

A novel solving ambiguity algorithm for uniform circular phase interferometer

Kai-Qiang Ren and Zheng-Bo Sun

*National Key Laboratory of Science and Technology on Blind Signal Processing,
Chengdu, 610041, P.R China
E-mail: rkq308597871@126.com*

The direction finding accuracy of phase interferometer is severely impaired in the presence of phase ambiguity. In this paper, a novel solving ambiguity algorithm is proposed to solve this problem, which is suitable for uniform circular array with arbitrary number of sensors without the need of unambiguous baseline. The basic idea of the proposed algorithm is searching for an optimal solution within the range of ambiguity number. Several computer simulations are carried out to illustrate the correctness and validity of the proposed algorithm.

Keywords: Phase Ambiguity; Uniform Circular Array; Solving Ambiguity.

1. Introduction

Interferometer direction finding is widely used in the field of passive electronic reconnaissance due to the advantage of high sensitivity and good real-time performance. Interferometer generally estimates DOA (direction-of-arrival) based on phase difference between antenna array. Phase ambiguity appears when the phase difference is beyond its cycle 2π . Phase ambiguity means multiple DOA estimations correspond to one real signal direction, which may cause invalid DOA estimation[1-2]. Therefore, Phase ambiguity is the key problem that must be solved in interferometer direction finding.

In recent decades, a variety of solving ambiguity algorithms have been proposed. At present, the most used solving ambiguity algorithms include long-short baseline algorithm, stagger baseline algorithm, virtual baseline algorithm and unambiguous long-baseline algorithm[3-5]. All of the above algorithms need an unambiguous baseline to solve the ambiguity of other baseline in the antenna array, which limits the application of these algorithms in practice. As a result, algorithm solving the ambiguity without the need of unambiguous baseline is a problem worth studying. This paper proposes a novel solving ambiguity algorithm for UCA (uniform circular array) with arbitrary number of

sensors. The algorithm proposed still has high DF (direction-finding) accuracy and high solving ambiguity probability on the condition of low SNR and serious phase ambiguity.

2. Problem Formulation

Assume that a far-field narrowband signal impinges on an UCA of N sensors from the azimuth angle α and pitching angle β . The radius of UCA is r .

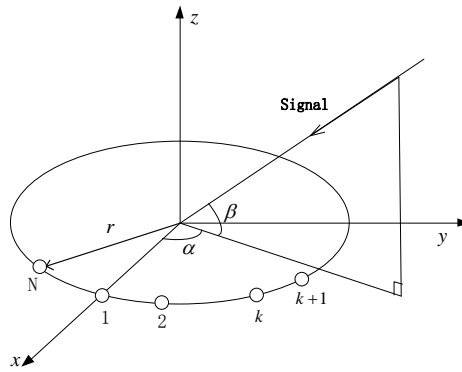


Fig. 1 Illustration of signal impinging on N sensors uniform circular antenna array

The output of the k th sensor can be written as

$$x_k(t) = s(t - \tau_k) + n_k(t) \quad (1)$$

Where $s(t)$ is the signal received by the center of UCA, $n_i(t)$ is additive white Gaussian noise with zero mean and σ_N^2 variance in the i th sensor, τ_k is time delay of the k th sensor relative to the center of UCA. According to the propagation characteristics of electromagnetic wave and the geometry of array, τ_k can be given by

$$\tau_k = \frac{r}{c} \cos \beta \cos \left(\alpha - \frac{2\pi(k-1)}{N} \right) \quad (2)$$

Thus the phase difference between any two adjacent sensors can be written as

$$\begin{aligned} \varphi_{k+1,k} &= \varphi_{k+1} - \varphi_k = (-2\pi f \tau_{k+1}) - (-2\pi f \tau_k) \\ &= \frac{4\pi r}{\lambda} \cos \beta \sin \left(\alpha - \frac{\pi}{N} (2k-1) \right) \sin \frac{\pi}{N} \end{aligned} \quad (3)$$

Because the cycle of phase difference is 2π , we have the following expression

$$\varphi_{k+1,k} = \widehat{\varphi}_{k+1,k} + 2p\pi \quad (4)$$

Where $\widehat{\varphi}_{k+1,k}$ is the measurement of $\varphi_{k+1,k}$, p is the ambiguity number of phase difference. It is obvious that p is a integer. Considering the fact that

$$\widehat{\varphi}_{k+1,k} \in (-\pi, \pi] \quad (5)$$

Substituting Eq. (3) and Eq. (4) into Eq. (5), we can obtain the following formula

$$-\pi < \frac{4\pi r}{\lambda} \cos \beta \sin \left(\alpha - \frac{\pi}{N} (2k-1) \right) \sin \frac{\pi}{N} + 2p\pi \leq \pi \quad (6)$$

Considering that $\alpha \in [-\pi, \pi)$ and $\beta \in [0, \pi/2]$, the range of p can be obtained from Eq. (6) as following

$$-\frac{1}{2} - \frac{2r}{\lambda} \sin \frac{\pi}{N} < p \leq \frac{1}{2} + \frac{2r}{\lambda} \sin \frac{\pi}{N} \quad (7)$$

Thus the aim of solving ambiguity algorithm is to estimate p and estimate accurate DOA of the signal further.

