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A special class of constacyclic codes over a non-chain ring

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In this paper, we study a special class of constacyclic codes over a non-chain ring. Firstly, a new Gray map is given. Then we investigate some properties of this class of constacyclic codes. Finally, we present an example to illustrate the research results.

Keywords: Constacyclic Codes; Gray Map; Dual Codes; Negacyclic Codes.

1. Introduction

Since 1950s, constacyclic codes have been well studied over finite fields. Constacyclic codes have rich practical applications and rich algebraic structures for efficient error detection and correction. Constacyclic codes have been studied over various finite chain rings. This has motivated the study of constacyclic codes over non-chain rings.

Recently, the structures and some results of cyclic codes over $F_2+vF_2(v^2=v)$ were studied by Zhu et al. in [7]. In [5], optimal p-ary codes from constacyclic codes over a non-chain ring $F_p+vF_p(v^2=1)$ were investigated. Shi et al. investigated cyclic codes and the weight enumerator of linear codes over $F_2+vF_2+v^2F_2(v^3=v)$ in [6]. Mostafanasab and Karimi studied $(1-2u^2)$-constacyclic codes over $F_p+uF_p+u^2F_p$ $(u^3=u)$ in [3]. In [4], Raka and Kathuria discussed $(1-2u^3)$-constacyclic codes and quadratic residue codes over $F_p[u]/\langle u^4-u\rangle(u^4=u)$. Aydin et al. studied some new binary quasi-cyclic codes from codes over the ring $F_2+uF_2+vF_2+uvF_2$ $(u^2=v^2=0, uv=vu)$ in [1]. Karadeniz and Yildiz investigated $(1+v)$-constacyclic codes over $F_2+uF_2+vF_2+uvF_2$ in [2]. As we know, constacyclic codes over different alphabets form an important class of codes that include cyclic codes and negacyclic codes as special cases. Following the above trend, the purpose of the present paper is devoted to studying $(1-2v^2)$-constacyclic codes over the ring R and their Gray images.

The material of the paper is organized as follows. The next section introduces some preliminary results on linear codes over the ring R that we need. In Section 3, we investigate the properties of $(1-2v^2)$-constacyclic codes over R. In Section 4, to illustrate some results of $(1-2v^2)$-constacyclic codes over R, we present an example.

2. Preliminary

In this paper, let $R=F_p[u,v]/\langle u^2-1, v^3-v, uv-vu\rangle$, where $u^2=1, v^3=v$ and p are odd primes, we study $(1-2v^2)$-constacyclic codes over R. R is a ring of characteristic p and of size p^6. For any positive integer a, if there is an integer $b\left(0<b<p\right)$ such that $ab\equiv 1\left(\operatorname{mod} p\right)$, we write $b=a^{-1}=1/a$. It follows that $v^3-v == v(v+1)(v-1)$. Let $y_1=v, y_2=v+1$, $y_3=v-1$ and $\overline{y_i}\equiv(v^3-v)/y_i$, for $i=1,2,3$. Then there exist $a_i, b_i\in R_1[v]$ such that $a_iy_i+b_i\overline{y_i}=1$, where $R_1=F_p+uF_p$. Let $\varepsilon_i=b_i\overline{y_i}$. Through a direct calculation, we can obtain $R=(1-v^2)R_1\oplus 2^{-1}(v^2-v)R_1\oplus 2^{-1}(v^2+v)R_1=2^{-1}(1-u)(1-v^2)F_p\oplus 2^{-1}(1+u)(1-v^2)F_p\oplus 4^{-1}(1-u)(v^2-v)F_p\oplus 4^{-1}(1+u)(v^2-v)\oplus 4^{-1}(1-u)(v^2+v)F_p\oplus 4^{-1}(1+u)(v^2+v)F_p$. Denoted by $\eta_1,\eta_2,\eta_3,\eta_4,\eta_5,\eta_6$ respectively the following elements of R:

$$\eta_1=2^{-1}(1-u)(1-v^2), \eta_2=2^{-1}(1+u)(1-v^2), \eta_3=4^{-1}(1-u)(v^2-v),$$
$$\eta_4=4^{-1}(1+u)(v^2-v), \eta_5=4^{-1}(1-u)(v^2+v), \eta_6=4^{-1}(1+u)(v^2+v).$$

From a simple calculation, then we have the following direct results: (1) η_i are non-zero idempotents in R, and $\eta_i\eta_j=0$, if $i\neq j$ for $i,j\in\{1,2,3,4,5,6\}$; (2) $\sum_{i=1}^{6}\eta_i=1$.

