Consensus problem of Markov jump multi-agent systems with time-varying delay under sampled-data

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The consensus problem of Markov jump multi-agent systems with time-varying delay under sampled-data control is investigated. Based on the extended Wirtinger inequality, a discontinuous Lyapunov functional is given, which makes full use of the sawtooth structure characteristic of sampling input delay. A less conservative consensus condition is obtained to ensure that the master systems are consensus with the slave systems. Finally, many numerical examples are given to illustrate the effectiveness of proposed methods.

Keywords: Markov Jump; Multi-Agent Systems; Consensus, Sampled-Data Control.

1. Introduction

Multi-agent systems exist widely in the real world where typical examples include intelligent robots, sensor networks, animal groups. There are considerable researches refer to this topic due to its extensive applications in various fields. Many researchers devoted their effort to develop distributed control protocols based on the information communication among the agents. In the past decade, the consensus problem of multi-agent systems has attracted a growing number of attention , which means that all the agents will eventually reach an agreement on certain quantities of interest, that is to say, the multi-agent systems can achieve consensus. The consensus issue has a long-standing record, especially, since the simple multi-agent model proposed by Vicsek et al. From then on, a large number of related research work emerged to abundant research data, see [1-4].

It is known to all that feedback control is an important step to understand the state of systems. However, owing to the limited network resources, such as unreliability of information channels, the capability of transmission bandwidth of networks, sampled-data strategy is more practical compared with continuous feedback control, and has been widely developed and applied in many areas. It has been introduced into consensus problems [5-6]. Moreover, based on sampled-data control, [7] proposed a protocol which used to guarantee consensus of the system. The paper [8] considered the consensus problem of second-order multi-agent dynamical systems with sampled position data. By using data-sampling technique and a zero-order hold circuit, a new consensus protocol was induced in [9] for directed networks of multiple agents with intrinsic nonlinear dynamics and sampled-data information.

On the other hand, it is necessary to consider the influence triggered by stochastic factors, especially, some abrupt phenomena may make a system jump from one mode to another, such as random failures and sudden environmental changes. Therefore, switching dynamical systems are studied by more and more people [10-15]. The paper [13] investigated the consensus problem of data-sampled multi-agent systems with Markovian switching topologies and presented two different consensus algorithms. The event-triggered consensus problem was discussed for multiple agents connected by a directed network in [12]. Based on stochastic sampled-data, the event-triggered function was designed. The paper[14]studied the consensus problem of second-order Markovian jump multi-agent systems with delays. And the paper [11] presented the consensus of multi-agent system with random governed by a Markov chain. Additionally, in [15], leader-following consensus problem was researched for multi-agent systems with Markovian switching topologies in a sampled-data setting.

2. Preliminaries

Fix a probability space (Ω, F, P) and consider the following Markov jump multi-agent system with time-varying delay:

$$\dot{x}(t) = f(x(t)) + A(r(t))x(t) + B(r(t))x(t-d(t)) + \upsilon(t) \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \cdots, x_n(t)]^T$, $f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \cdots, f_n(x_n(t))]$, $x_i(t)$ is the state of the i th agent at time t, $f_i(x_i(t))$ stands for the nonlinear term. Each $f_i(\cdot)$ is continuous and bounded, and there exist constants F_1 and F_2 such that $F_1 \le \frac{f_i(x) - f_i(y)}{x-y} \le F_2$, $i = 1, 2, \cdots, n$.

$\upsilon(t)=[\upsilon_1(t),\cdots,\upsilon_n(t)]^T$ is an external input vector; $d(t)$ is the time-varying delay and satisfies $0\le d(t)\le d$ and $\dot d(t)\le\mu$ in which d and μ are known constants. $\{r(t),t\ge 0\}$ is a continuous-time Markov process with right continuous trajectories and taking values in a finite set $S=\{1,\cdots,s\}$ with transition rate matrix $\Lambda=\{\pi_{ij}\}$ given by $\Pr\{r_{+h}=j\,|\,r_t=i\}=\begin{cases}\pi_{ij}h+o(h), & j\ne i\\ 1+\pi_{ii}h+o(h), & j=i\end{cases}$ where $h>0$, $\lim_{h\to 0}\frac{o(h)}{h}=0$, and $\pi_{ij}\ge 0$ for $i\ne j$, is the transition rate from mode i to mode j at time $t+h$ and $\pi_{ii}=-\sum_{j=1,i\ne j}^{s}\pi_{ij}$.

In this paper, system (1) is regarded as the master system and a slave system for (2.1) can be described by the following equation:

$$\dot y(t)=f(y(t))+A(r(t))y(t)+B(r(t))y(t-d(t))+\upsilon(t)+u(t) \tag{2}$$

Where $u(t)=[u_1(t),u_2(t),\cdots,u_n(t)]^T$ is the appropriate control input that will be designed in order to obtain a certain control objective.

