

# The Simulation of Tire Dynamic Performance Based on “Magic Formula”

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**Abstract.** Based on “Magic Formula”, the tire dynamics simulation model was established in the Matlab/Simulink, contraposed many conditions like braking, steering and steering-braking combination. The main load of tire and the wheel aligning torque were simulated, then got the relationship curve between the longitudinal force/lateral force and slip rate and the relationship curve between the wheel aligning torque and the side slip angle in corresponding conditions. The research results show that the tire dynamic characteristic curve is in good agreement with the actual situation, which not only verifies the correctness of the tire model, but also lays the foundation for the subsequent vehicle dynamics simulation based on the tire model.

## Introduction

Tire, the transfer components between the road and the vehicle, has a great impact on the vehicle's handling stability, and both vehicle motion and active safety control are achieved through the control of tire force, so the simulation accuracy of the full vehicle's dynamics characteristic is directly influenced by the accuracy of tire model<sup>[1]</sup>.

Theoretical model and empirical-semi-empirical model are the two main construction methods of the tire models<sup>[2]</sup>. Generally speaking, theoretical model uses specific analytical expressions to represent the mechanical properties of tires, but it get complex form, difficult calculation and poor accuracy, which partly limit its application. Compared with the theoretical model, through the analysis of abundant test data which reflect the tire strength characteristics. The empirical model or semi-empirical model can effectively expresses the mechanical properties between ground and tire by using a mathematical formula which includes fitting parameters. Empirical-semi-empirical model is broader used due to its higher accuracy.

As one of the semi-empirical models, "Magic Formula" tire model is widely recognized as the most outstanding tire-computing model in the simulation analysis of vehicle handling stability for its good calculation accuracy<sup>[3]</sup>. For the "Magic Formula" model applied better during the analysis of vehicle handling stability, a tire dynamic simulation model is established in Matlab / Simulink based on "Magic Formula" model, and the simulation of braking, turning and turning braking combination conditions are carried out, turning out the tire dynamic characteristic curves are in good agreement with the actual situation.

## Tire model

The concrete form of the “Magic Formula” shows in Eq. (1). On the basis of it, the force and torque of the tire can be obtained by the fitting calculation from the experiment parameters. Each fitting parameter is shown in Table 1.

$$Y = D \sin\{C \arctan[Bx - E(Bx - \arctan Bx)]\} + S_v \quad (1)$$

Where  $Y$  are respectively the  $F_x$  (lateral force),  $F_y$  (longitudinal force), and  $M_z$  (aligning torque),  $x$  represents the side slip angle or longitudinal slip ratio according to different environments,  $D$  = the

crest factor,  $B$  = the curve origin slope,  $E$  = the curve form factor,  $C$  = the shape characteristic factor,  $S_h, S_v$  are respectively the Horizontal drift and the Vertical drift.

The “Magic Formula” uses combination formula of trigonometric function to fit the experimental data of the tire, and it perfectly expresses the tire's longitudinal force  $F_x$ , lateral force  $F_y$ , aligning torque  $M_z$ , turning torque  $M_x$ , resistance torque  $M_y$  and the combined effect condition of longitudinal force and lateral force<sup>[4,5]</sup> with the same formula. In this formula, the relation between input variables and output variables of the tire is shown in Fig.1.

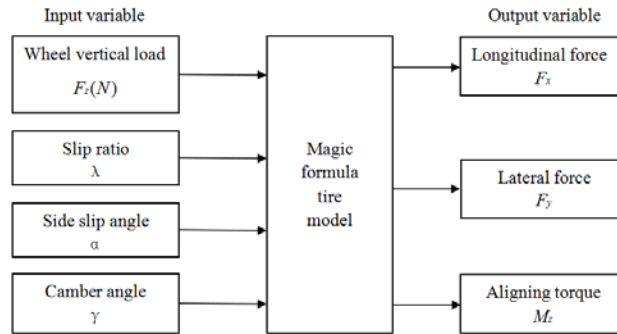


Fig. 1. Diagram of tire relationships

### The longitudinal force of the steady-state braking environment

The slip ratio has a direct influence on the angular speed of the tire while the vehicle is moving. The angular speed also changes under the same driving force or brake force<sup>[5]</sup>. With Eq. (1), the relationship between tire longitudinal force and vertical load and slip ratio is shown as follows.

