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Chaotic sequences generated by chaotic quantification under finite precision

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Finite precision is the first problem to be solved when chaos is applied to practical communication systems. In this paper, the influence of calculation precision on the properties of chaotic sequences is studied, and some valuable results are given. And a new method of chaotic quantification aiming to solve finite precision problem is introduced, the behavior of the sequences generated by this method being further studied and the advantages of this method being shown through some analysis and simulation results.

Keywords: Chaotic Sequence; Finite Precision; Chaotic Quantification.

1. Introduction

Chaos is a complicated phenomenon of nonlinear dynamic system with the properties of pseudorandom and high sensitivity to initial value[1], and it is widely used in communication, signal processing and so on[2-7]. But when applied to practical systems, chaos will be periodic after several iterations because of the finite precision of the hardware, which is called finite precision problem[8]. Since it is evitable in chaotic application, some research about the influence of calculation precision on properties of chaotic sequences based on logistic map is done in this paper. And a new method of chaotic quantification is introduced aiming to solve this problem and get better properties[9]. The behavior of sequences generated by this method is further studied. And some comparisons of chaotic quantization with the previous method multilevel quantification[8] are also given in this paper to show the advantages of this method through analysis and some simulation results.

2. Influence of Calculation Precision to Chaotic Sequences

Logistic map is a widely used one-dimensional chaotic map, and it is formulated as follows[1],

$$x_{k+1} = 4x_k(1-x_k) \qquad 0 < x_k < 1 \tag{1}$$

and its possibility density function is

$$\rho(x) = \begin{cases} \dfrac{1}{\pi\sqrt{x(1-x)}} & x \in (0,1) \\ 0 & \text{others} \end{cases} \tag{2}$$

Under finite precision, we can modify the logistic map to

$$\begin{cases} y_k = \lceil x_k \rceil_{bit\_num} \\ x_{k+1} = 4y_k(1-y_k) \end{cases} \qquad 0 < x_k < 1 \tag{3}$$

Here, $\lceil\ \rceil$ transform a decimal fraction to *bit_num* bits precision with floor operation.
Then, we can see the influence of calculation precision on the period of chaos from the normalized auto-correlation of sequences generated with infinite precision, 16 bits finite precision and 20 bits finite precision in fig.1.
Choose 10 initial values distributed symmetrically in (0,1), set calculation precision as from 4 to 32, generate chaotic sequences of 10000 length separately after discarding the first 10000 points. Then plot all these points in fig.2, it can be found that with the increase of bit precision, the chaotic sequences change from short periodic state to long periodic state till to nonperiodic state in some zone.

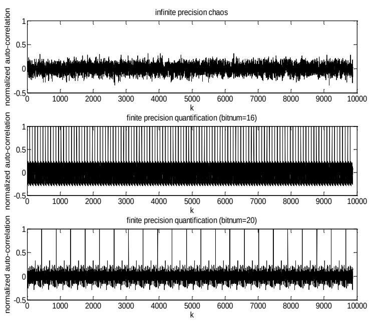

Fig. 1 Normalized auto-correlation

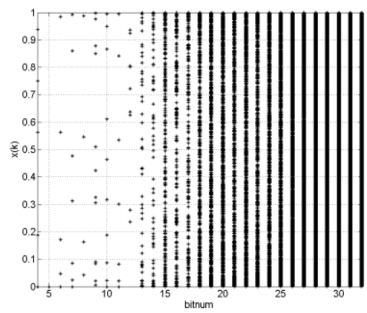

Fig. 2 Track points with different precision

3. Chaotic Quantification Method

To solve the finite precision problem described above, a new method of chaotic quantification method is introduced[9].

First, with the initial value x_{0a}, generate chaotic real sequence $\{x_a(i):i=1,2,\ldots\}$ using formula (3). And then, transform $x_a(i)$ to $x_{aT}(i)$.

$$x_{aT}(i) = b_0(i)b_1(i)\cdots b_{bit\_num-1}(i) \tag{4}$$

With another initial value x_{0b}, also generate chaotic real sequence $\{x_b(i):i=1,2,\ldots\}$ based on formula (3), use another method to generate $x_{bT}(i)$. The possibility distribution function of Logistic map can be deduced from (2):

$$F(x) = \int_0^x \rho(t)dt \overset{t=\frac{1+\sin\theta}{2}}{=} \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\arcsin(2x-1)} d\theta = \frac{1}{\pi}[\arcsin(2x-1) + \frac{\pi}{2}], \quad x \in (0,1) \tag{5}$$

Its inverse function is

$$F^{-1}(x) = \frac{\sin[\pi(x-0.5)]+1}{2}, x \in (0,1) \tag{6}$$

To make $x_{bT}(i)$ distribute symmetrically, let $x=k/bit\_num$, $k=0,1,\ldots,bit\_num$ in (6), we can get bit_num quantization zone in (0,1):

$$\{q_b(i) = k \mid F^{-1}(\frac{k}{bit\_num}) \le x_b(i) < F^{-1}(\frac{k+1}{bit\_num})\}, \quad k = 0,1,\cdots,bit\_num-1 \tag{7}$$

The possibility of $x_b(i)$ to occur in each zone is $1/bit\_num$. According to these zones, $x_b(i)$ can be quantized into integer sequence$\{q_b(i):i=0,1,\ldots\}$, $q_b(i) \in \{0,1,\ldots, bit\_num\}$ with symmetrical distribution.

