\tilde{C}d(q)$ for the different angles, we can see that the value of $d^H(q)\tilde{C}d(q)$ is the smallest when q locates in the interval $\Theta=[0^{\circ},10^{\circ}]$; the value will become larger outside of the range. Therefore, if Δ_0 is selected to be equal to the maximum value of the term $d^H(q)\tilde{C}d(q)$ within the presumed angular sector Θ,the constraint (13)guarantees that the estimate of the desired signal steering vector does not converge to any of the interference steering vectors and their linear combinations, but only fall within the interval Θ.

Based on the above example, we give constraints on the estimated vector $\hat{a}$,

$$\hat{a}^H\tilde{C}\hat{a}\le\Delta_0 \tag{13}$$

Where Δ_0 is a uniquely selected value for a given angular sector Θ, that is,

$$\Delta_0 \triangleq \max_{\theta\in\Theta} d^H(\theta)\tilde{C}d(\theta) \tag{14}$$

Using the definition of Δ_0 (14) together with (12), we can find that,

$$\Delta_0 \triangleq \max_{\theta\in\Theta} v^H(\theta)\Lambda_2v(\theta)\le M\lambda_{K+1} \tag{15}$$

Taking into account the normalization constraint and the constraint (13), the problem of estimating the desired signal steering vector based on the knowledge of the sector Θ can be formulated as the following optimization problem，

$$\min_{\hat{a}}\ \hat{a}^H\hat{R}^{-1}\hat{a} \tag{16a}$$

$$\text{subject to}\ \ ||\hat{a}||^2=M \tag{16b}$$

$$\hat{a}^H\tilde{C}\hat{a}\le\Delta_0 \tag{16c}$$

Compared to other MVDR RAB methods,which require the knowledge of the presumed steering vector, array geometry, propagation media, and signal source characteristics. The proposed algorithm only requires less accurate array geometry and approximate knowledge of the angular sector Θ, so the proposed algorithm requires less prior information.

Steering Estimation via Convex Optimization. The first step is to make sure the constraint (16) is feasible. It is easy to prove the constraint (16) is feasible if and only if Δ_0/M is greater than or equal to the smallest eigenvalue of the matrix $\tilde{\mathbf{C}}$.Indeed, if the smallest eigenvalue of the matrix $\tilde{\mathbf{C}}$ is larger than Δ_0 / M ,then the constraint (16c) can not be satisfied for for any $\hat{\mathrm{a}}$.

If the problem (16) is feasible, the equation $\hat{\mathrm{a}}^H \hat{\mathrm{R}}^{-1} \hat{\mathrm{a}} = \mathrm{Tr}(\hat{\mathrm{R}}^{-1} \hat{\mathrm{a}} \hat{\mathrm{a}}^H)$ and $\hat{\mathrm{a}}^H \tilde{\mathbf{C}} \hat{\mathrm{a}} = \mathrm{Tr}(\tilde{\mathbf{C}} \hat{\mathrm{a}} \hat{\mathrm{a}}^H)$ can be used to rewrite it as

$$\min_{\hat{\mathrm{a}}} \quad Tr(\hat{\mathrm{R}}^{-1} \hat{\mathrm{a}} \hat{\mathrm{a}}^{\mathrm{H}}) \tag{17a}$$

$$\text{subject to} \quad Tr(\hat{\mathrm{a}} \hat{\mathrm{a}}^{\mathrm{H}}) = M \tag{17b}$$

$$Tr(\tilde{\mathbf{C}} \hat{\mathrm{a}} \hat{\mathrm{a}}^{\mathrm{H}}) \le \Delta_0 \tag{17c}$$