$$F_{x0} = D_x \sin \{ C_x \arctan [ B_x \lambda - E_x (B_x \lambda - \arctan B_x \lambda) ] \} \quad (2)$$

Where  $C_x = 1.65$ ;  $D_x = a_1 F_z^2 + a_2 F_z$ ,  $B_x = B_x C_x D_x / C_x D_x$ ;  $E_x = a_6 F_z^2 + a_7 F_z + a_8$ ,  $B_x C_x D_x = (a_3 F_z^2 + a_4 F_z) \times e^{a_5 F_z}$ ;  $a_1, \dots, a_8$  are the fitting parameters shown in Table 1.

### The lateral force of the steady-state steering environment

With Eq. (1), the relationships between lateral force  $F_{y0}$  and side slip angle and vertical load  $F_z$  are shown as follows.

$$\begin{cases} F_{y0} = D_y \sin \{ C_y \arctan [ B_y x - E_y (B_y x - \arctan B_y x) ] \} + S_v \\ x = \alpha + S_h \end{cases} \quad (3)$$

Where  $C_y = 1.3$ ;  $D_y = a_1 F_z^2 + a_2 F_z$ ;  $B_y C_y D_y = a_3 \sin [a_4 \arctan (a_5 F_z)] (1 - a_{12} |\gamma|)$ ;  $B_y = B_y C_y D_y / C_y D_y$ ;  $E_y = a_6 F_z^2 + a_7 F_z + a_8$ ;  $S_h = a_9 \gamma$ ;  $S_v = (a_{10} F_z^2 + a_{11} F_z) \gamma$ ;  $\gamma$  = the vehicle camber angle;  $a_1, \dots, a_8$  are fitting parameters. Ignoring the effects of  $S_h$  and  $S_v$ , all fitting parameters are shown in Table 1.

### Steering-braking in the environment of swerving with braking

According to 1.1 and 1.2, the tire force is solved in each single working condition, which turns out the relations among different tire forces in the combined condition of turning and braking are as follows.

$$\begin{cases} F_x = \frac{\sigma_x}{\sigma} F_{x0} \\ F_y = \frac{\sigma_y}{\sigma} F_{y0} \end{cases} \quad (4)$$

$$\text{Where } \sigma = \sqrt{\sigma_x^2 + \sigma_y^2}; \sigma_x = -\frac{\lambda}{1 + \lambda}; \sigma_y = -\frac{\tan \alpha}{1 + \lambda}$$

### Aligning torque

On the basis of Eq.1, the mathematical model of tire aligning torque can be represented as follows.

$$\begin{cases} M_z = D_z \sin \{ C_z \arctan [ B_z x - E_z ( B_z x - \arctan B_z x ) ] \} + S_v \\ x = \alpha + S_h \end{cases} \quad (5)$$

Where  $C_z = 2.4$ ;  $D_z = a_1 F_z^2 + a_2 F_z$ ;  $B_z C_z D_z = (a_3 F_z^2 + a_4 F_z) \times (1 - a_{12} |\gamma|) e^{a_5 F_z}$ ;  $B_z = B_z C_z D_z / C_z D_z$ ;  $S_h = a_9 \gamma$ ;  $S_v = (a_{10} F_z^2 + a_{11} F_z) \gamma$ ;  $E_z = a_6 F_z^2 + a_7 F_z + a_8$ .

Table. 1. The fitting parameters

|       | $a_1$ | $a_2$ | $a_3$    | $a_4$    | $a_5$    | $a_6$    | $a_7$  |
|-------|-------|-------|----------|----------|----------|----------|--------|
| $F_x$ | -21.3 | 1144  | 49.6     | 226      | 0.069    | -0.006   | 0.056  |
| $F_y$ | -22.1 | 1011  | 1078     | 1.82     | 0.208    | 0        | -0.354 |
| $M_z$ | -2.72 | -2.28 | -1.86    | -2.73    | 0.11     | -0.07    | 0.643  |
|       | $a_8$ | $a_9$ | $a_{10}$ | $a_{11}$ | $a_{12}$ | $a_{13}$ |        |
| $F_x$ | 0.486 | -     | -        | -        | -        | -        |        |
| $F_y$ | 0.707 | 0.028 | 0        | 14.8     | 0.022    | -        |        |
| $M_z$ | -4.04 | 0.015 | 0.066    | 0.945    | 0.03     | 0.07     |        |

### Tire dynamics model on the basis of Simulink

According to Eq.(2) to (5), the tire model and aligning torque model in the combined working condition are built based on the MATLAB/Simulink, which is as shown in Fig.2 and Fig.3.