Let $\sigma(r_0,r_1,\cdots,r_{n-1})=(r_{n-1},r_0,\cdots,r_{n-2})$, $\gamma(r_0,r_1,\cdots,r_{n-1})=(-r_{n-1},r_0,\cdots,r_{n-2})$, $\rho(r_0,r_1,\cdots,r_{n-1})=((1-2v^2)r_{n-1},r_0,\cdots,r_{n-2})$, and C be a linear code of length n over R. Then C is said to be cyclic if $\sigma(C)=C$, negacyclic if $\gamma(C)=C$ and $(1-2v^2)$ -constacyclic if $\rho(C)=C$. The Euclidean inner product of $x=(x_0,x_1,\cdots,x_{n-1})$ and $y=(y_0,y_1,\cdots,y_{n-1})$ in R^n is defined as $xy=x_0y_0+x_1y_1+\cdots+x_{n-1}y_{n-1}$, where the operation is performed in R. For a code C over R, it's dual $C^{\perp}$ is defined as $C^{\perp}=\{x\in R\mid xy=0,\forall y\in C\}$. Recall that a code C is said to be self-dual if $C=C^{\perp}$ and self-orthogonal if $C\subseteq C^{\perp}$.

For $(1-2v^2)$ being unit in R, it is well known that a $(1-2v^2)$ -constacyclic code of length n over R can be identified as an ideal of the quotient ring $R[x]/\langle x^n-(1-2v^2)\rangle$ via the R -module isomorphism $\varphi: R^n\to R[x]/\langle x^n-(1-2v^2)\rangle$, namely $(a_0,a_1,\cdots,a_{n-1})\mapsto a_0+a_1x+\cdots+a_{n-1}x^{n-1}(\mathrm{mod}\langle x^n-(1-2v^2)\rangle)$.

Expressing elements of R as $a+bu+cv+duv+ev^2+fuv^2=m+vl+v^2q$, where $m=a+bu, l=c+du$ and $q=e+fu$ are in R_1. We define a Gray map $\Phi: R\to R_1^2$ by $\Phi(a+duv+ev^2+fuv^2)=\Phi(m+vl+v^2q)=bu+cv+(-q,2m+q)=(-e-fu,2a+e+(2b+f)u)$. One can verify that Φ is a linear map, which can be extended to R^n naturally, as follows: $\Phi(r_0,r_1,\cdots,r_{n-1})=(-q_0,-q_1,\cdots,-q_{n-1},2m_0+q_0,2m_1+q_1,\cdots,2m_{n-1}+q_{n-1})$, where $r_i=m_i+vl_i+v^2q_i, 0\le i\le n-1$.

The polynomial correspondence of the Gray map can be defined as $\Phi: R[x]/\langle x^n-(1-2v^2)\rangle\to R_1[x]/\langle x^{2n}-1\rangle$ given by $\Phi(a(x)+b(x)u+c(x)v+d(x)uv+e(x)v^2+f(x)uv^2)=\Phi(m(x)+vl(x)+v^2q(x))=-q(x)+x^n(2m(x)+q(x))$, where $m(x)=a(x)+b(x)u, l(x)=c(x)+d(x)u$ and $q(x)=e(x)+f(x)u$ in $R_1[x]$.

Let C be a linear code of length n over R. For $1\le j\le 6$, we define $C_j=\{x_j\in F_p^n:\exists x_i\in F_p$ for $i=\{1,2,3,4,5,6\}\setminus\{j\}$ such that $\sum_{i=1}^{6}\eta_i x_i\in C\}$. It is easy to verify that C_j are linear codes of length n over F_p, $C=\bigoplus_{i=1}^{6}\eta_i C_i$ and $|C|=\prod_{i=1}^{6}|C_i|$.

