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Reachable set estimation for fuzzy cellular neural networks with bounded disturbances

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This paper is concerned with the problem of reachable set estimation for fuzzy cellular neural network with unit-peak bounded. Based on the Lyapunov-Krasovskii functional approach, delay -dependent conditions for estimating the reachable set of the considered system are derived. Moreover, an example is given to demonstrate the advantages of our method.

Keywords: FCNN; Bounded; Reachable Set; Time Delay.

1.Introduction

In the past decades, cellular neural network, as a special case of artificial neural networks, and their dynamical characteristics have been extensively investigated. There are numerous excellent results related to the subject have been generated due to its wide application in various fields such as image processing, solving partial differential equations, reducing non-visual problems to geometric maps. Consider the fact that uncertainty or vagueness is inevitable in many practical systems, fuzzy theory is devoted to deal with vagueness. Since fuzzy cellular neural network (FCNN) was proposed by [1]. Therefore, the research of fuzzy cellular neural network has attracted an increasing number of attention and research results have been present by many investigators.

The exponential stability problem of delayed fuzzy cellular neural networks with constant and time-varying delays was considered in [2]. With respect to white blood cell detection, [3] proposed A new detection algorithm based on fuzzy cellular neural networks. [4] dealt with the exponential stability problem of delayed fuzzy cellular neural networks with diffusion. [5] studied the global

asymptotic stability problem of fuzzy cellular neural networks with time delay in the leakage term under impulsive perturbations, some sufficient conditions were present to guarantee global asymptotic stability of the equilibrium point. [6] and [8] studied the global asymptotic stability problem of a class of bidirectional associative memory delayed fuzzy cellular neural networks. [7] investigated the global exponential synchronization problem of delayed fuzzy cellular neural networks.

On the other hand, the problem of reachable set estimation has received considerable attention. The objective of this subject is to construct a bounding ellipsoid that contains all the reachable states of the considered system under zero conditions. Moreover, there is no doubt that the study of time-delay systems in the field of research is vital part of it. Thus, a number of researchers have devoted their efforts to the issue of reachable set estimation for time-delay systems. As a consequence, many works have been devoted to the issue of reachable set estimation for time-delay systems and its related fields, see [9-17]. the reachable set estimation problem was concerned in [9] for discrete-time linear systems with multiple constant delays and bounded peak inputs. [10-11] concerned with the problem of reachable set estimation for a class of linear systems in the presence of both discrete and distributed delays. Furthermore, the problem of reachable set estimation for a class of delayed neural networks suffered by polytopic uncertainties was addressed in [12].

To the best of our knowledge, the reachable set estimation problem for FCNN with delays has not yet been fully investigated, which is still open. Based on the above discussion, our objective in this paper is to analyze FCNN and presents some sufficient conditions to solve this problem. Moreover, an example is given to demonstrate the advantages of our method.

2.Preliminaries

Let $C([-\tau,0],R)$ be the Banach space of continuous functions which map $[-\tau,0]$ into R^n with the topology of uniform convergence. Consider the following FCNN with delays

$$\begin{cases} \dot{x}_i(t) = -a_i x_i(t) + \bigwedge_{j=1}^{n} \alpha_{ij} f_j(x_j(t-\tau)) + \bigvee_{j=1}^{n} \beta_{ij} f_j(x_j(t-\tau)) \\ + \bigwedge_{j=1}^{n} T_{ij} u_j + \bigvee_{j=1}^{n} H_{ij} u_j, \qquad t \geq 0, \\ x_i(t) = \phi_i(t), \qquad -\tau < t < 0. \end{cases} \tag{1}$$

Where α_{ij}, β_{ij}, T_{ij} and H_{ij} are elements of fuzzy feedback MIN template, fuzzy feedback MAX template, fuzzy feedforward MIN template and fuzzy

feedforward MAX template, respectively. $\wedge$ and $\vee$ denote the fuzzy AND and fuzzy OR operation, respectively. x_i , u_i denote state and input of the i th neuron, respectively. $f_i(\cdot)$ is the activation function, $i, j = 1, 2, \cdots, n$. $\omega_i(t) \in R$ is called to be unit-peak bounded if it satisfies $|\omega_i(s)|^2 \le 1$ for all $t \ge 0$. The initial data $\phi_i(t)$ is a $C([-\tau, 0], R^n)-$ valued variable. Assume that system (2.1) has a unique global solution on $t \ge 0$, which is denoted by $x(t)$, where $x(t) = [x_1(t), x_2(t), \cdots, x_n(t)]^T$.

The reachable sets of system (1) with unit-peak bounded disturbance inputs are defined by $R_{up} = \left\{ x(t) \mid x(t), \omega(t) satisfy (2.1), |\omega(t)|^2 \le 1 \right\}$ and the ellipsoid to be determined is defined by $\varepsilon(P) = \{\xi \in R^n \mid \xi^T P \xi \le 1\}$, where $P > 0$ is a real constant matrix. As this is an estimation problem, we want the ellipsoid $\varepsilon(P)$ to be as small as possible. This purpose can be achieved by maximizing a positive scalar δ subject to

$$P \ge \delta I > 0 . \tag{2}$$

Assume that the nonlinear functions $f_j(u)$ satisfies the following condition.