Introducing the following positive semi-definite matrix variable $\mathrm{A} \triangleq \hat{\mathrm{a}} \hat{\mathrm{a}}^{\mathrm{H}}, \mathrm{A} \succeq 0$,the problem (17) can be recast as

$$\min_{\mathrm{A}} \quad Tr(\hat{\mathrm{R}}^{-1} \mathrm{A}) \tag{18a}$$

$$\text{subject to} \quad Tr(\mathrm{A}) = M \tag{18b}$$

$$Tr(\tilde{\mathbf{C}} \mathrm{A}) \le \Delta_0 \tag{18c}$$

$$rank(\mathrm{A}) = 1 \tag{18d}$$

Where $rank(\cdot)$ stands for the rank of a matrix, the only non-convex constraint in (18) is the rank-one constraint, the others are linear in A , using the semi-definite programming (SDP) relaxation technique [14]. The relaxed problem can be obtained by dropping the non-convex rank-one constraint (18d) and requiring that $\mathrm{A} \succeq 0$,Thus the (18) can be transformed into the following convex problem.

$$\min_{\mathrm{A}} \quad Tr(\hat{\mathrm{R}}^{-1} \mathrm{A}) \tag{19a}$$

$$\text{subject to} \quad Tr(\mathrm{A}) = M \tag{19b}$$

$$Tr(\tilde{\mathbf{C}} \mathrm{A}) \le \Delta_0 \tag{19c}$$

$$\mathrm{A} \succeq 0 \tag{19d}$$

The equalities (19) are convex which can be solved by MATLAB CVX toolbox efficiently.

Simulation Results

Let us consider a ULA of 10 omni-directional antenna elements with a half-wavelength spacing, the two interfering sources are assumed to impinge on the antenna from the directions 30° and 50°, and the desired signal direction is assumed to be $\theta_{\mathrm{p}} = 3^{\circ}$. In all simulation examples, the interference-to-noise ratio(INR) equals 30dB and the desired signal is always present in the training data. The experimental results were obtained from 100 independent Monte Carlo experiments.

The proposed beamformer is compared with the following four methods in terms of the output SINR:(i) the eigenspace-based beamformer(ESB) of [4], (ii) the worst-case performance optimization beamforming method; (iii) the mismatch error orthogonal decomposition method (EOD)[13], (iv) the diagonally loaded SMI beamformer(LSMI) [3]. Among them, the algorithm proposed in this paper and the EOD algorithm in [13] , the angular sector Θ are assumed to be $\Theta = [\theta_{\mathrm{p}} - 5^{\circ}, \theta_{\mathrm{p}} + 5^{\circ}]$, and the MATLAB CVX toolbox is used to solve the convex optimization of the algorithm. The value $\delta = 0.1$ in the EOD algorithm is used and the value $\varepsilon = 0.3M$ in the

Worst-Case based beamformer is used, the diagonal loading level is set to twice the noise power (recommended by Cox in [7]).

Simulation Example 1: Desired Signal Steering Vector Mismatch Due to Wavefront Distortion. We consider the situation that the waveform propagates in an inhomogeneous medium, the phase increment accumulation is caused by the steering vector mismatch. It is assumed that the phase increments are held constant in each Monte Carlo experiment and randomly generated from the Gaussian distributed random number generator with zero mean and standard 0.04 . Fig. 2 shows the relationship between the output SINR and SNR, where the number of snapshots is 30.

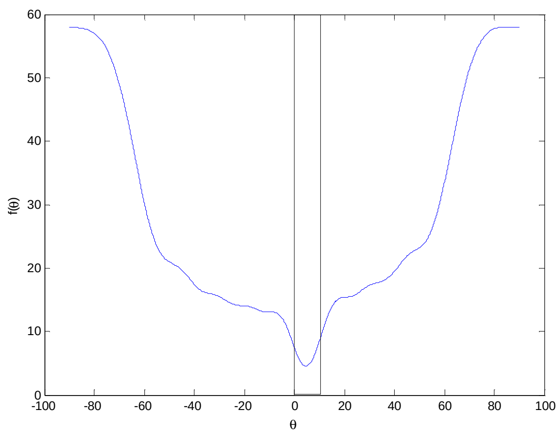

Fig.1 Values of term $f(q) = \mathrm{d}^{\mathrm{H}}(\theta)\tilde{C}\mathrm{d}(\theta)$ for different angles