3. $(1-2v^2)$ –constacyclic codes over $F_p[u,v]/\langle u^2-1,v^3-v,uv-vu\rangle$

With the notations introduced above, the following proposition and corollary can be obtained by similar proof as [3], so we omit it here.

Proposition 3.1 *If* Φ *is the Gray map from* R^n *into* R_1^{2n}, *then* $\Phi\rho = \sigma\Phi$.

As a consequence of Proposition 3.1, we have the following corollary.

Corollary 3.2 *The Gray image of a* $(1-2v^2)$*-constacyclic code over* R *of length* n *is a cyclic code over* R_1 *of length* $2n$.

Proposition 3.3 *Let* C *be a code of length* n *over* R. *Then* C *is a* $(1-2v^2)$*-constacyc-lic code if and only if* $C^{\perp}$ *is a* $(1-2v^2)$*-constacyclic code.*

Proof. "$\Rightarrow$" Note that $\rho(r_0, r_1, \cdots, r_{n-1}) = \rho_{\lambda}(r_0, r_1, \cdots, r_{n-1}) = (\lambda r_{n-1}, r_0, r_1, \cdots, r_{n-2})$ if C is λ-constacyclic. Let C be a code of length n over R. For $x \in C^{\perp}$ and $y \in C$. Since C is $(1-2v^2)$-constacyclic, $\rho_{1-2v^2}^{n-1}(y) \in C$. Therefore $0 = x\rho_{1-2v^2}^{n-1}(y) = (1-2v^2)\rho_{(1-2v^2)^{-1}}(x)y$, i.e. $\rho_{(1-2v^2)^{-1}}(x)y = 0$, which means $\rho_{(1-2v^2)^{-1}}(x) \in C^{\perp}$. Thus, $C^{\perp}$ is closed under the $\rho_{(1-2v^2)^{-1}}$-shift. Since $(1-2v^2)^{-1} = 1-2v^2$, $C^{\perp}$ is a $(1-2v^2)$-constacyclic code. "$\Leftarrow$" Obviously. □

Proposition 3.4 *Let* C *be a code of length* n *over* R *such that* $C \subseteq (R_1 + v^2 R_1)^n$. *If* C *is self-orthogonal, then so is* $\Phi(C)$.

Proof. The proof is similar to Proposition 2.4 in [3]. □

Then we have the following theorem.

Theorem 3.5 *Let* $C = \bigoplus_{i=1}^{6} \eta_i C_i$ *be a code of length* n *over* R. *Then* C *is a* $(1-2v^2)$-con*stacyclic code of length* n *over* R *if and only if* C_1, C_2 *are cyclic and* C_3, C_4, C_5, C_6 *are negacyclic codes of length* n *over* F_p.

Proof. Note that $(1-2v^2)\eta_j = \eta_j$ for $1 \le j \le 2$, and $(1-2v^2)\eta_k = -\eta_k$ for $3 \le k \le 6$. Let $r = (r_0, r_1, \cdots, r_{n-1}) \in C$. Then $r_i = \sum_{t=1}^{6} \eta_t a_i^t, a_i^t \in F_p, 0 \le i \le n-1$. Let $a^t = (a_0^t, a_1^t, \cdots, a_{n-1}^t)$, then $a^t \in C_t$. Assume that C_1, C_2 are cyclic and C_3, C_4, C_5, C_6 are negacyclic codes, then $\sigma(a^i) \in C_i, \gamma(a^j) \in C_j, 1 \le i \le 2$, $3 \le j \le 6$. Thus $\rho(r) = ((1-2v^2)r_{n-1}, r_0, r_1, \cdots, r_{n-2}) = \sum_{i=1}^{2} \sigma(a^i)\eta_i + \sum_{j=3}^{6} \gamma(a^j)\eta_j \in C$. Hence, C is a $(1-2v^2)$-constacyclic code over R. For the converse, let $a^t = (a_0^t, a_1^t, \cdots, a_{n-1}^t) \in C_t$. For $r = (r_0, r_1, \cdots, r_{n-1}) \in C$, let $r_i = \sum_{t=1}^{6} \eta_t a_i^t, 0 \le i \le$