(H1) There are positive constants L_i $(i = 1, 2, \cdots, n)$, such that $|f_i(u) - f_i(v)| \le L_i |u - v|$ for any $u, v \in R$.

(H2) For $i = 1, 2, \cdots, n$, the parameters of system (1) satisfy $-a_i + \sum_{j=1}^{n} (|\alpha_{ij}| + |\beta_{ij}|) L_j < 0$.

Lemma 1. Suppose $x = (x_1, x_2, \cdots, x_n)^T$ and $y = (y_1, y_2, \cdots, y_n)^T$ are two states of system (2.1), then we have

$$|\bigwedge_{j=1}^{n} \alpha_{ij} f_j(x_j) - \bigwedge_{j=1}^{n} \alpha_{ij} f_j(y_j)| \le \sum_{j=1}^{n} L_j |\alpha_{ij}| |x_j - y_j|$$

$$|\bigvee_{j=1}^{n} \alpha_{ij} f_j(x_j) - \bigvee_{j=1}^{n} \alpha_{ij} f_j(y_j)| \le \sum_{j=1}^{n} L_j |\beta_{ij}| |x_j - y_j|$$

Where L_j is shown as (H1).

Lemma 2. If (H1) and (H2) hold, and $\omega(t) = 0$. Then system (1) has a unique equilibrium point $x^* = (x_1^*, x_2^*, \cdots, x_n^*)^T$.

Thus, system (1) with $\omega(t)=0$ admits an equilibrium point $x^*=(x_1^*,x_2^*,\cdots,x_n^*)^T$. Let $y_i(t)=x_i(t)-x_i^*$, $\varphi_i(t)=\phi_i(t)-x_i^*$, then system (1) becomes

$$\begin{cases} \dot{y}_i(t)=-a_i y_i(t)+\bigwedge_{j=1}^{n}\alpha_{ij}[f_j(y_j(t-\tau)+x_j^*)-f_j(x_j^*)] \\ +\bigvee_{j=1}^{n}\beta_{ij}[f_j(y_j(t-\tau)+x_j^*)-f_j(x_j^*)]+\sum_{j=1}^{n}b_{ij}\omega_j(t), \qquad t\geq 0, \\ \qquad y_i(t)=\varphi_i(t), \qquad -\tau<t<0. \end{cases} \tag{3}$$

3.Main Results

Theorem 1. Consider system (1) with unit-peak bounded disturbance $\omega(t)$. Given a series of scalars $\tau>0$, $\sigma>0$, $p_i>0$ and $q_i>0$ for $i=1,2,\cdots,n$. For $P=diag\{p_1,p_2,\cdots,p_n\}$ and $Q=diag\{q_1,q_2,\cdots,q_n\}$, if there exist a scalar $\sigma>0$ such that

$$\Omega=\begin{pmatrix} -2PA+Q & PL & PB \\ * & -Q & O \\ * & * & -\sigma I \end{pmatrix}<0 \tag{4}$$

Then the ellipsoid $\varepsilon(P)$ contains the reachable set R_{up}.

Proof. Taking $V(y(t),t)=e^{-\sigma t}\sum_{i=1}^{n}p_i y_i(t)^2+e^{-\sigma t}\sum_{i=1}^{n}q_i\int_{t-\tau}^{t}y_i(s)^2ds$, we have

$$\dot{V}(y(t),t)=-\sigma e^{-\sigma t}\sum_{i=1}^{n}p_i y_i(t)^2+e^{-\sigma t}\sum_{i=1}^{n}p_i\dot{y}_i(t)y_i(t)$$

$$-\sigma e^{-\sigma t}\sum_{i=1}^{n}q_i\int_{t-\tau}^{t}y_i(s)^2ds+e^{-\sigma t}\sum_{i=1}^{n}q_i[y_i(t)^2-y_i(t-\tau)^2]$$

$$\leq -\sigma V(y(t),t)+2e^{-\sigma t}\sum_{i=1}^{n}p_i\,|\,y_i(t)\,|\left\{-a_i\,|\,y_i(t)\,|+\bigwedge_{j=1}^{n}|\,\alpha_{ij}\,||\,f_j(y_j(t-\tau)+x_j^*)\right.$$

$$-f_j(x_j^*)|+\bigvee_{j=1}^{n}|\beta_{ij}||f_j(y_j(t-\tau)+x_j^*)-f_j(x_j^*)|+\sum_{j=1}^{n}|b_{ij}\omega_j(t)|\Big\}$$