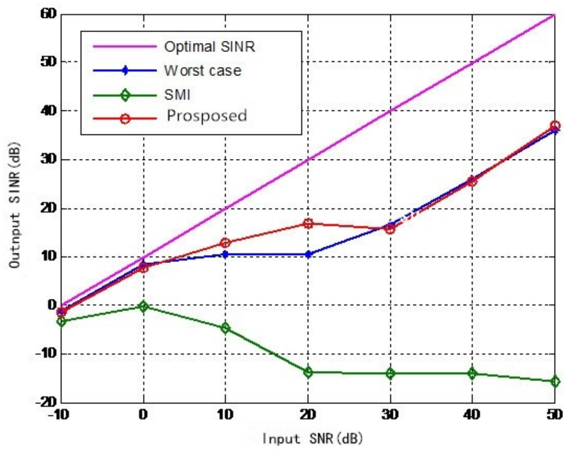

Fig.2 Output SINR versus SNR for training data size of K=30 and INR=30dB

It can be seen that the proposed algorithm has better performance than the Worst-Case method in the low signal-to-noise ratio range. However, when the signal-to-noise ratio is greater than the interference-to-noise ratio, the performance tends to be the same. In fact, this also ensures that the estimate of the desired signal does not converge to the interfering signal interval

Simulation Example 2: Effect of The Error in The Knowledge of The Antenna Array Geometry. In this example, we study how the elements spacing error affect the proposed beamformer performance. It is assumed that the elements are undisturbed on a straight line, but the pitch is not uniform half-wavelength but errors occur, the errors are randomly distributed between the interval $[-0.05l, 0.05l]$, which will lead to the the steering vector mismatch of the desired signal. Figs.3 and 4 depict the output SINR curves (SNR = 20dB, INR = 30dB) versus the number of training snapshots and versus the different SNR (snapshots K = 30, INR = 30dB), respectively. It can be seen that the proposed algorithm has better performance even if there is an error in the knowledge of antenna array geometry.

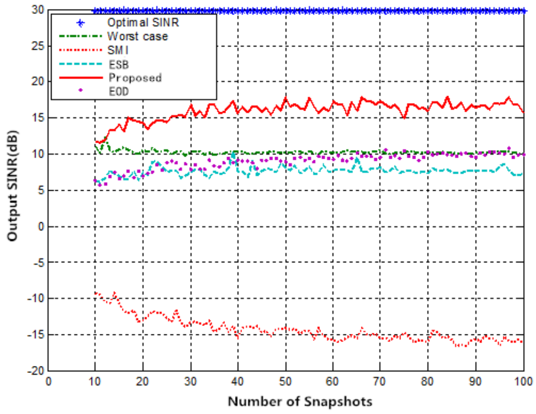

Fig.3 Output SINR versus training sample size K for fixed SNR=20 dB and INR=30 for the case of perturbations in antenna array geometry

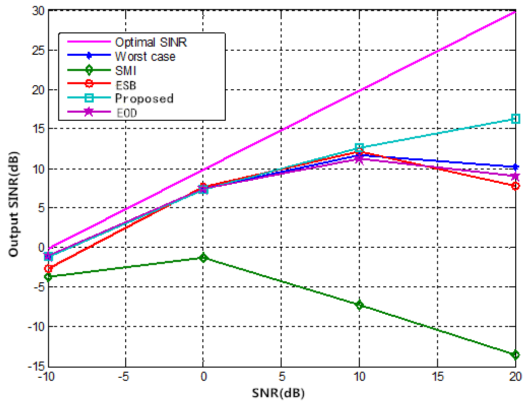

Fig.4 Output SINR versus SNR for training data size of K=30 and INR=30dB for the case of perturbations in antenna array geometry

Conclusions

The robustness of adaptive beamforming is discussed in this paper when there is little prior information. An adaptive beamforming algorithm based on the desired signal subspace complement is proposed in this paper. The algorithm is described in detail, and the solution based on convex optimization is given after the mathematical equivalence conversion, thus the rationality of the solution is guaranteed theoretically. The simulation experiments verifies the effectiveness of the algorithm, the presented algorithm is robust to many types of array errors, and has better performance compared to current algorithms.

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