$n-1$. Thus $\rho(r)=\sum_{i=1}^{2}\sigma(a^i)\eta_i+\sum_{j=3}^{6}\gamma(a^j)\eta_j\in C$, which implies $\sigma(a^i)\in C_i$, $\gamma(a^j)\in C_j, 1\le i\le 2, 3\le j\le 6$. So C_1, C_2 are cyclic and C_3, C_4, C_5, C_6 are negacyclic. □

Lemma 3.6 *Let* $x^n-(1-2v^2)=g(x)h(x)$ *in* $R[x]$ *and let* C *be a* $(1-2v^2)$*-constacyclic code generated by* $g(x)$. *If* $f(x)$ *is relatively prime with* $h(x)$, *then* $C=\langle g(x)f(x)\rangle$.

Proof. The proof is similar to that of Lemma 2 in [1]. □

Theorem 3.7 *Let* C *be a* $(1-2v^2)$*-constacyclic code of length* n *over* R, *then we have*

(1) $C=\langle \eta_1 g_1(x),\eta_2 g_2(x),\eta_3 g_3(x),\eta_4 g_4(x),\eta_5 g_5(x),\eta_6 g_6(x)\rangle$, *where* $g_i(x)$ *are monic and* $C_i=\langle g_i(x)\rangle, 1\le i\le 6$, *respectively. Further* $g(x)=\sum_{i=1}^{6}\eta_i g_i(x)$ *is the unique polynomial such that* $C=\langle g(x)\rangle$. Moreover, $|C|= p^{6n-\sum_{i=1}^{6}\deg(g_i(x))}$;

(2) *Suppose that* $g_j(x)h_j(x)=x^n-1, 1\le j\le 2$ *and* $g_k(x)h_k(x))=x^n+1$, $3\le k\le 6$. *Let* $h(x)=\sum_{i=1}^{6}\eta_i h_i(x)$, *then* $g(x)h(x)=x^n-(1-2v^2)$;

(3) *If* $\gcd(f_i(x),h_i(x))=1$ for $1\le i\le 6$, *then* $gcd(f(x),h(x))=1$ *and* $C=\langle g(x)f(x)\rangle$, *where* $f(x)=\sum_{i=1}^{6}\eta_i f_i(x)$;

(4) $C^{\perp}=\bigoplus_{i=1}^{6}\eta_i C_i^{\perp}=\langle \eta_1 h_1^{\perp}(x),\eta_2 h_2^{\perp}(x),\eta_3 h_3^{\perp}(x),\eta_4 h_4^{\perp}(x),\eta_5 h_5^{\perp}(x),\eta_6 h_6^{\perp}(x)\rangle=$ $\langle h^{\perp}(x)\rangle$, *where* $h^{\perp}(x)=\sum_{i=1}^{6}\eta_i h_i^{\perp}(x), h_i^{\perp}(x)$ *is the reciprocal polynomial of* $h_i(x)$, *and* $h^{\perp}(x)$ *is the reciprocal polynomial of* $h(x), 1\le i\le 6$. Moreover $|C^{\perp}|=p^{\sum_{i=1}^{6}\deg(g_i(x))}$;

(5) *Let* t_1 *denote the number of irreducible factors of* x^n-1 *over* F_p *and* t_2 *denote the number of irreducible factors of* x^n+1 *over* F_p. *Then the number of* $(1-2v^2)$*-constacyclic codes of length* n *over* R *is* $4^{t_1}16^{t_2}$.