Define the error signal as $e(t)=y(t)-x(t)$, the error system can be represented as follows:

$$\dot e(t)=g(e(t))+A(r(t))e(t)+B(r(t))e(t-d(t))+u(t) \tag{3}$$

In which $g(e(t))=f(y(t))-f(x(t))$. It can be found the functions $g_i(\cdot)$ satisfy the following condition: $F_1\le\frac{g_i(x)}{x}\le F_2$, $i=1,2,\cdots,n$.

Denote by t_k the updating instant time of the Zero-Oder-Hold (ZOH), and suppose that the updating signal (successfully transmitted signal from the sampler to the controller and to the ZOH) at the instant t_k has experienced a constant signal transmission delay η. It is assumed that the sampling intervals satisfy $t_{k+1}-t_k=h_k\le h$ for any integer $k\ge 0$, in which h is a positive scalar and represents the largest sampling interval. Thus, we have that

$$t_{k+1}-t_k+\eta\le h+\eta=\tau \tag{4}$$

Therefore, the state-feedback controller takes the following form:

$$u(t)=Ke(t_k-\eta),\quad t_k\le t<t_{k+1} \tag{5}$$

In which K is the sampled-data feedback controller gain matrix to be determined. By substituting Eq. (5) into (3) and defining $\tau(t) = t - t_k + \eta$, $t_k \le t < t_{k+1}$, the error systems can be rewritten $\dot{e}(t) = g(e(t)) + A(r(t))e(t) + B(r(t))e(t-d(t)) + Ke(t-\tau(t))$,

$$t_k \le t < t_{k+1} \tag{6}$$

It can be found from (2.4) that $\eta \le \tau(t) < t_{k+1} - t_k + \eta \le \tau$ and $\dot{\tau}(t) = 1$ for $t \ne t_k$.

3. Main Results

In this section, the sampled-data consensus problem of system (1) and slave system (2) will be studied via a discontinuous Lyapunov functional approach.

Lemma 3.1. (Extended Wirtinger inequality) Let $z(t) \in W[a,b)$ and $z(a) = 0$. Then for any matrix $R > 0$, the following inequality holds

$$\int_a^b z(s)^T Rz(s)ds \le \frac{4(b-a)^2}{\pi^2}\int_a^b \dot{z}(s)^T Rz(s)ds. \tag{4}$$

Now, we show the following main result.

Theorem 3.2. If there exist a series of $P(i) > 0$, $R_k > 0$ $(i \in S, k = 1,2,3,4)$ and two constants $0 < \mu < 1$, $\gamma > 0$ such that

$$\Omega = \begin{bmatrix} \Omega_{11} & B(i)^T P(i) & O & K^T P(i) & \Omega_{15} & A^T(i) \\ * & (\mu-1)R_1 & O & O & O & B^T(i) \\ * & * & \Omega_{33} & \Omega_{34} & O & O \\ * & * & * & \Omega_{44} & O & K^T \\ * & * & * & * & -\gamma I & I \\ * & * & * & * & * & \Omega_{66} \end{bmatrix} < 0$$

Where

$$\Omega_{11} = P(i)A(i) + A(i)^T P(i) + R_1 + R_2 + \sum_{i=1}^{s} \pi_{ij} P(j) - \gamma F_1 F_2 I$$

$$\Omega_{15} = P(i) + \frac{\gamma}{2}(F_1 + F_2)I, \Omega_{33} = -\frac{\pi^2}{4} R_4 - R_2 - R_3 + R_4$$

$$\Omega_{34} = \frac{\pi^2}{4} R_4, \Omega_{44} = -\frac{\pi^2}{4} R_4, \Omega_{66} = -\frac{1}{h^2} R_4^{-1}$$

then the master system (1) and slave system (2) are consensus.

Proof. Consider the following discontinuous Lyapunov functional for error system (2.6)

$$V(e_t, r(t), t) = \sum_{i=1}^{5} V_i(e_t, r(t), t) \tag{5}$$

In which

$$V_1(e_t, r(t), t) = e(t)^T P(r(t)) e(t)$$

$$V_2(e_t, r(t), t) = \int_{t-d(t)}^{t} e(s)^T R_1 e(s) ds$$

$$V_3(e_t, r(t), t) = \int_{t-\eta}^{t} e(s)^T R_2 e(s) ds$$

$$V_4(e_t, r(t), t) = \int_{t-\tau}^{t-\eta} e(s)^T R_3 e(s) ds$$

$$V_5(e_t, r(t), t) = (\tau - \eta)^2 \int_{t_k-\eta}^{t} \dot{e}(s)^T R_4 \dot{e}(s) ds$$

$$-\frac{\pi^2}{4} \int_{t_k-\eta}^{t-\eta} [e(s) - e(t_k - \eta)]^T R_4 [e(s) - e(t_k - \eta)] ds$$