$$+e^{-\sigma t}\sum_{i=1}^{n}q_i[y_i(t)^2-y_i(t-\tau)^2]$$

$$\le -\sigma V(y(t),t)+2e^{-\sigma t}\sum_{i=1}^{n}p_i|y_i(t)|\Big\{-a_i|y_i(t)|+\sum_{j=1}^{n}L_j|\alpha_{ij}||y_j(t-\tau)|$$

$$\sum_{j=1}^{n}L_j|\beta_{ij}||y_j(t-\tau)|+\sum_{j=1}^{n}|b_{ij}\omega_j(t)|\Big\}+e^{-\sigma t}\sum_{i=1}^{n}q_i[y_i(t)^2-y_i(t-\tau)^2]$$

$$\le -\sigma V(y(t),t)+2e^{-\sigma t}\sum_{i=1}^{n}p_i|y_i(t)|\Big\{-a_i|y_i(t)|+\sum_{j=1}^{n}L_j|\alpha_{ij}||y_j(t-\tau)|$$

$$\sum_{j=1}^{n}L_j|\beta_{ij}||y_j(t-\tau)|+\sum_{j=1}^{n}|b_{ij}\omega_j(t)|\Big\}+e^{-\sigma t}\sum_{i=1}^{n}q_i[y_i(t)^2-y_i(t-\tau)^2]$$

$$+(\sigma-\sigma e^{-\sigma t})\omega(t)^2$$

Then, we can get

$$\dot{V}(y(t),t)+\sigma V(y(t),t)-\sigma\omega(t)^2\le Z\Omega Z^T\le 0$$

in which

$$Z=\left(|y_i(t)|\quad |y_i(t-\tau)|\quad |\omega_i(t)|\right).$$

Then, by utilizing Lemma 4 in [16], we have $V(y(t),t)\le 1$ under zero initial conditions. This further implies that $y(t)Py(t)^T\le 1$ since $V(y(t),t)\ge y(t)Py(t)^T$. Therefore, the reachable set R_{up} is contained in $\varepsilon(P)$. □

Remark. Theorem 1 gives a sufficient condition for the estimation of the reachable set R_{up}. We can obtain an ellipsoid $\varepsilon(P)$ with the shortest major principal axis by maximizing the positive scalar δ with respect to the conditions in Eqs. (2) and (4). It should be pointed out that the inequality $\Omega<0$ becomes an LMI when the scalar σ is fixed. As shown in [14-15], we can find a local optimum value of σ such that the scalar σ with respect to condition (2) and (4) is maximized.

4.Numerical examples

In this section, we provide an example to show the effectiveness of the result in Theorem 1 on the reachable set estimation problem for system (1) with unit-peak bounded disturbance inputs. To this end, we consider system (1) with

parameters given by $\omega_j = \dfrac{1}{1+n}$, $f_1(x_1) = \dfrac{1}{2}(|x_1+1|+|x_1-1|)$, $f_2(x_2) = x_2$, $i, j = 1, 2$. Obviously, $L_1 = L_2 = 1$, $\tau = 1.2$. Taking $a_1 = 0.75$, $a_2 = 0.9$, $\alpha_{11} = 0.25$, $\alpha_{12} = 0.15$, $\alpha_{21} = 0.25$, $\alpha_{22} = 0.35$, $\beta_{11} = 0.2$, $\beta_{12} = 0.25$, $\beta_{21} = 0.15$, $\beta_{22} = 0.25$, $T_{11} = 0.2$, $T_{12} = 0.15$, $T_{21} = 0.15$, $T_{22} = 0.25$, $H_{11} = 0.15$, $H_{12} = 0.1$, $H_{21} = 0.2$, $u_1(t) = \cos t$, $u_2(t) = \sin t$. By simple calculation, we can get that there exists $\sigma = 1$ such that $\delta_{\max} = 2.4335$ and $P = \begin{bmatrix} 2.4336 & 0 \\ 0 & 2.4336 \end{bmatrix}$. The simulations of the obtained bounding ellipsoid is shown in Fig. 1.

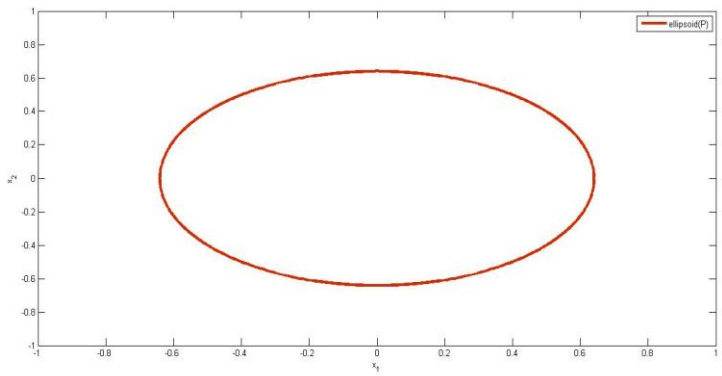

Fig. 1 Bounding ellipsoid for system (2.1)

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