3. Algorithm for Solving Ambiguity

3.1. DOA estimation algorithm

In order to estimate DOA of the signal, the following formulas are defined

$$\begin{aligned} \Sigma_{k_1 k_2} &= \varphi_{k_1+1, k_1} + \varphi_{k_2+1, k_2} \\ &= \frac{8\pi r}{\lambda} \cos \beta \sin \frac{\pi}{N} \sin \left(\alpha - \frac{\pi}{N} (k_1 + k_2 - 1) \right) \cos \left(\frac{\pi}{N} (k_2 - k_1) \right) \end{aligned} \quad (8)$$

$$\begin{aligned} \Delta_{k_1 k_2} &= \varphi_{k_1+1, k_1} - \varphi_{k_2+1, k_2} \\ &= \frac{8\pi r}{\lambda} \cos \beta \sin \frac{\pi}{N} \cos \left(\alpha - \frac{\pi}{N} (k_1 + k_2 - 1) \right) \sin \left(\frac{\pi}{N} (k_2 - k_1) \right) \end{aligned} \quad (9)$$

By the comparison of Eq. (8) and Eq. (9), we can further define

$$\begin{aligned} Z &= \left(\frac{\Delta_{k_1 k_2}}{\rho \sin \left(\frac{\pi}{N} (k_2 - k_1) \right)} + j \frac{\Sigma_{k_1 k_2}}{\rho \cos \left(\frac{\pi}{N} (k_2 - k_1) \right)} \right) e^{j \frac{\pi}{N} (k_1 + k_2 - 1)} \\ &= \cos \beta e^{j\alpha} \end{aligned} \quad (10)$$

where

$$\rho = \frac{8\pi r}{\lambda} \sin \frac{\pi}{N} \quad (11)$$

Thus DOA of the signal can be estimated as following

$$\begin{aligned} \hat{\alpha} &= \arg \{Z\} \\ \hat{\beta} &= \arccos \{|Z|\} \end{aligned} \quad (12)$$

3.2. Solving ambiguity algorithm

In order to estimate real value of ambiguity number, a method of searching in the range of ambiguity number is proposed as following

(i) According to frequency of the signal, radius and sensor number of the UCA, solve the range of ambiguity number p from Eq. (7).

(ii) By processing the signal received from antenna array sensors, we can obtain the measuring phase difference between any two adjacent sensors $\{\hat{\varphi}_{k+1,k}\}_{k=1,2,\dots,M}$ and define $\hat{\varphi}_{M+1,M} \square \hat{\varphi}_{1,M}$.

(iii) Choose a couple of baselines, for example, baseline (k_1+1, k_1) and baseline (k_2+1, k_2) . Then substitute all of the ambiguity numbers in Eq. (7) into Eq. (4) and we can obtain the corresponding real measuring phase difference of the choosing couple of baselines.

(iv) Substitute the real phase difference into Eq. (10), and reject the ambiguity numbers dissatisfying $\|Z\| \leq 1$. Then we can get a set of the estimations of DOA $\{(\hat{\alpha}, \hat{\beta})\}$ from Eq. (12).

(v) For every DOA estimation obtained in step (iv), calculate the phase difference using all the baselines composed of adjacent sensors except baseline (k_1+1, k_1) and baseline (k_2+1, k_2) , and we can get a set of phase difference $\{\bar{\varphi}_{k+1,k}\}_{k \neq k_1, k_2}$.

(vi) Solve the cost function defined as Eq. (13), thus we can finally get the real DOA estimation without ambiguity.

$$J = \arg \min_{(\hat{\alpha}, \hat{\beta})} \left[\sum_{k \neq k_1, k_2} \left| \text{mod}(\hat{\varphi}_{k+1,k} - \bar{\varphi}_{k+1,k}, 2\pi) \right|^2 \right] \quad (13)$$

4. Simulation Results

To illustrate the performance of the proposed solving ambiguity algorithm, several computer simulations are carried out with an UCA of 5 sensors. Assume that radius of the UCA is 2m, the azimuth angle is 60° , the pitching angle is 60° .

All of the following simulation results are obtained from 500 Monte Carlo experiments.

In the first example, we show the performance of the proposed algorithm versus SNR on the condition that frequency of the signal is 1GHz. Fig. 2 shows the DOA estimate error versus SNR. The results illustrate that the DOA estimate accuracy is very high if we can solving the phase ambiguity correctly. Fig. 3 shows the good performance of the proposed algorithm in solving ambiguity probability. The probability of solving ambiguity correctly can reach as high as 90% when SNR>-5dB, and solving ambiguity probability can reach nearly 100% when SNR>0dB.

