

## Study about the design of merge after toll

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**Abstract.** The barrier toll is a crucial part of a freeway. The bottleneck of a freeway about safety and throughput is usually the fan-in area. In this paper, we put forward the merging model of barrier tolls (MBT) based on cellular automaton considering various factors which influence the merging vehicles. we analyze the influencing factors of going straightly and changing lanes in fan-in area to determine the criteria of going straightly(update the site, vary speed, record track), the criteria of changing lanes(changing-lane condition, update the site, keep speed, record track), and release criteria(release randomly, interval time).The model adopts dynamic simulative emulation and transfers continuous problem into discrete system. When the number of lanes and throughput are given, our model can obtain a optimal solution of the fan-in area.

### Introduction

Toll plazas play an important role in the freeway traffic. A lot of scholars pay attention to solve the problem of toll plaza. Caldas, Marco Antonio de Farah and Sacramento, Karina Tarina Thiebaut proposes a mesoscopic simulation to analyze service levels of different physical and operational arrangements of a particular toll plazas<sup>[1]</sup>. The model considers speed functions, toll gate arrivals and departures, as well as events as possible accidents and track change functions when the vehicles arrive at the gate, and offers data for the HCM indexes, which produces the respective service levels. In this paper, we approach a model about the fan-in area in a creative way. The idea about the safety comes to us after analyzing the article which called Effects of Open Road Tolling on Safety Performance of Freeway Mainline Toll Plaza.<sup>[2]</sup>Min HAN, Jinliang LIU also point out a fact that the problem of toll plazas is the most challenging issue for freeway management.<sup>[3]</sup>We can draw the conclusion that a suitable proportion of conventional tollbooths, exact-change tollbooths and electronic toll collection booths can solve the congestion near the toll plaza in some degree. So in this paper, we will also try to find the influence of this proportion.

Typically, a barrier toll has several advantages, such as the control of traffic volume and the quality of traffic flow. However, the fan-in area after the barrier toll can be a bottleneck of the freeway. As we all know, a much larger number of the barrier tolls is provided than the number of travel lanes leaving the toll plaza. So, the fan-in area is a crucial factor that influences the throughput. It will be significant that if we can make a optimum proposal about the design of the fan-in area. In this paper, we make a deeply research on the fan-in area. Taking accident prevention, throughput and cost into account, we aim to decide the optimal shape, size and merging pattern of the fan-in area.

### Assumptions

To simplify the problem, we make some assumptions.

1. *The time that Highway tollbooth operator serves motorists is within a certain range. We also regard this time as a stochastic figure.* Due to a man is subjective, Highway tollbooth operator can not do the same action in the same time and the speed level of payment action is different with each motorist.

2. All motorists tend to change lane as soon as possible while meeting conditions (If motorists need to change lane). Nearly every motorist wants drive in the middle of the freeway.

3. The speed of leaving the toll barrier is the equivalent of each. when motorists pay for tolls, they pull up. After exiting the toll barriers, motorists start the vehicles. At this moment, the speed is negligible compared with that running on road.

4. Most vehicles accelerate with the same acceleration in the fan-in area. According to physics, we can obtain this assumption easily.

5. Vehicles chooses to decelerate or pull up when the distance between itself and the former vehicles is too short or the change lane condition is not satisfied when vehicles will reach the mandatory lane changing point at present speed.

## MBT Model

### Influencing Factors.

*Going Straightly:* In order to keep the safety of vehicle, we have to take the gap between the former vehicle and the latter vehicle, the speed of the former vehicle and the latter vehicle and the vehicles that are changing lane into consideration. According to common sense, motorists decelerate when the distance between themselves and the former vehicle or vehicle preparing to change lane is short. Thus, We can guarantee safety.

*Changing lanes:* For vehicles which need change lines, they enter formal lanes step by step. motorists not only need to consider the speed, but also the gap between itself and the latter vehicle of the neighbor lane. If motorists decelerate, vehicles face the possibility of collision with the latter vehicle of the neighboring lane<sup>[4]</sup>. If motorists accelerate, vehicles face the possibility of collision with the former vehicle.

In order to study the characteristics of lane changing, we consult and analyze a large amount of data. We find a relationship between mandatory lane changing point and the number of vehicles which is mandatory to change lane. The possibility of changing lanes is increase with the decrease of the distant of mandatory lane changing point<sup>[5]</sup>.

### Notations.

Table 1. Symbol

| Symbol             | Definition  | Units    |
|--------------------|---|----------|
| $x(i, j)$          | The site of the vehicle is i row and j columns                    | unitless |
| $v(i, j)$          | The speed of the vehicle in i row and j columns                   | cell/s   |
| Road-length<br>$h$ | The length of the road in CA model                                | cell     |
| $mp$               | The distance of mandatory lane changing point in CA model         | cell     |
| $x(i, j)'$         | The site of the former vehicle                                    | unitless |
| $p$                | The possibility of acceleration                                   | unitless |
| $N$                | the number of blue cells appearing at the exit                    | number   |
| $t$                | Equal to 2000   | second   |
| $M$                | the number of mandatory lane changing vehicles day and night      | number   |
| $y$                | throughput  | number/s |
| $I$                | the number of mandatory lane changing vehicles in unit throughput | unitless |
| $A$                | The track of vehicles   | unitless |

### The MBT Model Based on Cellular Automaton.

Cellular Automaton(CA) modeling is extended to study the heterogeneous traffic observed in developing countries. In heterogeneous traffic, the physical and mechanical characteristics of different vehicles vary widely which in turn leads to complex traffic. This nature of the heterogeneous traffic is modeled with the help of CA model<sup>[6]</sup>. Cellular automaton is mathematical model in which time, space and other variables are all discrete.