According to Lemma 3.1, one can obtain that $V_5(e_t, i, t) \geq 0$. Let L be infinitesimal generator of the Markov process acting on $V(e_t, r(t), t)$. Then one can get

$$LV_1(e_t, i, t) = sym\{e(t)^T P(i)[g(e(t)) + A(r(t))e(t)$$

$$+B(r(t))e(t-d(t)) + Ke(t-\tau(t))]\} + \sum_{j=1}^{s} \pi_{ij} e(t)^T P(j) e(t) \tag{6}$$

$$LV_2(e_t, i, t) = e(t)^T R_1 e(t) - (1-\mu) e(t-d(t))^T R_1 e(t-d(t)) \tag{7}$$

$$LV_3(e_t, i, t) = e(t)^T R_2 e(t) - e(t-\eta)^T R_2 e(t-\eta) \tag{8}$$

$$LV_4(e_t,i,t) = e(t-\eta)^T R_3 e(t-\eta) - e(t-\tau)^T R_3 e(t-\tau) \tag{9}$$

$$LV_5(e_t,i,t) = h^2 \dot{e}(t)^T R_4 \dot{e}(t) - \frac{\pi^2}{4}\begin{bmatrix} e(t-\eta) \\ e(t-\tau(t)) \end{bmatrix}^T \begin{bmatrix} R_4 & -R_4 \\ * & R_4 \end{bmatrix} \begin{bmatrix} e(t-\eta) \\ e(t-\tau(t)) \end{bmatrix} \tag{10}$$

On the other hand, one has from (3) that there exists a scale constant $\gamma > 0$ such that for any $i = 1, 2, \cdots, n$,

$$\gamma[g_i(e_i(t)) - F_1 e_i(t)][g_i(e_i(t)) - F_2 e_i(t)] \le 0 \tag{11}$$

Then, adding the right side of (11)to $LV(e_t,i,t)$, one has from (5)-(10)

$$LV(e_t,i,t) \le \chi(t)\bar{\Omega}\chi(t)^T + h^2 \dot{e}(t)^T R_4 \dot{e}(t) < 0 \tag{12}$$

where

$$\chi(t) = [e(t)^T, e(t-d(t))^T, e(t-\eta)^T, e(t-\tau(t))^T, g(e(t))^T]$$

$$\bar{\Omega} = \begin{bmatrix} \Omega_{11} & B(i)^T P(i) & O & K^T P(i) & \Omega_{15} \\ * & (\mu-1)R_1 & O & O & O \\ * & * & \Omega_{33} & \Omega_{34} & O \\ * & * & * & \Omega_{44} & O \\ * & * & * & * & -\gamma I \end{bmatrix}$$

Which implies system (6) is stable, that is, the master system (1) and slave system (2) are consensus. The proof is completed.

4. Numerical Simulation

In this section, a numerical example is given to show the validity of the obtained results. Consider the master system (1) and slave system (2) with the following parameters:

$$A(1) = \begin{bmatrix} -2.4 & 1 \\ -1 & -3 \end{bmatrix}, B(1) = \begin{bmatrix} -2.1 & 1.3 \\ 1.6 & -2.5 \end{bmatrix}, K = \begin{bmatrix} -4 & 1.4 \\ 1.2 & -3 \end{bmatrix}$$

$$A(2) = \begin{bmatrix} -2.6 & 1 \\ -1.4 & -3.2 \end{bmatrix}, B(2) = \begin{bmatrix} -2.4 & 1.2 \\ 1.3 & -2.5 \end{bmatrix}, \Lambda = \begin{bmatrix} -2 & 2 \\ 1.4 & -1.4 \end{bmatrix}$$

And $f_1(x_1(t)) = \sin(x_1(t)),\ f_2(x_2(t)) = \cos(x_2(t))$. It is easy to find that $F_1 = -I, F_2 = I$. It is assumed that $\upsilon(t) = 0$ and time-varying delay $d(t) = \frac{e^t}{1+e^t}$. A straightforward calculation is that $d = 1$ and $\mu = 0.25$.

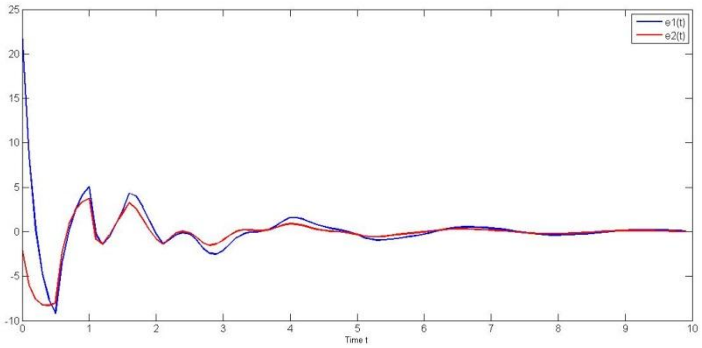

Fig. 1 State responses of error system (6).

In view of Theorem 3.2, one can get $\gamma = 2.4$ obtained by the Matlab LMI control toolbox. Under the above conditions, the response curves of error system (6) can be found in Fig. 1. It is obviously that the system (1) is consensus with the master system (2).

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