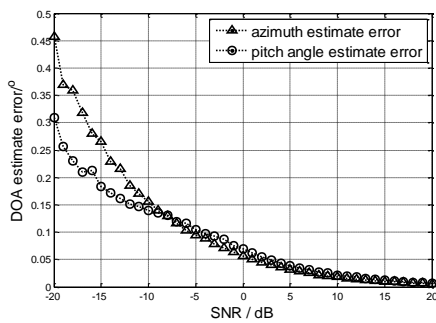


Fig. 2 DOA estimate error versus SNR

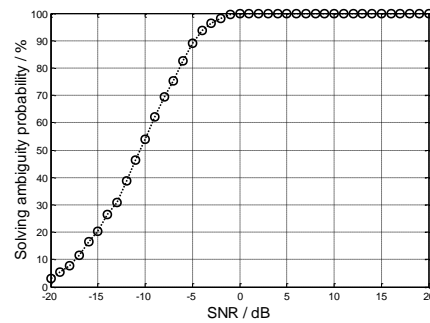


Fig. 3 Solving ambiguity probability versus SNR

Considering that the higher the frequency is, the more serious the phase ambiguity is. In the second example, we carry out a simulation about the performance of the proposed algorithm versus frequency of the signal on the condition of SNR=5dB. We can know that there is no phase ambiguity when frequency of the signal is less than 64MHz from Eq. (7). Therefore, we set the frequency starting from 64MHz in the simulation. Fig 4 shows the truth that the higher the frequency is, the more accurate the DOA estimation is. It is because the length of baseline relative to wavelength becomes bigger when the frequency becomes higher. Fig 5 shows that it becomes more difficulty to solve phase ambiguity when frequency increases. It is to be observed that solving ambiguity probability is down to 90% when frequency of the signal increases to 2GHz. Moreover, the range of the ambiguity number is $p \in [-16,16]$ when frequency of the signal is 2GHz.

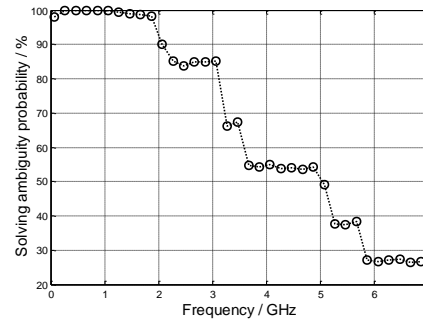
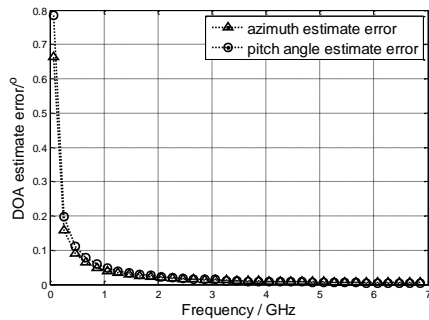


Fig. 4 DOA estimate error versus frequency Fig. 5 Solving ambiguity probability versus frequency

5. Conclusion

A novel solving ambiguity algorithm is proposed in this paper, which suits for UCA with arbitrary number of sensors. In order to demonstrate the performance of the proposed algorithm, several computer simulation results are presented. The simulation results show that the proposed algorithm still has high DF accuracy and high solving ambiguity probability on the condition of low SNR and serious phase ambiguity.

References

1. A. Flieller, P. Larzabal, H. Clergeot, Study of ambiguities in array manifold: a general framework, *Statistical Signal and Array Processing, 1996. Proceedings, 8th IEEE Signal Processing Workshop on*, 1996, pp. 574-577.
2. W. S. McCormick, J. B. Y. Tsui, V. L. Bakkie, A noise insensitive solution to an ambiguity problem in spectral estimation [J], *IEEE Trans. on AES*, 1989, 25(5): 729-732.
3. Si Wei-jian, Chu Ping, Research on the ways of solving direction finding ambiguity for interferometer [J], *Applied Science and Technology*, 2007, 24(9): 54-57.
4. Zhou Ya-qiang, Chen Zhu, Huangfu Kan, Sun Zhong-kang, Algorithm of Solving Multi-baseline Interferometer Phase Difference Ambiguity in Noisy Circumstance [J], *Journal of Electronics & Information Technology*, 2005, 27(2): 259-261.
5. E. Jacobs, W. E. Ralston. Ambiguity resolution in interferometry [J], *IEEE Trans. on AES*, 1981, 17(2): 766-779.
6. Lu An-nan. Direction finding ambiguity resolution by combination of long baselines [J]. *Mathematics in Practice and Theory*, 2008,38(10): 89-96.