In fan-in area, because of the influence of vehicles in neighboring lane and the shape of the lane, the rules of going straightly and changing lanes vary according to different condition. We can refer to the NaSch Model<sup>[7]</sup>. Cells, like the squares of a checkerboard in the model are divided into three modes containing idle, occupied and impenetrable. Idle mode represent there is no vehicle. Occupied mode represent there is a vehicle. Impenetrable mode represent that vehicles can not get through.

For Model Based on Cellular Automaton, we make some explanations:

1. Each cell is a 3.65m\*6m area in reality. According to the width of lane and the length of vehicle in reality, we can surely make it.
2. We operate procedure 2000 times. We can obtain a fairly correct answer.
3. Vehicles are in the middle of each lane.
4. We suppose that the performance of changing lane occur in a moment.
5. We suppose the speeds of the vehicles are positive integer.
6. Unit time is one second.

The criteria of going straightly:

- 1) Update the site of the former vehicle.

$$x(i, j) = 0, \begin{cases} x(i, j+v(i, j)) = 1 & j+v(i, j) < road\_length \\ x(i, mp-1) = 1 & mp \leq x(i, j)+v(i, j) \leq road\_length \\ disappear & x(i, j)+v(i, j) > road\_length \end{cases} \quad (1)$$

- 2) Update the site of the latter vehicle.

$$x(i, j) = 0, \begin{cases} x(i, j+v(i, j)) = 1 & j+v(i, j) < x(i, j)' \\ x(i, j-1) = 1 & j+v(i, j) \geq x(i, j)' \end{cases} \quad (2)$$

- 3) Vary the speed of the former vehicle.

$$v(i, j) = \begin{cases} v(i, j)+1 & p \\ v(i, j) & 1-p \end{cases} \quad (3)$$

- 4) Vary the speed of the latter vehicle.

$$v(i, j) = \begin{cases} v(i, j)' & x(i, j) = x(i, j) - 1 \\ v(i, j)+1 & p \\ v(i, j) & 1-p \end{cases} \quad (4)$$

- 5) Record the track of vehicle. For example:

If a vehicle go straightly, the site of it is  $xt(i, j1)$  at  $t$  moment and the site of it is  $xt+1(i, j2)$  at  $t+1$  moment. So the track of the vehicle is from  $j1$  to  $j2$ . We define a matrix  $A$  to record the track. As shown in the following Figure.

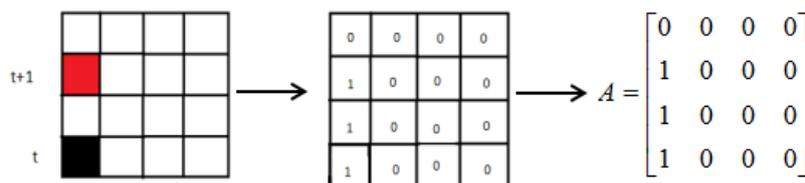


Fig1. The track record of vehicles going straightly

The criteria of changing lanes

- 1) Judge the condition of changing lanes. We can derive the condition of changing-lane (the range of gap between themselves and the latter vehicle in neighboring lane is between 0 and 30 meters and the range of relative speed when vehicles are changing lane is between -6m/s and 6m/s) from section 4. 1.
- 2) update the site matrix. If  $A(i+1, j)=0 \& A(i+1, j+1)=0$ , then  $x(i+1, j+1)=1$ .
- 3) Keep speed. Vehicles have the same speed as before changing lanes.
- 4) Record the track of vehicle. For example

If a vehicle is changing lane, the site of it is  $x^t(i, j1)$  while the speed of it is  $v^t(i, j1)$  at  $t$  moment and the site of it is  $x^{t+1}(i, j2)$  while the speed of it is  $v^{t+1}(i, j2)$  at  $t+1$  moment. So, We define a matrix  $A$  to record the track in one second. As shown in the following Figure.

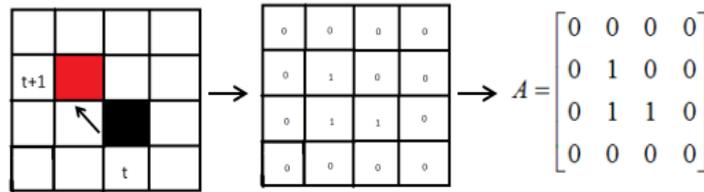


Fig2.The track record of vehicles changing lanes

*Determine the optimal number of barrier tolls*

When the number of lanes is determined, there will be unnecessary and wasteful if the barrier toll is more and there will be congested and illogical if the barrier toll is less. Thus, we search for a rational number of barrier toll which can ensure the throughput and decrease the cost.