The final chaotic sequences $\{c(i)\}$ are generated by abstracting one bit from $x_{aT}(i)$ according to $q_b(i)$, that is to say, for each i, if $q_b(i)=k$, $k\in\{0,1,\ldots, bit\_num-1\}$, then chose the bit k of $x_{aT}(i)$ as $c(i)$.

4. Period of Chaotic Sequences

Let P1 be the period of {xa(i)}, and if use single-level quantization, the chaotic sequences generated will still has a period of P1.Let P2 be the period of {qb(i)}, and when use the method of chaotic quantization, it is easy to know that the period of the chaotic sequences is the lease common multiple of P1 and P2. So, the chaotic quantization can lengthen the period of the original sequences, and solve the finite precision to some extent.

For practical use, after further study of finite precision behavior of chaos, we have found that with the increasing of bitnum, the period of chaotic sequences generated gets longer by and large, while the actual period of a chaotic sequence not only depends on the bitnum, but also depends on the initial value. We have summarized the period of different bitnum as table.1, here, the initial values are chosen from (0, 1) one value each 1/101 symmetrically, totally 100 values.

Tab. 1 Period of different bitnum

Bit_num	Min_period	More_period	Times/100	Max_period
16	18	79	72	119
17	8	199	84	199
18	588	588	100	588
19	16	656	75	656
20	125	451	61	451
21	49	256	52	393
22	7	931	72	931
23	771	771	100	771
24	272	272	68	993
25	898	898	74	1109
26	558	912	45	2271
27	433	4128	85	4128
28	130	13404	96	13404
29	379	6876	96	6876
30	370	12472	67	12472
31	10972	10972	55	14575
32	1046	18675	97	18675
33	472	10952	95	11284

From table.1 it can be seen that we can choose proper bitnum for $x_a(i)$ and $x_b(i)$, and the period of sequences generated using chaotic quantification will be quite prolonged. For example, if we choose the precision of $x_a(i)$ to be 16 and the precision of $x_b(i)$ to be 17, then the P1 could be 79 and P2 could be 199, finally the period will be the lease common multiple of P1 and P2, which is the value of 15721. Of course, there exist many different combinations of choice in chaotic quantification. So long as we choose proper bitnum and proper initial value with table.1, we can get more advantages of chaotic quantification.

5. Property Advantages of Sequences Generated by Chaotic Quantification

It has been studied that based on Logistic map, use multilevel quantization and chaotic quantization to generate 50 groups of chaotic sequences with different initial values distributing in (0, 1) and different length from 1024 to 102400 separately with the calculation precision of the same computer, and compare their properties of balance and correlation as follows[9].

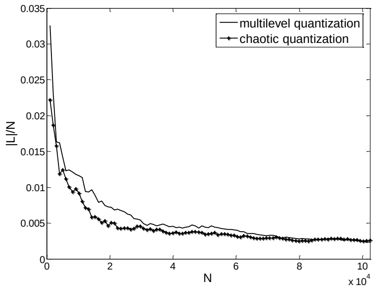

Fig. 3 Property of balance

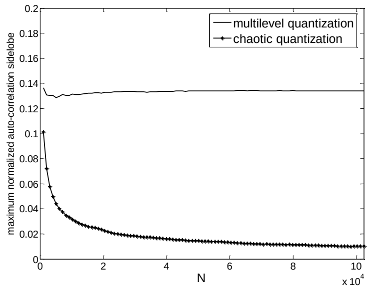

Fig. 4 Property of normalized auto-correlation

The results in Fig 3, 4 and 5 show that with increasing the length of sequence the properties of balance and correlation become better, and the sequences generated by chaotic quantization have better properties of balance and correlation.

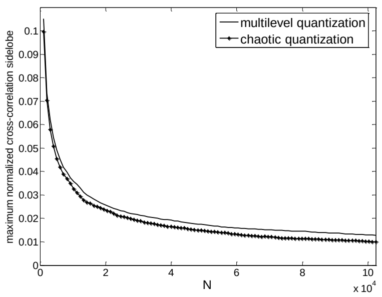

Fig. 5 Property of normalized cross-correlation

6. Conclusion

Finite precision problem is the key problem in chaotic application. In this paper, we have done some research work on the influence of calculation precision to chaotic sequences, and we proposal a new chaotic quantification method to generate chaotic sequences, which can not only solve the finite precision but also get better balance and correlation properties than multilevel quantification which is chose to solve finite precision problem before. So, this method is suitable for chaotic practical use especially for communication application and it will have bright future.

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