Proof. (1) By Theorem 3.5, $C_j=\langle g_j(x)\rangle\subseteq F_p[x]/\langle x^n-1\rangle$ for $1\le j\le 2$, $C_k=\langle g_k(x)\rangle\subseteq F_p[x]/\langle x^n+1\rangle$ for $3\le k\le 6$. Since $C=\bigoplus_{i=1}^{6}\eta_i C_i$, then $C=\{c(x)\mid$

$c(x)=\sum_{i=1}^{6}\eta_i f_i(x),\ f_i(x)\in C\}$. Thus $C\subseteq\langle\eta_1 g_1(x),\eta_2 g_2(x),\eta_3 g_3(x),\eta_4 g_4(x),$ $\eta_5 g_5(x),\eta_6 g_6(x)\rangle\subseteq R_n=R[x]/\langle x^n-(1-2v^2)\rangle$. Conversely, suppose $f(x)=\sum_{i=1}^{6}\eta_i g_i(x)r_i(x)\in\langle\eta_1 g_1(x),\ \eta_2 g_2(x),\ \eta_3 g_3(x),\ \eta_4 g_4(x),\ \eta_5 g_5(x),\ \eta_6 g_6(x)\rangle$, where $r_i(x)\in R_n$, for $1\le i\le 6$. There exist $a_i(x),b_i(x),c_i(x),d_i(x),e_i(x),\ f_i(x)\in F_p[x]$ such that $r_i(x)=\eta_1 a_i(x)+\eta_2 b_i(x)+\eta_3 c_i(x)+\eta_4 d_i(x)+\eta_5 e_i(x)+\eta_6 f_i(x)$. Note that $\eta_i^2=\eta_i,\eta_i\eta_j=0,1\le i\ne j\le 6$. Then $f(x)=\sum_{i=1}^{6}\eta_i g_i(x)a_i(x)\in C$, thus $\langle\eta_1 g_1(x),$ $\eta_2 g_2(x),\eta_3 g_3(x),\eta_4 g_4(x),\eta_5 g_5(x),\eta_6 g_6(x)\rangle\subseteq C$. Hence $C=\langle\eta_1 g_1(x),\eta_2 g_2(x),$ $\eta_3 g_3(x),\eta_4 g_4(x),\eta_5 g_5(x),\eta_6 g_6(x)\rangle$. Since $\eta_i g_i(x)=\eta_i g(x)$, then $C=\langle g(x)\rangle$. Clearly $|C|=|C_1||C_2||C_3||C_4||C_5||C_6|=p^{6n-\sum_{i=1}^{6}\deg(g_i(x))}$. This proves (1).

(2) By assumption we have that

$$g(x)h(x)=g(x)\left(\sum_{i=1}^{6}\eta_i h_i(x)\right)=\sum_{i=1}^{6}\eta_i g_i(x)h_i(x)=\sum_{j=1}^{2}\eta_j(x^n-1)+\sum_{k=3}^{6}\eta_k(x^n+1)$$
$$=\left(\sum_{i=1}^{6}\eta_i\right)x^n-(\eta_1+\eta_2-\eta_3-\eta_4-\eta_5-\eta_6)=x^n-(1-2v^2).$$

Hence, $g(x)h(x)=x^n-(1-2v^2)$.

(3) Suppose that $\gcd(f_i(x),h_i(x))=1$ for $1\le i\le 6$ and $f(x)=\sum_{i=1}^{6}\eta_i f_i(x)$. Thus there exist $a_i(x),b_i(x)\in R[x]$ such that $a_i(x)f_i(x)+b_i(x)h_i(x)=1$. Set $a(x)=\sum_{i=1}^{6}\eta_i a_i(x)$ and $b(x)=\sum_{i=1}^{6}\eta_i b_i(x)$. Then we have $a(x)f(x)+b(x)h(x)=\sum_{i=1}^{6}\eta_i(a_i(x)f_i(x)+b_i(x)h_i(x))=1$, which implies $\gcd(f(x),h(x))=1$. According to Lemma 3.6, we have $C=\langle g(x)f(x)\rangle$.

(4)-(5) can be similarly proved. □

Similar to Theorem 6 of [4], we have the following theorem.