According to the throughput, we simulate the most crowded condition by design the throughput about 90000 per day and night. Take no account of the shape of fan-in area and the mandatory lane changing point, we can obtain Figure3(14 barrier tolls and 4 lanes)and Table 2 below.

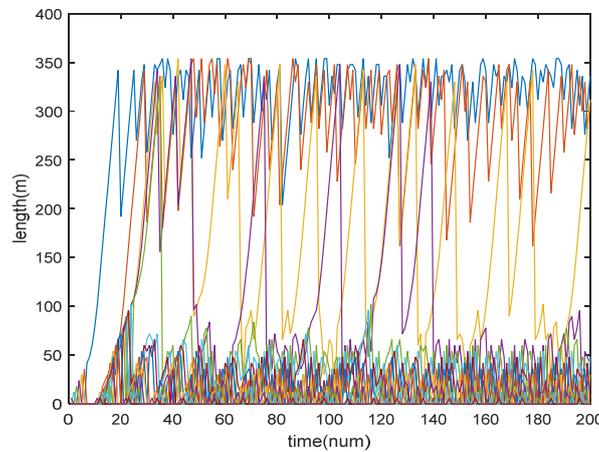


Fig3.Barrier tolls-lanes diagram

Figure3 demonstrates that 14 barrier tolls match with 4 lanes at least. Each color represents one lane. The value of Y-axis represents the distance between the exiting of toll barrier and the forehead vehicles. That is to say, the high value of Y-axis is the formal freeway. In this way, we can get a congruent relationship between barrier tolls and lanes. The congruent relationship is showed in Table.

Table2.barrier tolls-lanes

|                         |   |   |    |    |    |    |    |    |    |
|-------------------------|---|---|----|----|----|----|----|----|----|
| Number of barrier tolls | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| Number of lanes         | 3 | 3 | 4  | 4  | 4  | 4  | 4  | 4  | 5  |

*Calculate the Throughput*

In 2000-time-step, we can count the number of cells appearing at the exit. We suppose this number is N. Thus, the throughput y can be calculated:

$$y = \frac{N}{t} \cdot 100\% \tag{5}$$

t=2000 in the formula(5).

We can obtain Table according to (5).

Table3.Throughput-barrier tolls

|                             |       |       |        |        |        |        |
|-----------------------------|-------|-------|--------|--------|--------|--------|
| The number of barrier tolls | 10    | 11    | 12     | 13     | 14     | 15     |
| Throughput(n/day)           | 83937 | 92275 | 100396 | 109166 | 117201 | 126014 |

*Index of danger*

We can regard the number of mandatory lane changing vehicles in unit time as the index of danger. In order to compare the danger index of different solution under same criteria, we come up with the

conception of the proportion of the number of mandatory lane changing vehicles and the amount of vehicles in a day and night.

$$I = \frac{M}{N} \cdot 100\% \quad (6)$$

M:the number of mandatory lane changing vehicles day and night.

N:the number of blue cells appearing at the exit

Table4.Index of danger-barrier tolls

| The number of barrier tolls    | 10     | 11     | 12     | 13     | 14     | 15     |
|--------------------------------|--------|--------|--------|--------|--------|--------|
| Index of danger in fan-in area | 1. 51% | 1. 52% | 1. 54% | 2. 09% | 2. 61% | 2. 77% |

## Conclusion

If we want to design a merge after toll,we can get a good result by analyzing the behavior of the vehicles in practice.The model adopts dynamic simulative emulation and transfers continuous problem into discrete system. When the number of lanes and throughput are given, our model can obtain a optimal solution of the fan-in area.it finally proposes two suitable relationship between the number of lanes and barrier toll.

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## References

- [1] Caldas, Marco Antonio de Farah & Sacramento, Karina Thiebaut Simulation model of discret events applied to the planning and operation of a toll plaza. *Journal of Transport Literature*, 2016, 40-44.
- [2] Yang, H&Ozbay, K&Bartin, B Effects of Open Road Tolling on Safety Performance of Freeway Mainline Toll Plaza. *TRANSPORTATION RESEARCH RECORD*.
- [3] Min HAN, Jinliang LIU. The congestion condition and solution of Tian Jin. *Urban Roads Bridges & Flood Control*, 2013(9):169-171.
- [4] Zhizhao Zhang. Research on Traffic Conflict of Freeway Diverging And Merging Sections And Toll Booth[D]. *HARBIN INSTITUTE OF TECHNOLOGY*, 2014.
- [5] Baili Wang, Xiaofang Yang, Hongxing Liu Lane-changing Behavior Simulation Research in Traffic Merging Area. *COMPUTER SIMULATION*, 1006-9348(2016)02-0211-05.
- [6] Mallikarjuna, C&Rao, KR Cellular Automata Model for Heterogeneous Traffic. *JOURNAL OF ADVANCED TRANSPORTATION*, 2009:321-345.
- [7] K Nagel, M Schreckenberg. A cellular automaton model for freeway traffic[J]. *Journal de physique I*, 1992, 2 ( 12 ) : 2221 – 2229.