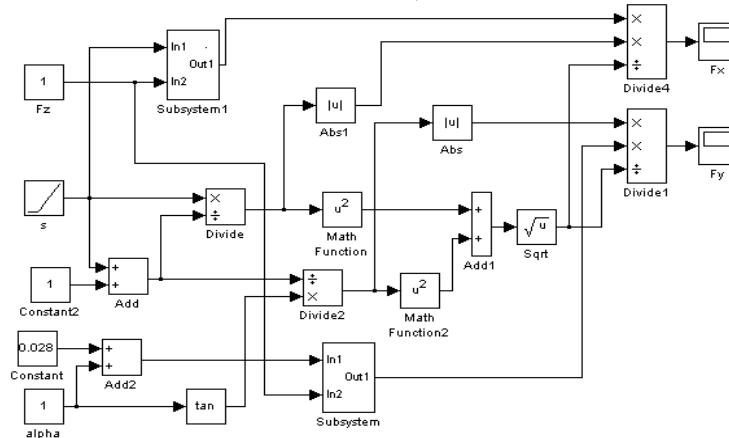


Fig. 2. Tire simulation model under the combined working condition

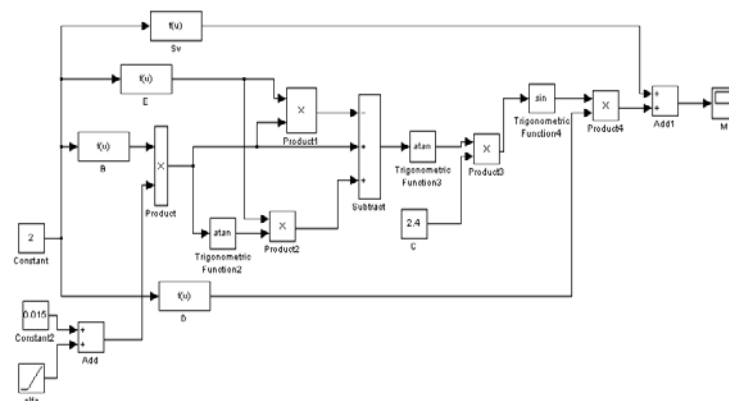


Fig. 3. Aligning torque computational model

### Simulation analysis

In order to clearly show the relationship between the tire force / torque and slip rate, side slip angle, the vertical loads are respectively set up as 1KN, 3KN, 5KN, 7KN to simulate the tire models in various working conditions, and the curves got from simulation are as shown from Fig.4 to Fig.7.