Theorem 3.8 *Let C be a $(1-2v^2)$-constacyclic code of length n over R generated by $g(x)=\sum_{i=1}^{6}\eta_i g_i(x)$, where $g_i(x)\in F_p[x]$ are the monic generator polynomials of C_i. Then the Gray image $\Phi(C)$ of C is a cyclic subcode of $\langle g_1(x)\varpi(x),g_2(x)g_3(x)\rangle$ of length $2n$ over R_1, where $\varpi(x)=gcd(g_3(x),g_4(x),g_5(x),g_6(x))$ in $F_p[x]$.*

In the following, we study $(1-2v^2)$-constacyclic codes over R when n is odd by introducing the following isomorphism from R_n to T_n.

Proposition 3.9 *Let* $\phi: R_n = R[x]/(x^n-1) \to T_n = R[x]/(x^n-(1-2v^2))$ *such that* $\phi(c(x)) = c((1-2v^2)x)$. *If* n *is odd, then* ϕ *is a ring isomorphism from* R_n *to* T_n.

Proof. The result follows from $\phi(x^n-1) = (1-2v^2)^n x^n - 1 = (1-2v^2)$ $(x^n-(1-2v^2)) = 0.$ □

According to Proposition 3.9, we obtain the following corollaries.

Corollary 3.10 *Let* n *be an odd number. Then we have:* (1) I *is an ideal of* R_n *if and only if* $\phi(I)$ *is an ideal of* T_n; (2) *A* $(1-2v^2)$*-constacyclic code of length* n *over* R *is equivalent to a cyclic code of length* n *over* R *by the ring isomorphism* Φ .

Let $\overline{\phi}: R^n \to R^n$ be defined as $\overline{\phi}(c_0, c_1, \cdots, c_{n-1}) = (c_0, (1-2v^2)c_1,$ $(1-2v^2)^2 c_2, \cdots, (1-2v^2)^{n-1} c_{n-1})$. In the light of the definition, we have the following corollary.

Corollary 3.11 C *is a cyclic code over* R *of odd length* n *if and only if* $\overline{\phi}(C)$ *is a* $(1-2v^2)$*-constacyclic code of length* n *over* R .

Proof. "$\Rightarrow$" Let C be a cyclic code over R of odd length n . Then $\overline{\phi}(c_0, c_1, \cdots, c_{n-1}) = (c_0, (1-2v^2)c_1, (1-2v^2)^2 c_2, \cdots, (1-2v^2)^{n-1} c_{n-1}) = (c_0, (1-2v^2)c_1,$ $c_2, \cdots, c_{n-1}) \in \overline{\phi}(C), \overline{\phi}(c_{n-1}, c_0, c_1, \cdots, c_{n-2}) = (c_{n-1}, (1-2v^2)c_0, (1-2v^2)^2 c_1, \cdots, (1-2v^2)^{n-1} c_{n-2}) =$ $(c_{n-1}, (1-2v^2)c_0, c_1, \cdots, c_{n-2}) \in \overline{\phi}(C)$. So $\rho(\overline{\phi}(c_0, \cdots, c_{n-1})) = \rho(c_0, (1-2v^2)c_1, c_2, \cdots,$ $(1-2v^2)c_{n-2}, c_{n-1}) = ((1-2v^2)c_{n-1}, c_0, (1-2v^2)c_1, \cdots, (1-2v^2)c_{n-2}) \in \overline{\phi}(C)$. Hence $\overline{\phi}(C)$ is a $(1-2v^2)$-constacyclic code of length n over R . "$\Leftarrow$" We can use the similar way to prove. □

Definition 3.12 *Let* τ *be the following permutation on* $\{0, 1, \cdots, 2n-1\}$ *with* n *being odd such that* $\tau = (1, n+1)(3, n+3)\cdots(2i+1, n+2i+1)\cdots(n-2, 2n-2)$. *The Nechaev permutation is the permutation* π *defined by* $\pi(c_0, c_1, \cdots, c_{2n-1})$ $= (c_{\tau(0)}, c_{\tau(1)}, \cdots, c_{\tau(2n-1)})$.

According to Proposition 2.16 and Corollary 2.17 in [3], we can get the following similar results.

Proposition 3.13 *Let* $\bar{\phi}$ *be defined as above. If* π *is the Nechaev permutation and* n *is odd, then we have:* (1) $\Phi\bar{\phi}=\pi\Phi$; (2) *If* χ *is the Gray image of a cyclic code over* R *, then* $\pi(\chi)$ *is a cyclic code.*

4. Examples

In this section, we present an example to illustrate the main results obtained in previous sections.

Example 4.1 *For* $p=3, n=5, (x^5-1)$ *and* (x^5+1) *can be factorized as* $x^5-1=(x-1)(x^4+x^3+x^2+x+1), x^5+1=(x+1)(x^4+2x^3+x^2+2x+1)$, *which give* 4 *cyclic codes,* 4 *negacyclic codes over* F_3 *and thus* $4^2 16^2$ $(1-2v^2)$ *-constacyclic codes over* R *. Let* $g_1(x)=g_2(x)=\sum_{i=0}^{4} x^i$ *and* $g_3(x)=g_4(x)=g_5(x)$ $=g_6(x)=x^4+2x^3+x^2+x^2+2x+1$, *then* $C=\langle \eta_1 g_1, \eta_2 g_2, \eta_3 g_3, \eta_4 g_4, \eta_5 g_5, \eta_6 g_6\rangle$ *is* a $(1-2v^2)$ *-constacyclic code of length* 5 *over* R *.* $C_j=\langle g_j(x)\rangle, j=1,2$ *are the repetition codes of length* 5 *and* $C_k=\langle g_k(x)\rangle,\ k=3,4,5,6$ *are negacyclic codes of length* 5 *over* F_3 *. Then* $g(x)=\sum_{i=1}^{6}\eta_i g_i(x)=(x^4+x^3+x^2+x+1)+$ $(x^3+x)v^2$ *and* $\langle g(x)\rangle$ is a $(1-2v^2)$ *-constacyclic code of length* 5 *over* R *.* C *is equivalent to a cyclic code generated by* $\sum_{j=1}^{2}\eta_j g_j(x)+\sum_{k=3}^{6}\eta_k g_k(-x)=\sum_{i=0}^{4}x^i$. *Further* $\Phi(g(x))=-x^3-x+x^5(-x^4-x^3-x^2-x-1+x^3+x)=-x^9-x^7-x^5-x^3$ $-x$ *and* $\Phi(C)$ is *a cyclic subcode of* $\langle x^4+x^3+x^2+x+1, x^8+x^6+x^4+x^2+1\rangle$ *of length* 10 over F_3+uF_3.

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References

1. Aydin N, Karadeniz S and Yildiz B. Some new binary quasi-cyclic codes from codes over the ring $F_2+uF_2+vF_2+uvF_2$. *App. Algebra Eng. Commun. Comp*., 24, 5(2013), pp. 355-367.
2. Karadeniz S, Yildiz B. $(1+v)$ -constacyclic codes over $F_2+uF_2+vF_2+uvF_2$. *Joumal of the Franklin Institute*, 348, 9(2011), pp. 2625-2632.
3. Mostafanasab H, Karimi N. $(1-2u^2)$ -constacyclic codes codes over $F_p+uF_p+u^2F_p$. arXiv: 1506.07273vl「cs.IT」(2015).
4. Raka M, Kathuria L. $(1-2u^3)$ -constacyclic codes and quadratic residue codes over $F_p[u]/\langle u^4-u\rangle$. *Cryptography and Communications Discrete Structures, Boolean Functions and Sequences*, (2016), pp. 1-15.
5. Shi M J. Optimal p -ary Codes from Constacyclic Codes over a Nonchain Ring R. *Chinese Journal of Electronics*, 23, 4(2014), pp. 773-777.
6. Shi M J, Solé P and Wu B. Cyclic codes and the weight enumerator of linear codes over $F_2+vF_2+v^2F_2$. *Appl. Comput. Math*, 12, 2(2013), pp. 247-255.
7. Zhu S X, Wang Y and Shi M J, Some results on cyclic codes over F_2+vF_2. *IEEE Trans. Inform. Theory*, 56, 4(2010), pp. 1680-1684.