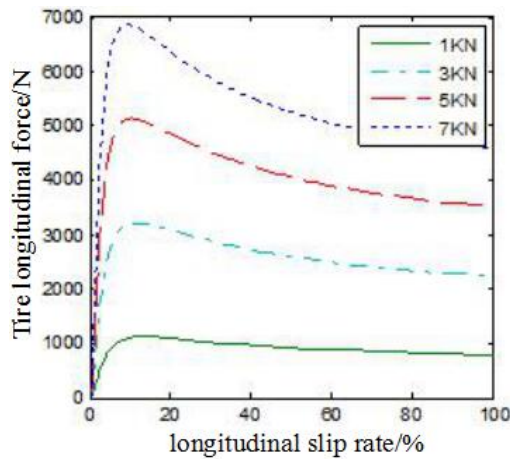


Fig. 4. Longitudinal force- Slip rate curve

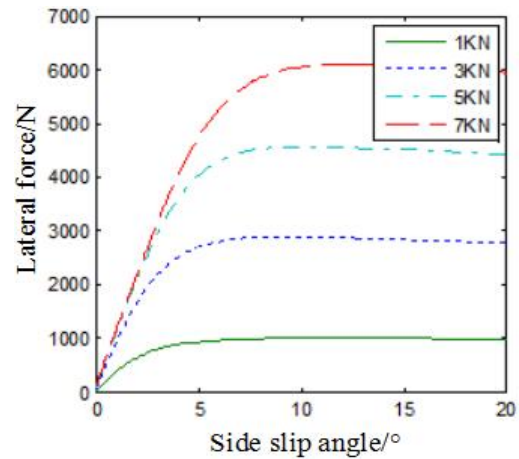


Fig. 5. Lateral force- Slip rate curve

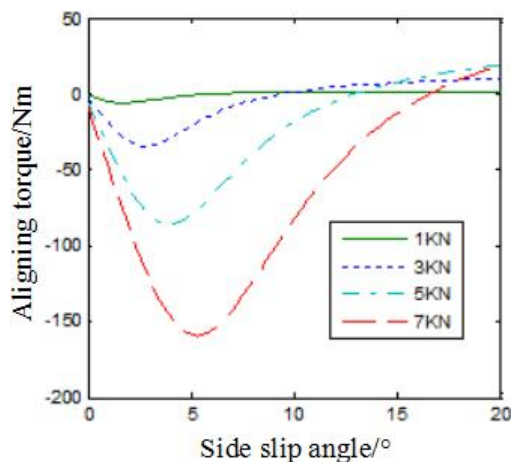


Fig. 6. Aligning torque- Side slip angle curve

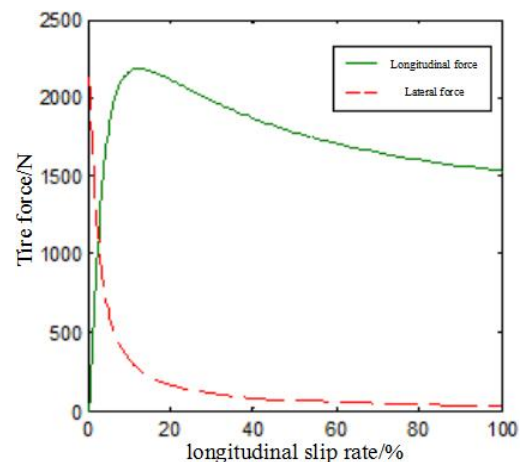


Fig. 7. Longitudinal force & Lateral force- Slip rate curve

Fig.4 shows the response curves of the tire longitudinal force in pure braking working condition. With the slip rate changes gradually, longitudinal force increases sharply. The longitudinal force reaches the peak when slip rate is about 10%, and then it decreases gradually when it beyond 10%. It concluded that the braking performance of the wheel is best when the slip rate is 10%. Fig.5 shows the response curves of the lateral force in pure steering working condition. It is observed that the lateral force increases linearly when the side slip angle in  $0\sim 5^\circ$ , and then the lateral force changes nonlinearly when the side slip rate is beyond  $0\sim 5^\circ$ . Fig.6 shows the response curves of aligning torque. It is visible the aligning torque reaches the peak when the side slip angle is about  $5^\circ$ , and with the increasing of side slip angle, the aligning torque decreases gradually, and it tends to zero finally. Generally, the influence of the aligning torque will not be taken into consideration during the analysis of the vehicle dynamics. Fig.7 shows the relationship between the lateral force/ longitudinal force and the side slip ratio in the combined working condition of turning and braking. It is visible with the increasing of side slip ratio, the tire longitudinal force changes slightly, but the lateral force changes greatly. From the above analysis, the tire model simulation of the force and torque changes have a high degree of consistency with the actual situation, which verifies the correctness of the tire model.

## Conclusion

The tire nonlinear characteristic has an important effect on the steering characteristic and the vehicle riding stability. Therefore, it is necessary to study the tire dynamics. This paper carried on the simulation modeling to tires based on “Magic Formula”. The models built in this paper can show a variety of tire states changes such as load, side slip, roll and longitudinal slip rate, and it also can effectively analyze the tire mechanics characteristics for any loads, any tire movement states.

Through the analysis of the simulation results, it can be seen the tire dynamics model based on "Magic Formula" exhibits a high fitting accuracy for the longitudinal force, the lateral force and the aligning torque, which can fully reflect the vehicle tire dynamic characteristics. It also can be a reference in future study.

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